| Friday 5 June 2009 | 2 to 4.30 |
| :--- | :--- |
| 2 to 3.30 Foreign Language Option |  |

Paper 8

## SELECTED TOPICS

Answer one question from Section A. In addition:

If you are not taking the Foreign Language option, answer four questions, taken from only two of Sections $B-H$. Not more than two questions from each section may be answered.
If you are taking the Foreign Language option, answer two questions from one of Sections $B-H$.

All questions carry the same number of marks.
The approximate number of marks allocated to each part of a question is indicated in the right margin.

Answers to questions in each section should be tied together and handed in separately.
Section A (Introductory Business Economics) ..... 2
Section B (Civil and Structural Engineering) ..... 3
Section C (Mechanics, Materials and Design) ..... 7
Section D (Aerothermal Engineering) ..... 11
Section E (Electrical Engineering) ..... 15
Section F (Information Engineering) ..... 17
Section G (Engineering for the Life Sciences) ..... 20
Section H (Manufacturing, Management and Design) ..... 23Attachments: Data Sheet for Section B (6 pages)Data Sheet for Section E (2 pages)
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A Introductory Business Economics

Answer not more than one question from this section.

1 (a) Using an appropriate diagram, describe the kinked demand curve theory.
(b) Describe the circumstances under which monopolies might be beneficial for consumers.
(c) What impact would the following have on the level of investment in the macroeconomy:
(i) a fall in the rate of interest;
(ii) a rise in uncertainty of consumers;
(iii) a Government subsidy on investment.

2 (a) Explain the concept of profit maximisation. [4]
(b) Under what circumstances may firms not seek to maximise profits?
(c) Using the Keynesian Consumption Function Model, explain the potential impact of cutting indirect taxes (such as VAT) on the level of aggregate demand.
(d) Using either the Life Cycle Model or the Permanent Income Model, explain the potential impact of cutting indirect taxes (such as VAT) on the level of aggregate demand.

# SECTION B Civil and Structural Engineering 

Answer not more than two questions from this section.
You may refer to the data sheet as necessary at the end of the examination paper.

3 A city council is considering constructing a metro system comprising two lines, one on each side of the river. The river runs through the city in a north-south direction. To the east of the river the ground conditions can be summarised as a stiff clay extending from the ground surface with an undrained shear strength of 150 kPa at a depth of 12.5 m and 250 kPa at a depth of 25 m . To the west of the river the ground conditions can be summarised as 15 m of sand overlying a soft clay with an undrained shear strength of approximately 30 kPa constant with depth. On both sides of the river there are many historic masonry buildings and the water table is close to the ground surface.

You are to make a presentation of a feasibility study for the metro to the city councillors. The scheme is likely to comprise bored tunnels between stations constructed by the cut-and-cover technique.
(a) Identify suitable construction techniques for the bored tunnels on each side of the river if the axes of the tunnels were at depths of (i) 12.5 m and (ii) 25 m . Illustrate your answers, where appropriate, by considering the stability ratio of the tunnels, assuming the unit weight for all the soils is $20 \mathrm{kN} \mathrm{m}^{-3}$. Under what circumstances could sprayed concrete linings be used?
(b) What would be key considerations in construction of the stations on each side of the river? Suggest how the walls supporting the soils could be constructed, giving two alternatives.
(c) If settlements are likely to be significant, why might buildings be damaged and what measures could be taken to prevent this?

4 A 12 m long diaphragm wall is constructed as one wall of an 8 m deep shaft for the start of a shallow tunnel drive, as shown in Fig. 1. A force $P$ will be applied at the mid-height of the wall resulting from the thrust from the tunnelling machine. A temporary surcharge $q$ may be applied at times at the ground surface outside the shaft. The soil profile comprises 8 m of dry sand overlying a stiff clay and the water table is at the top of the clay. The sand has unit weight $\gamma=18 \mathrm{kN} \mathrm{m}^{-3}$ and its critical state angle of friction $\varphi^{\prime}=30^{\circ}$. The clay has unit weight $\gamma=20 \mathrm{kN} \mathrm{m}^{-3}$, undrained shear strength $S_{u}=100 \mathrm{kPa}$ and a critical state angle of friction $\varphi^{\prime}=20^{\circ}$.

