ENGINEERING TRIPOS PART IB

Tuesday 2 June 2009 2 to 4

Paper 4

THERMOFLUID MECHANICS

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

*Answer not more than **two** questions from this section.*

1 Figure 1 shows a co-flow heat exchanger. The temperatures of the hot flow at inlet and exit are T_{h1} and T_{h2} ; the temperatures of the cold flow at inlet and exit are T_{c1} and T_{c2}; A is the area available for heat exchange and U the overall heat transfer coefficient.

(a) Assume that the overall heat transfer is $UA\Delta T_m$. By considering appropriate heat balances show that the Log Mean Temperature Difference ΔT_m is

$$\Delta T_m = \frac{\left[(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})\right]}{\log_e\left[(T_{h2} - T_{c2})/(T_{h1} - T_{c1})\right]}$$ [15]

(b) Explain why this holds also for counter-flow and sketch the temperature vs. area plot for such a device. [2]

(c) Why might counter-flow be a preferred design? [3]

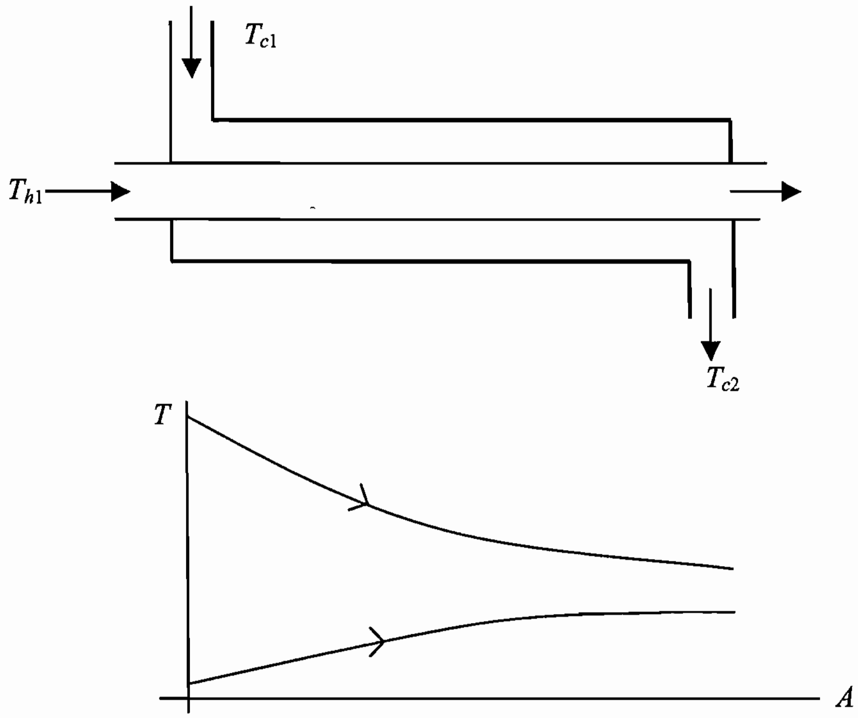

Fig. 1

2 (a) Consider a cyclic heat power plant drawing heat from a reservoir at constant temperature, T_1, and discharging heat to another reservoir at constant temperature, T_0. Write down an equation for the flow of exergy associated with the heat transfer and define the *rational efficiency* of the power plant. Show that the *cycle efficiency* may be simply computed as the product of the *rational* and *Carnot* efficiencies. [5]

(b) An ideal, superheated Rankine cycle has a temperature and pressure of 540 °C and 18 bar at turbine inlet. Sketch the Rankine and Carnot cycles on a *T-s* diagram. The turbine exhausts into the condenser at a pressure of 0.05 bar. Using steam tables and the chart compute both the cycle and rational efficiencies. You may neglect the feed pump work. What would be the efficiency of the associated Carnot cycle? Assume that the maximum temperature of the cycle is equal to 540 °C and the lowest temperature is equal to the condenser temperature. [10]

(c) Write down an equation for the exergy destruction associated with irreversible entropy production. Compute the value of this entropy production for the Rankine cycle in part (b) compared to its associated Carnot cycle and comment on the result. [5]

3 (a) Sketch a *T-s* diagram of the ideal Joule cycle. Show that, for a perfect gas, the cycle efficiency is given by

$$\eta_{CY} = 1 - r_p^{-(\gamma-1)/\gamma}$$

where r_p is the cycle pressure ratio. [5]

(b) Sketch the variation of η_{CY} with r_p. What sets a practical upper limit to the efficiencies that can be achieved? [5]

(c) Describe, with appropriate sketches, the cooling arrangements adopted commonly for gas turbine blades. [5]

(d) List five ways in which the real-world Joule cycle differs from the ideal. [5]

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SECTION B

*Answer not more than **two** questions from this section.*

4 Figure 2 shows a vertical plate moving steadily upwards at speed U drawing a constant flow rate of lubricant viscous fluid from a large tank past a fixed wall at a fixed distance d. The plate is of infinite depth into the paper. The inlet and outlet regions are exposed to atmospheric pressure. The fluid has density ρ and viscosity μ. Assume that the flow is steady, the streamlines are straight, and that, within the lubricant film, momentum terms are negligible.

