ENGINEERING TRIPOS PART IB

Thursday 4 June 2009 2 to 4

Paper 6

INFORMATION ENGINEERING

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig. 4.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

Answer not more than two questions from this section.

1 (a) For a linear system characterised by a proper rational transfer function, explain how the locations of the poles determine the stability and response. Illustrate your answer using a diagram showing the typical impulse response for various pole locations. [6]

(b) For the system shown in Fig. 1, show that the closed loop transfer function relating $\bar{y}(s)$ to $\bar{x}(s)$ is

$$\frac{4(s-1)}{(s+1)^2}.$$ [3]

(c) Determine the open-loop and closed-loop poles for the system in Fig. 1. Comment on the open-loop and closed-loop stability of the system. [3]

(d) Find the response $y(t)$ to a unit step input on $x(t)$, paying particular attention to the initial slope, any turning points and the final value of the response. Sketch the response $y(t)$. [8]

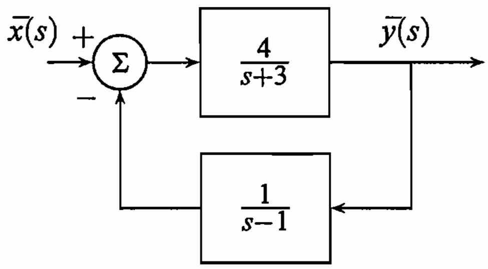

Fig. 1

2 (a) How can a *Nyquist diagram* for a linear system be constructed experimentally? [3]

(b) For the feedback system shown in Fig. 2, the controller $K(s) = K$, and the plant $G(s)$ is given by

$$G(s) = \frac{1}{s(s^2 + 2s + 1)} .$$

(i) Sketch the Nyquist diagram for the plant, showing any asymptotes. [5]

(ii) Describe how the magnitude of the closed loop frequency response can be determined from the Nyquist diagram. [2]

(iii) What is the gain margin of the feedback system if $K = 1$? [2]

(iv) For a particular value of K, the phase margin is $60°$. The frequency at which the phase margin is defined is ω_c. Find the value of ω_c, the corresponding value of K, and the magnitude of the closed loop frequency response at ω_c. Comment on how these values would change for a phase margin of less than $60°$. [8]

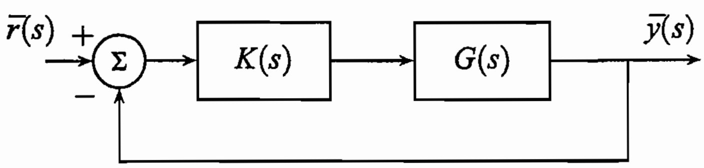

Fig. 2

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3 (a) Explain the meaning of the terms *gain margin* and *phase margin* of a feedback system. [3]

(b) For the feedback system shown in Fig. 3, $G(s)$ is asymptotically stable and of the form

$$G(s) = \frac{12}{(1+sT_1)(1+sT_2)(1+sT_3)}$$

where T_1, T_2 and T_3 are constants. The corresponding Bode diagram for $G(s)$ is given in Fig. 4. For the case $K(s) = 1$, find the gain margin and phase margin of the closed loop system. [4]

(c) $K(s)$ is now set to to be a compensator with transfer function

$$K(s) = \frac{0.5(1+0.1s)}{(1+0.025s)}.$$

Draw the Bode diagram of the compensated loop on the additional copy of Fig. 4 attached at the end of the paper. Estimate the phase margin and gain margin for the compensated loop. [9]

(d) Describe the differences in response to a unit step for the uncompensated and compensated loops. What is the the steady-state error for both loops? [4]

