ENGINEERING TRIPOS PART IB

Tuesday 2 June 2009 9 to 11

Paper 3

MATERIALS

Answer not more than ***four*** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments to this paper.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Single-sided graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 Annealed pure copper is cold-extruded between two flat converging dies which subtend an angle 2α as shown in Fig. 1. The dies of length ℓ are rigid with inlet and exit heights $2h_1$ and $2h_2$, respectively. The copper can be assumed to only deform in the (x, y) plane as shown in Fig. 1. An extrusion force F is required to push the strip through the dies, against the combined resistance of the normal pressure $p(x)$ and the frictional shear stress $\tau(x) = \mu\, p(x)$, where the co-efficient of friction μ is a constant and x is the horizontal co-ordinate measured from the mouth of the die (Fig. 1). The copper may be assumed to be a rigid ideally-plastic solid of yield strength Y, and obeying the simplified Tresca yield criterion $\sigma_{xx} + p = Y$, where σ_{xx} is the normal stress component in the x-direction as shown in Fig. 1.

(a) Draw a free-body diagram to show the forces acting on a typical vertical strip of the copper of thickness δx, height $2h(x)$ and unit depth at a typical horizontal co-ordinate x. [5]

(b) By considering the horizontal equilibrium of forces on the element sketched in part (a), show that

$$p \tan\alpha + \tau = \frac{d}{dx}(h\sigma_{xx})$$

You may assume that the stress components σ_{xx} and σ_{yy} vary only with the co-ordinate x and are independent of the co-ordinate y. [6]

(c) Calculate the distribution of the stress σ_{xx} within the deforming strip by integrating the equilibrium relation for the strip. In order to simplify the integration you may assume that the strip semi-thickness is constant and of magnitude $\bar{h} = (h_1 + h_2)/2$. [6]

(d) Obtain an expression for the extrusion force F per unit depth. [3]

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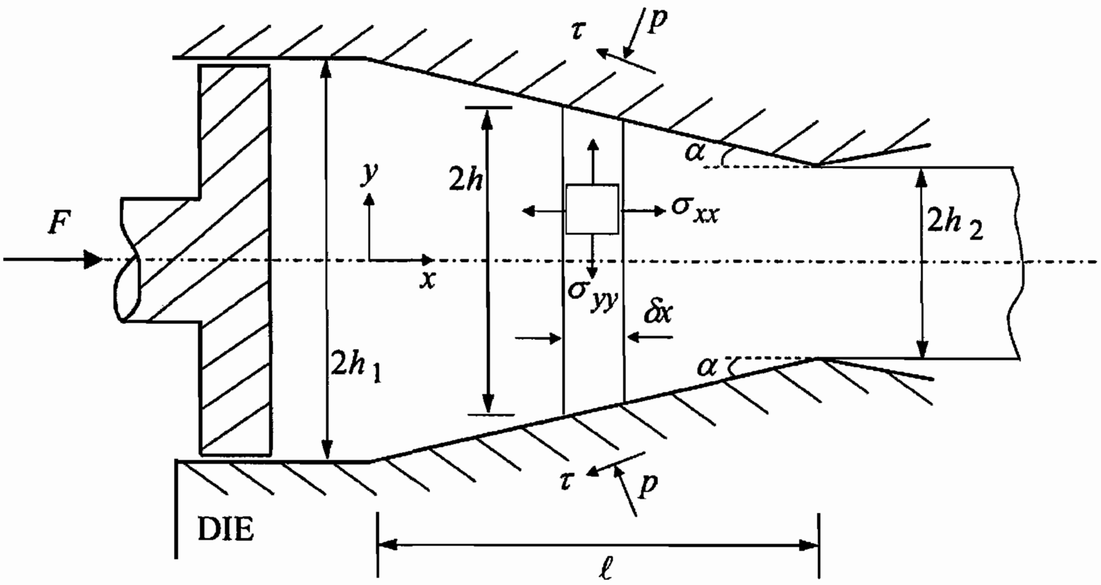

Fig. 1

2 The phase diagram for the silver (Ag) – copper (Cu) system is shown in Fig. 2. The numbers in parenthesis on the diagram refer to weight percent copper which are given on the upper scale.

(a) On a rough sketch of the phase diagram indicate the phases that are present in each region of the diagram. [4]

(b) An alloy containing 40 wt% Ag and 60 wt% Cu is heated to 1100 °C and then slowly cooled to room temperature. Make a labelled sketch to show the resulting microstructure, and give the amount of each phase present. [6]

(c) Explain how segregation can occur during the non-equilibrium cooling of the alloy described in (b) from the liquid state. [5]

(d) Describe, using the phase diagram, how you would choose an alloy composition and a heat treatment schedule suitable for producing an age-hardened silver-rich alloy. [5]