(a) Justify that the vertical velocity is only a function of x, and that the pressure p is constant everywhere in the film. [5]

(b) Show that the governing equation for the vertical velocity is:

$$\mu \frac{d^2 V}{dx^2} = \rho g \qquad [5]$$

(c) Apply the given boundary conditions to find the velocity profile as a function of the given parameters. [5]

(d) Obtain the value of the flow rate Q per unit depth as a function of the given parameters, and an expression for U which results in no net flow rate out of the tank. [5]

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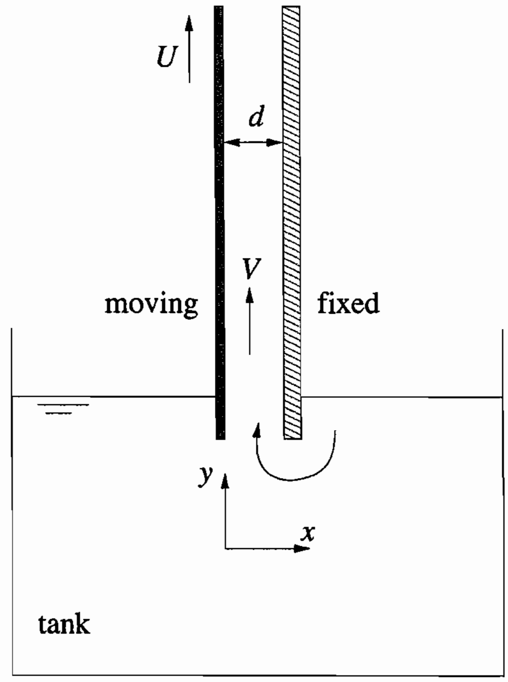

Fig. 2

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5 (a) (i) In a steady, two-dimensional flow of density ρ and velocity V the 'normal' component of the Euler equations is

$$\frac{\partial p}{\partial n} = \frac{\rho V^2}{R},$$

where p is the pressure, R the streamline radius of curvature and n the coordinate perpendicular to the streamline. What are the physical interpretations of the two terms?

(ii) If the flow is also inviscid, and uniform upstream, what can you say about its stagnation pressure? [4]

(b) Figure 3 shows steady, incompressible, two-dimensional flow through an (effectively infinite) array of circular arc vanes. The upstream flow is uniform, with velocity V_∞. Each vane has radius R and included angle 2ϕ. The vane spacing is h.

(i) What is the total volumetric flow rate between two vanes?

(ii) It is assumed that the flow streamlines have the same shape as the vanes (see dotted lines). The velocity of the fluid at coordinate n at the centre-line is $V_C(n)$. What are the inlet and outlet velocities for the streamtube that crosses the centre-line at n ?

(iii) If the flow is inviscid, how does the centre-line velocity $V_C(n)$ vary between the vanes? Express your answer in terms of the mid-point velocity, $V_C(0)$.

(iv) Derive an approximate expression for $V_C(n)$ that is valid for small h/R and, for this case, find the link between $V_C(0)$ and V_∞.

(v) What are the force components on each vane? Assume that your small h/R results for part (iv) are applicable. [14]

(c) In reality, viscous boundary layers will develop on the vane surfaces. Would you expect these boundary layers to be liable to separate upstream or downstream, of the centre-line? Briefly explain your reasoning. [2]

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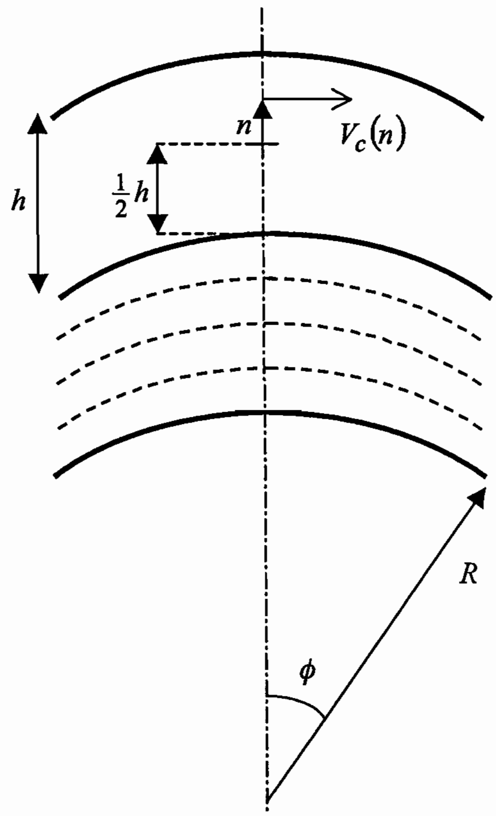

Fig. 3

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6 Figure 4 shows a design for a water fountain of height 10 m. The reservoir, pump and supply pipe are all at the same level.