Ignoring any soil or water pressures on the inside of the shaft, and assuming plane strain conditions and the wall to be rigid and smooth, calculate the maximum possible value of $P$ and sketch the corresponding pressure diagrams for each of the following conditions.
(a) Short-term, undrained conditions in the clay, with zero surcharge.
(b) The same as part (a) above, but including a surcharge $q=30 \mathrm{kPa}$.
(c) The same as part (a) above, with zero surcharge, except that the sand has been inundated with water up to ground level, caused by flooding due to a burst water main. Assume that the unit weight of the saturated sand $\gamma$ increases to $20 \mathrm{kN} \mathrm{m}^{-3}$.
(d) Dry sand, long-term conditions in the clay, with zero surcharge.


Fig. 1

5 A long reinforced concrete swimming pool is 12 m wide and 4 m deep, as shown in Fig. 2. The critical loading case for a pool is when it is empty, with the water table on the outside at ground level. Uplift is prevented by ground anchors around the perimeter. The anchors provide no moment resistance.

The applied loading on the side wall (which include soil pressure, hydrostatic pressure, and suitable partial safety factors) varies from zero at the top to 70 kPa at the base. The loading on the base has a uniform magnitude of 40 kPa .

The concrete has a cube strength of $40 \mathrm{~N} \mathrm{~mm}^{-2}$ and the reinforcement has a strength of $460 \mathrm{~N} \mathrm{~mm}^{-2}$. Ignore the effect of axial force in the slab base, and the slab's own dead weight. Consider a 1 m strip across the width of the pool.
(a) Draw a diagram of the bending moment in the walls and the slab, indicating critical values.
(b) Determine the thickness of the base slab if it is to be singly reinforced and to have a uniform depth.
(c) Design the reinforcement at the corners and other critical sections.
(d) To what value could the thickness of the slab be reduced if short lengths of the slab could be doubly reinforced?


Fig. 2

## SECTION C Mechanics, Materials and Design

Answer not more than two questions from this section.

6 (a) A horizontal-axis wind turbine with rotor diameter of 5 m has a rated power of 5 kW when operating in undisturbed air of density $1.3 \mathrm{~kg} \mathrm{~m}^{-3}$. Assume that the turbine operates at optimum efficiency.
(i) Explain briefly with a sketch what is meant by "optimum efficiency".
(ii) Find the rated wind speed.
(iii) Find the thrust at rated wind speed.
(b) Explain briefly why:
(i) wind turbines almost always have three blades;
(ii) large wind turbines have a gearbox but small ones do not;
(iii) the capacity factor of wind turbines is low, typically $25 \%$ to $30 \%$.
(c) A wind turbine is generating 0.5 MW when the rotor speed is 87 rpm . The turbine drives a gearbox which in turn drives a 4-pole generator at 501 rpm .
(i) Find the torque in the input and output shafts of the gearbox.
(ii) List the frequencies at which wind turbine noise might be heard.
(iii) On a diagram of the drive train show where the emergency brake might be located, giving reasons for your answer.

7 Figure 3 shows a horizontal axis wind turbine. A cross-section through an annular control volume is included. The graphs below show the axial and tangential flow velocities throughout the control volume.
(a) Find an expression for the mass flow rate $\delta \dot{m}$ in the annular control volume.
(b) (i) Show that the axial force $\delta N$ acting on the blades in the control volume is given by

$$
\delta N=B F_{N} \delta r=4 \pi r \rho V_{0}^{2} a(1-a) \delta r
$$

where $B$ is the number of blades, $F_{N}$ is the normal force per unit length acting on a blade and $\rho$ is the density of air.
(ii) Find an equivalent expression for the torque acting on the blades in the control volume.
(c) The wind turbine will operate with a tip speed ratio $\lambda=\omega R / V_{0}=8$. By making the assumption that the induction factors $a=0.2$ and $a^{\prime}=0.005$ over the whole length of the blades from $r=0$ to $r=R$, find an estimate for the coefficient of performance $C_{p}$ of the turbine.

$$
\begin{equation*}
C_{p}=\frac{P}{\frac{1}{2} \rho V_{0}^{3} \pi R^{2}} \tag{7}
\end{equation*}
$$

(d) Comment on your result for $C_{p}$ and the accuracy of the assumptions in your calculation.