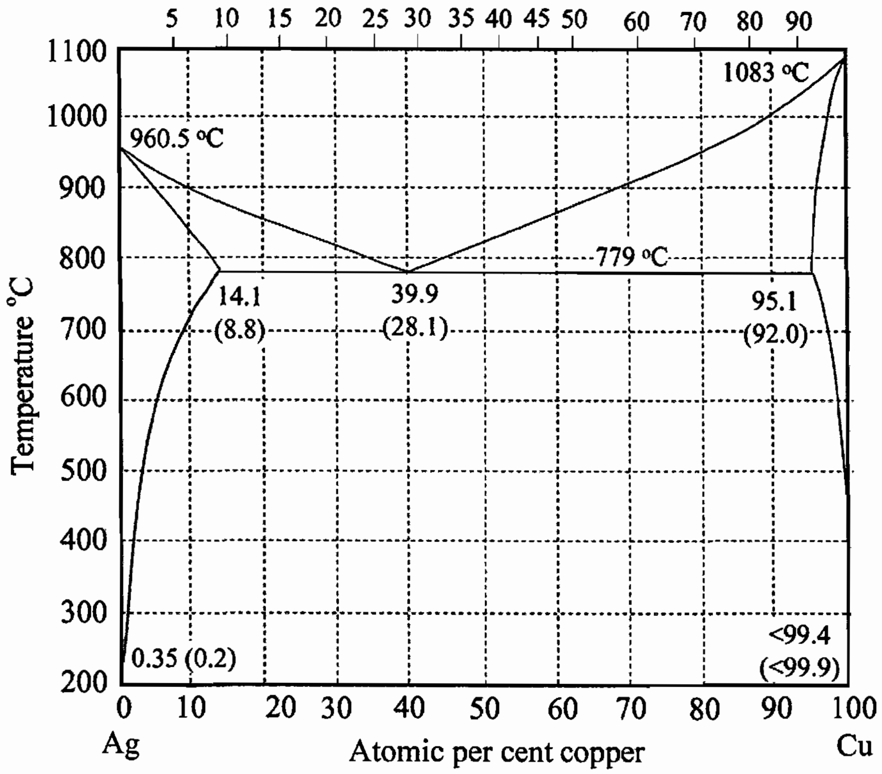

Fig. 2

3 (a) Define the glass transition temperature T_g for polymers. What controls the value of T_g? [3]

(b) The shear stress τ versus shear strain γ response of a rubber used in an anti-vibration mounting can be modelled by a combination of two linear springs and a linear dashpot, as shown in Fig. 3. The spring constants are G_1 and G_2 as shown in the figure. The shear stress is related to the shear strain rate $\dot{\gamma}_i$ for the dashpot via $\tau_i = \eta\dot{\gamma}_i$ where the subscript i is used to denote the stress and strain rate in the dashpot.

(i) Explain the physical basis for the inclusion of the dashpot. [2]

(ii) Determine the governing differential equation between the overall shear stress τ and overall shear strain γ in terms of time t. [5]

(iii) A constant overall stress τ_0 is applied at time $t = 0$ to the initially unloaded rubber. Calculate the overall strain γ immediately after the application of this stress (i.e. at time $t = 0^+$). [3]

(iv) Determine an expression for the time variation of the overall shear strain after application of the stress τ_0 and draw a rough sketch of this variation. [7]

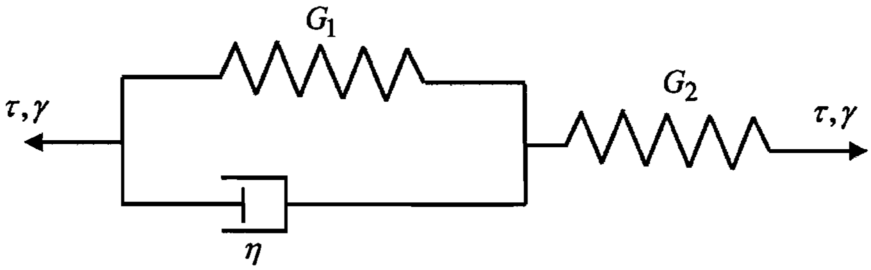

Fig. 3

SECTION B

4 Account for the following by writing explanatory notes along with sketches where appropriate.

(a) Titanium powder, when added to a steel melt, promotes a fine grain size. [4]

(b) When a 1.5 wt% Ni-Cr-Mo steel is furnace cooled from 800 °C to 300 °C over a period of 1 hour, the resulting microstructure is strong and reasonably tough. In contrast, a water quench from 800 °C to 20 °C produces a microstructure that is very hard and brittle. [5]

(c) The tread of car tyres is made of high-loss rubber, whereas the walls of the tyres are made from low-loss rubber. [2]

(d) A PET soft drinks bottle shrinks in size and whitens when it is heated to 150 °C for 1 hour and then cooled slowly. [4]

(e) The grain structure of aluminium can be controlled by suitable plastic deformation and heat treatment. Giving reasons, explain how you would produce aluminium with a grain size that is (i) very small and (ii) very large. [5]

5 (a) Giving reasons, describe briefly the type of components that are case-carburised. [4]

(b) A cylinder of low alloy steel containing 0.2 wt% carbon is to be case-carburised to provide a hardness of 7 GPa to a depth of 0.2 mm after quenching from the carburising temperature of 950 °C. The variation of hardness with carbon content for quenched steel is shown in Fig. 4.