(a) Calculate the gauge stagnation pressure that is required at point C, immediately upstream of the nozzle. You may neglect the effects of viscosity on the nozzle flow. [4]

(b) The flow in the pipe may be assumed steady and fully developed. It is subject to a frictional shear stress at the pipe walls. Show, *from first principles,* that the stagnation pressure drop is given by

$$P_{0B} - P_{0C} = \frac{32}{\pi^2}\rho Q^2 c_f \frac{l}{d^5},$$

where ρ is the water density, Q the volumetric flow rate, and c_f the wall skin friction coefficient. [4]

(c) The pump flow characteristic curve at maximum speed is given in Table 1. Neglecting any stagnation pressure losses in the pipe feeding the pump and in the bend at point C, estimate the volumetric flow rate of the fountain for $d = 0.1$ m, $l = 5$ m and $c_f = 0.005$. A plot of the data in Table 1 is shown in Fig. 5. An accuracy of 5% in the result is acceptable. [6]

Q ($m^3 s^{-1}$)	0.02	0.0225	0.025	0.0275	0.03	0.0325	0.035	0.0375
P_{0B}-P_{0A} (kPa)	166	158	149	139	127	114	98	80

Table 1

(d) The pump stagnation pressure rise is found also to depend on its size, D, rotation speed, N, and the water density. Deduce the functional form of the dimensionless curve underlying the data for the pump characteristic. [2]

(e) It is proposed to save power and water by reducing the pump speed from its maximum, 3500 rpm, to 3000 rpm. The fountain height is to remain unchanged. Estimate the new volumetric flow rate. [4]

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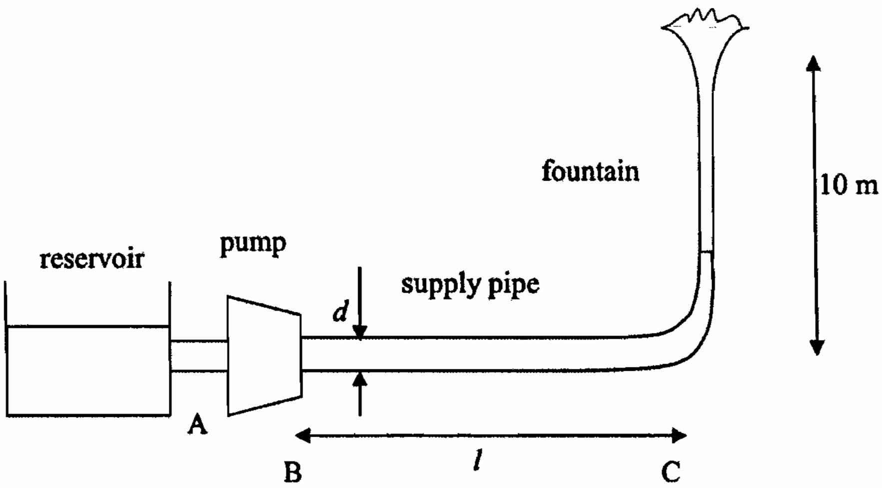

Fig. 4

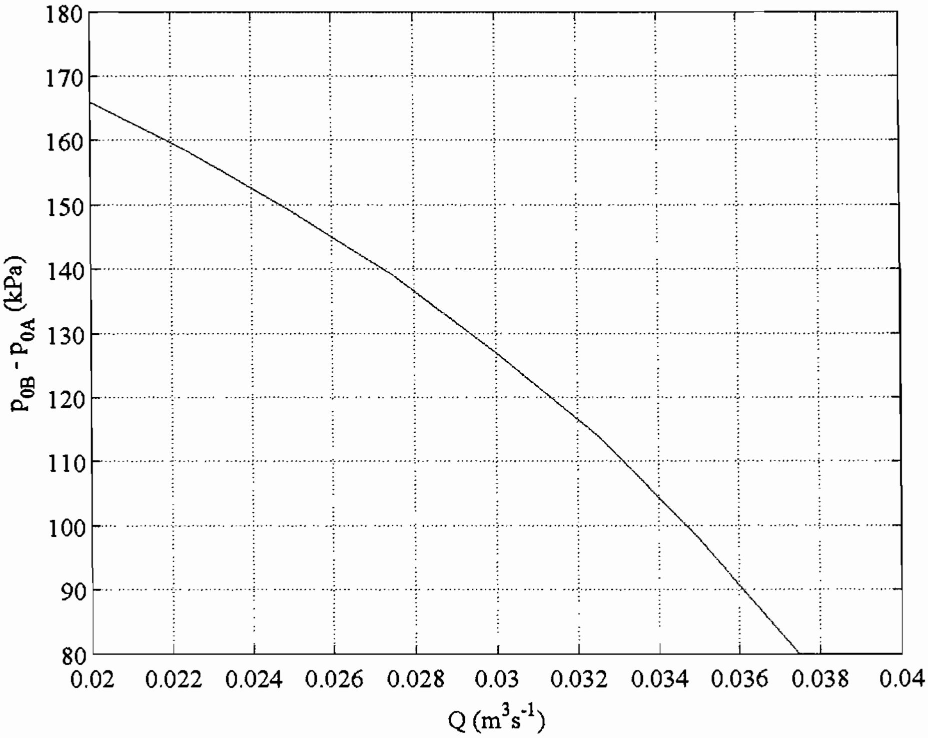

Fig. 5

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