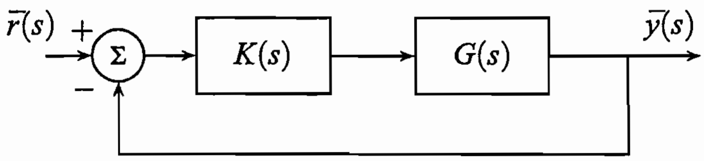

Fig. 3

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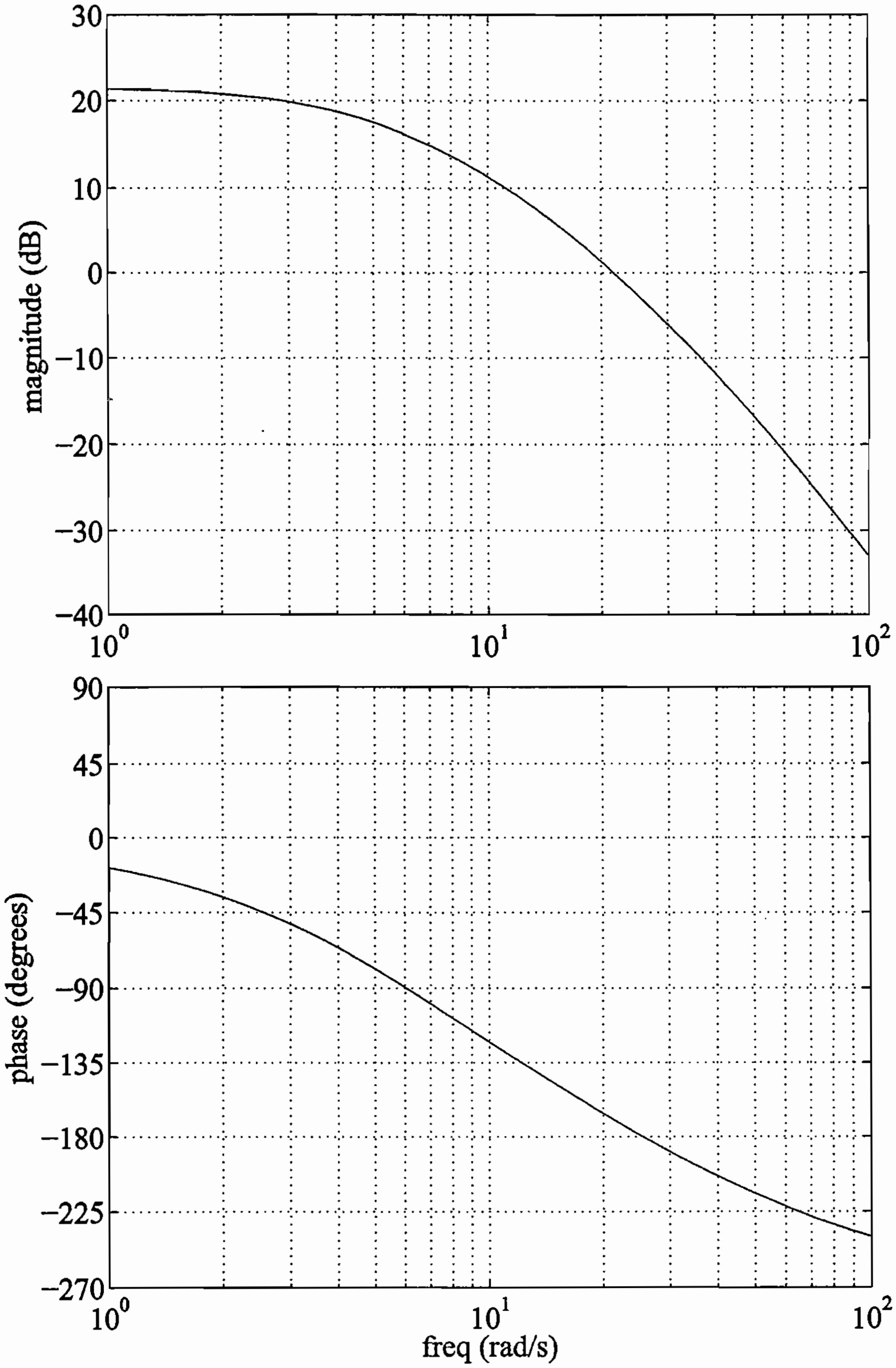

Fig. 4

Note: an additional copy of Fig. 4 is attached at the end of this paper. This should be annotated with your constructions and handed in with your answer to this question.

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SECTION B

Answer not more than two questions from this section.