Fig. 3

8 (a) Outline a typical structural design and manufacturing routes for wind turbine blades, illustrating your answer with appropriate sketches. Why does the design and manufacturing route typically depend on the turbine size?
(b) Figure 4 shows the probability density function $\phi$ of the load cycles suffered by a section of a turbine blade. The three-dimensional plot sketches the variation as a function of peak-to-peak stress range $S$ and mean stress $\bar{S}$ while the two-dimensional plot gives corresponding numerical data in a form suitable for extraction of a table. Two million load cycles are applied per month. The ultimate tensile strength $\sigma_{t s}$ of the material used in the blade is 300 MPa and, with zero mean stress, the peak-to-peak stress range $S$ and the number of cycles to failure $N$ are related by

$$
N=\left(\frac{S_{0}}{S}\right)^{m}
$$

with material properties $S_{0}=600 \mathrm{MPa}$ and $m=12$.
(i) Discuss the likely reasons for the shape of the probability density function $\phi$.
(ii) Use the data given to calculate the fatigue life of the blade section.
(iii) Comment on the factors likely to cause errors or inaccuracies in your estimate of the lifetime.

## Probability density <br>  <br> Alternating stress range $S$



Fig 4

## SECTION D Aerothermal Engineering

Answer not more than two questions from this section.

9 The table below gives key sets of variables and some of their dimensionless groupings for the characterisation of a turbojet engine. The groupings are important for describing a turbojet engine at a fixed operating condition.

| Relevant variables | Dimensionless groups (П) |
| :---: | :---: |
| $\Omega=f_{1}\left(r, R, c_{p}, c_{v}, T_{02}\right)$ | $\Omega r / \sqrt{\gamma R T_{02}}$ |
| $T_{02}=f_{2}\left(T_{04}\right)$ | $T_{04} / T_{02}$ |
| $F_{G}=f_{3}\left(p_{a,} A_{N}, p_{02}\right)$ | $\left(F_{G}+p_{a} A_{N}\right) /\left(A_{N} p_{02}\right)$ |
| $\dot{m}_{\text {air }}=f_{4}\left(A_{N}, c_{p}, T_{02}, p_{02}\right)$ | - |
| $\dot{m}_{f}=f_{5}\left(A_{N}, c_{p}, L C V, T_{02}, p_{02}\right)$ | - |

$\dot{m}_{\text {air }}$ is the air mass flow rate, $\dot{m}_{f}$ is the fuel mass flow rate, $A_{N}$ is the nozzle area, $c_{p}$ is the specific heat at constant pressure, $c_{v}$ the specific heat at constant volume, and $p_{02}$ and $T_{02}$ the engine stagnation inlet pressure and temperature, respectively. Also, $T_{04}$ is the turbine stagnation inlet temperature, $L C V$ the fuel's lower calorific value, $\gamma$ the ratio of specific heats, $R$ the perfect gas constant, $r$ radius, $p_{a}$ ambient pressure, $\Omega$ angular velocity and $F_{G}$ gross thrust.
(a) Complete the table above explaining the physical meaning of the two new groups that you add.
(b) What assumptions are needed for the dimensionless groups to be valid for engine performance analysis?
(c) Explain the difference between net and gross thrust and the significance of using $F_{G}+p_{a} A$ in the dimensionless group for thrust given in the table.
(d) A $50 \%$ scale static gas turbine rig test is performed with $p_{\mathrm{a}}=1.013 \mathrm{bar}$, $T_{\mathrm{a}}=288 \mathrm{~K}$, a combustor inlet temperature of 900 K and $\dot{m}_{\text {air }}=70 \mathrm{~kg} \mathrm{~s}^{-1}$. For the rig $F_{G}=50 \mathrm{kN}, \Omega=20,000 \mathrm{rpm}, A_{N}=0.175 \mathrm{~m}^{2}$ and $S F C=1.25 \mathrm{~kg} \mathrm{~h}^{-1} \mathrm{~kg}^{-1}$. For the full scale engine at $11,000 \mathrm{~m}\left(p_{\mathrm{a}}=0.227\right.$ bar and $\left.T_{\mathrm{a}}=217 \mathrm{~K}\right)$ with a flight Mach number of 0.8 , estimate the following: (i) $\dot{m}_{\text {air }}$; (ii) net thrust; (iii) $S F C$ and (iv) $\Omega$.