(i) Determine the carbon content range that will provide the required hardness up to the depth of 0.2 mm. [3]

(ii) The diffusion of carbon into steel is described by the time-dependent diffusion equation

$$C(x,t) = (C_s - C_0)\left[1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right] + C_0$$

where C is the concentration of carbon at a distance x below the surface after a time t, while C_0 and C_s are the initial and surface concentrations, respectively, of carbon in the cylinder. The diffusion co-efficient D at 950 °C is $4 \times 10^{-11}\ \mathrm{m^2 s^{-1}}$. Determine a suitable value for C_s and the time for the carburising process. [6]

(iii) Plot the resulting variation of hardness in the cylinder to a depth of 0.35 mm below the surface and hence determine the position of maximum hardness. [7]

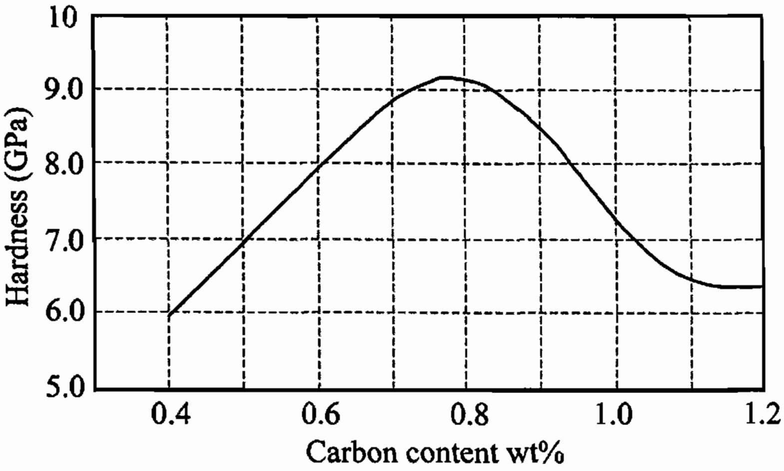

Fig. 4

(TURN OVER

6 The time-temperature-transformation (TTT) diagram of a plain carbon eutectoid steel has the following features: (i) at 650 °C transformation of the austenite starts after 10 s and the transformation is complete within 100 s; (ii) the fastest rate of transformation occurs at 550 °C where the transformation starts after 1 s and is complete within 10 s; (iii) at 360 °C transformation of the austenite starts after 10 s and is complete within 300 s; and (iv) the martensite start temperature is 210 °C and 90% of the austenite has transformed to martensite at 120 °C.

(a) Make a rough sketch of the TTT diagram of this eutectoid steel and explain briefly the shape of the lines drawn on the TTT diagram. [6]

(b) This steel is hot rolled above 850 °C and cooled slowly in air. Superimpose a suitable cooling curve on the TTT diagram drawn in part (a) and describe the final microstructure of the steel. [3]

(c) In service a shaft made from this steel is heated to 800 °C. With the help of the TTT diagram sketched in part (a), describe briefly, with the aid of labelled sketches the final microstructures of the steel on the surface and 6 mm below the surface of the shaft. The cooling rates in these two locations are as follows.

(i) The surface of the shaft cools to 400 °C in less than 1 s and to ambient temperature in under 10 s. [4]

(ii) An element of material 6 mm below the surface takes 1 s to cool to 500 °C, 100 s to 350 °C and 1000 s to ambient temperature. [4]

(d) Cracks frequently form on the surface of the shaft described in part (c) when it is subjected to thermal cycling. However, these cracks usually do not penetrate more than 3 to 4 mm below the surface. Discuss briefly the possible reasons for this observation. [3]

END OF PAPER

Numerical answers to Paper 3 (2009)

1. (c) $\sigma_{xx} = Y\left[1 - \exp\left(\frac{\ell - x}{L}\right)\right]$, where $L = \frac{\bar{h}}{\mu + \tan\alpha}$

(d) $F = 2\bar{h}Y\left[\exp\left(\frac{(\mu + \tan\alpha)\ell}{\bar{h}}\right) - 1\right]$

2. (b) Fraction of eutectic mix = 0.5
Weight fraction of α in eutectic mixture = 0.768
Weight fraction of β in eutectic mixture = 0.232

3. (b)(ii) $\frac{\dot{\tau}\eta}{G_2} + \left(\frac{G_1 + G_2}{G_2}\right)\tau = \eta\dot{\gamma} + G_1\gamma$

(b)(iii) $\gamma = \frac{\tau_0}{G_2}$

(b)(iv) $\gamma = \frac{\tau_0}{G_e} - \left[\frac{\tau_0}{G_e} - \frac{\tau_0}{G_2}\right]\exp\left(-\frac{G_1 t}{\eta}\right)$,

where $\frac{1}{G_e} = \frac{1}{G_1} + \frac{1}{G_2}$

5. (b)(i) Requires carbon in the range 0.5 to 1.02 wt%

(b)(ii) time = 10.18mins.

(b)(iii) peak at $x = 94$ μm