4 (a) Show, from first principles, the convolution property of the Fourier transform, i.e.,

$$\mathscr{F}\{x(t) * y(t)\} = \mathscr{F}\{x(t)\}\mathscr{F}\{y(t)\}$$

where $*$ and $\mathscr{F}\{.\}$ denote the convolution and Fourier transform operators, respectively. [4]

(b) Let the signal $y(t)$ be given by $y(t) = x_1(t) + x_2(t)$ where, for $i = 1,2$

$$x_i(t) = \begin{cases} \sin \omega_i t & -\frac{T}{2} \leq t \leq \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$

Assume $\omega_1 > \omega_2$. Spectral analysis is used to estimate ω_1 and ω_2.

(i) Calculate the Fourier transform, $X_i(\omega)$, of $x_i(t)$. [5]

(ii) Sketch $|Y(\omega)|$ for large and small values of T. What trade-offs between ω_1, ω_2 and T do you envisage in estimating ω_1 and ω_2? [7]

(iii) What advantage might be gained if the signals $x_i(t)$ are obtained as

$$x_i(t) = \Lambda(t) \sin \omega_i t$$

where $\Lambda(t)$ is a triangular pulse of duration $2T$ centered at $t = 0$? [4]

5 A signal of bandwidth B Hz is to be digitised.

(a) The signal is first sampled at a rate of $f_S = 1/T_S$ samples per second, where T_S is the sampling period. By taking the Fourier transform of the sampled signal, show that if $f_S \geq 2B$, it is possible to have perfect signal recovery from the sampled signal. Explain the steps involved. [5]

(b) The sampled signal is next quantised with a uniform quantiser of step size Δ. Show that the RMS quantisation noise is given by $\Delta/\sqrt{12}$, stating the assumptions made. [5]

(c) Calculate the minimum resolution (in bits) necessary to achieve an SNR of 30 dB when a sinusoidal signal with amplitude 5 V is quantised with a uniform quantiser. [4]

(d) Calculate the resulting minimum data rate of the digitised signal. [2]

(e) Fig. 5 shows the input-output characteristic of a non-uniform quantiser. Explain the advantages of this over a uniform quantiser. For what type of signals would this non-uniform quantiser be most effective? [4]

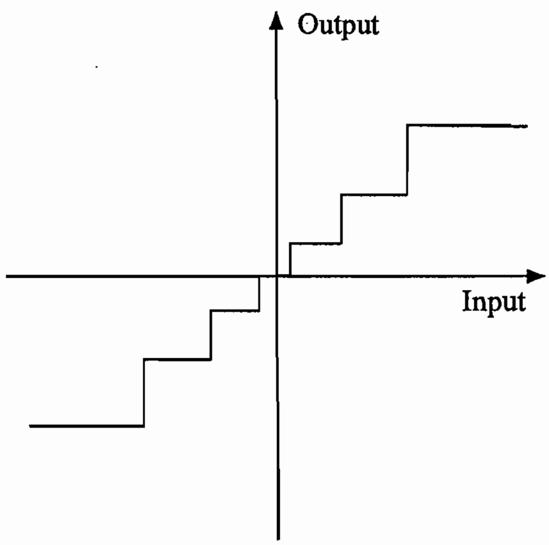


Fig. 5

6 (a) Describe *modulation* and explain why it is used in practical communications systems. [2]

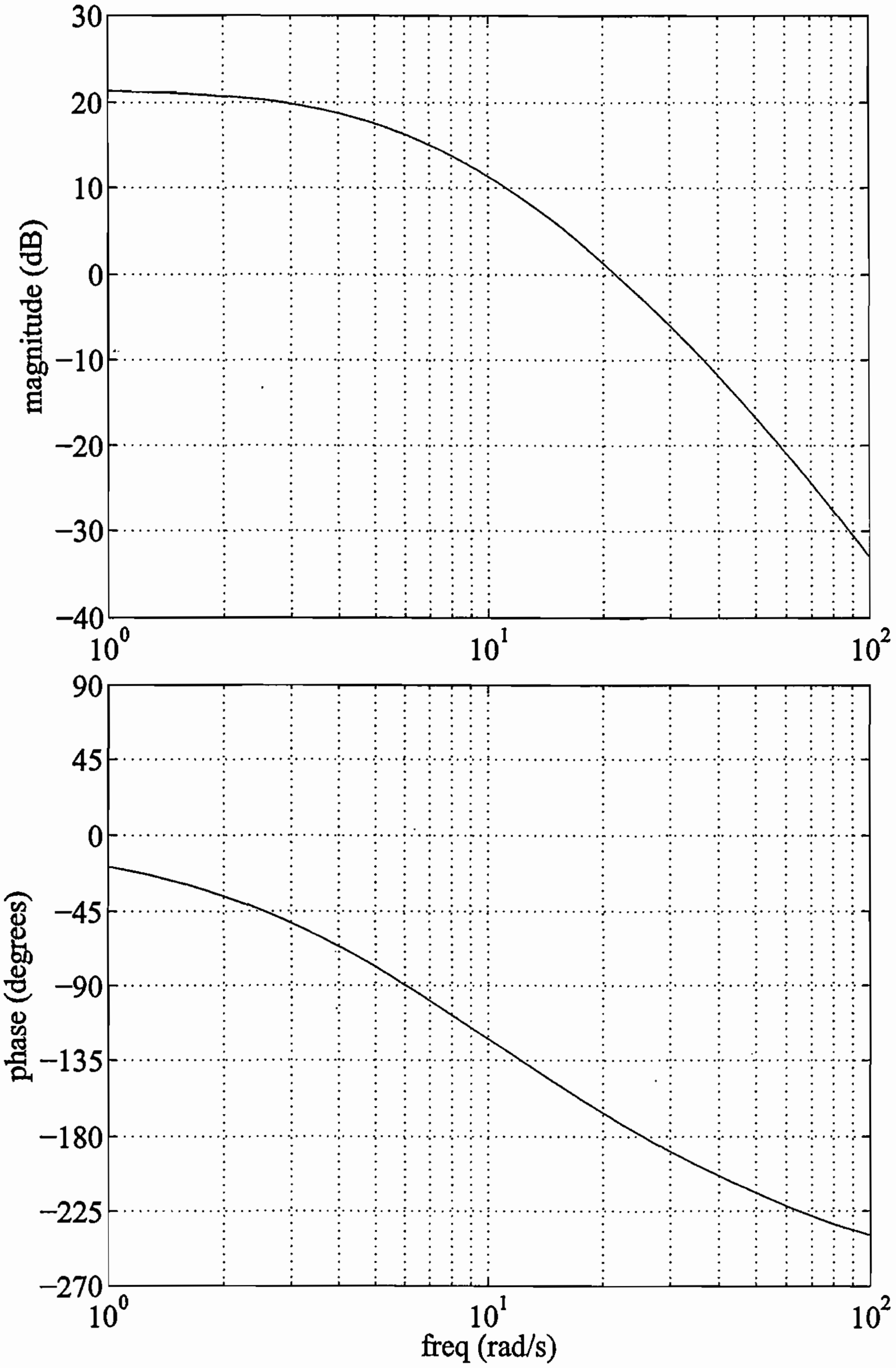
(b) Explain the differences in bandwidth consumption between the following modulation schemes: AM, DSB-SC, SSB-SC, FM and BPSK. [5]

(c) Show that the error probability of BPSK when transmitted over an additive white Gaussian noise (AWGN) channel is given by $P_e = Q(\sqrt{2\text{SNR}})$, where $Q(x) = 1 - \Phi(x)$, and $\Phi(x)$ is the cumulative distribution function of a zero mean unit variance Gaussian random variable. [5]

(d) Find the minimum SNR required to ensure that a BPSK modulated system has a probability of error $P_e \leq 5 \times 10^{-6}$. [4]

(e) Calculate the total bandwidth required to accommodate 20 BPSK users transmitting at rate $R = 50$ kbit/s with a rectangular pulse shape. Assume that no interference is caused beyond the second zero of the sinc function. Repeat the above calculation if now we need a guard band of 10 kHz between users. [4]

END OF PAPER



Copy of Fig. 4. This should be annotated with your constructions and handed in with your answer to question 3.