Take $\gamma=1.4, c_{p}=1.005 \mathrm{~kJ} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$ and $R=287 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$ for air.

10 An aircraft is cruising at 31000 ft where the ambient temperature is $T_{\mathrm{a}}=226.73 \mathrm{~K}$ and pressure, $p_{\mathrm{a}}=0.287$ bar. It has a Mach number of 0.85 . The aircraft has four turbofan (two-shaft) engines. Fig. 5 shows the general arrangement for one of these engines. The high pressure compressor (HPC), high pressure turbine (HPT) and low pressure turbine (LPT) have isentropic efficiencies of 0.9. Also, the inlet and nozzle flows can be viewed as isentropic processes. It can be assumed that the core and bypass jets have equal velocities ( $V_{\mathrm{jb}}=V_{\mathrm{jc}}$ ).
(a) Using the labelled stations in Fig. 5 draw separate $T-s$ diagrams for the core and bypass flows.
(b) Find the stagnation pressure $p_{02}$ and temperature $T_{02}$ at the engine inlet.
(c) If the stagnation temperature rise through the fan is 41.4 K and the pressure ratio for the HPC is 25 , find the fan exit stagnation temperature $T_{023}$. Also find the HPC exit temperature $T_{03}$.
(d) If the combustor exit temperature, $T_{04}=1450 \mathrm{~K}$ find the pressure $p_{045}$ and temperature $T_{045}$ at the HPT exit, stating all assumptions. Take the fan pressure ratio as 1.6.
(e) Assuming the stagnation temperature drop across the LPT is 361 K find the stagnation pressure $p_{05}$ and temperature $T_{05}$ after the LPT. Also find the velocity of the core jet, $V_{\mathrm{jc}}$, and the propulsive efficiency $\eta_{\mathrm{p}}$.
(f) Taking $T_{013}=T_{023}$ calculate the bypass ratio (BPR).
(g) How much does the assumption $V_{\mathrm{jc}}=V_{\mathrm{jb}}$ influence the estimated thrust and specific fuel consumption and more detailed parameters such as nozzle areas, temperatures and pressures at the fan and LPT exits?

Take the specific heat at constant pressure, $c_{p}=1.005 \mathrm{~kJ} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$ and the ratio of specific heats $\gamma=1.4$.


23

Fig. 5

11 (a) Assuming the relationship between pressure and density is isentropic ( $p / \rho^{\gamma}=$ constant) show the variation of the properties of air with altitude can be expressed as

$$
\frac{p}{p_{S L}}=\left[1-\left(\frac{\gamma-1}{\gamma}\right) \frac{g z}{R T_{S L}}\right]^{\frac{\gamma}{\gamma-1}}
$$

where $p_{S L}$ and $T_{S L}$ are sea level values of static pressure and temperature, respectively. Also, $\gamma$ is the ratio of specific heats, $R$ is the gas constant, $g$ the acceleration due to gravity and $z$ the height above sea level.
(b) For a Boeing 747, for the optimum flight Mach number $M=0.83$ it can be shown that

$$
\frac{M L}{D} \approx 1.2+51 C_{L}-47.3 C_{L}^{2}
$$

where $L$ and $D$ are lift and drag. From the above, estimate the optimum lift coefficient, $C_{L}$, for cruise.
(c) Cruise starts at an altitude of 9500 m . If the aircraft mass is 500 tonnes at the start of cruise, determine the wing area. Assume constant values of Mach number and lift coefficient taken from part (b). Also use the equation derived in part (a) with $T_{S L}=288.5 \mathrm{~K}$ and $p_{\mathrm{SL}}=1.013 \mathrm{bar}$.
(d) Assuming that the lift coefficient and Mach number do not change, find the altitude at the end of cruise. The aircraft mass at the end of cruise is 375 tonnes.
(e) The Breguet equation for range, $s$, is given below

$$
s=-\frac{V_{\infty}(L / D)}{g S F C} \quad \ln \left[\frac{W_{2}}{W_{1}}\right]
$$

where $V_{\infty}$ is the cruise speed, $W_{1}$ and $W_{2}$ the aircraft mass at the start and end of cruise, and $S F C$ the specific fuel consumption. Estimate $s$ if $S F C=0.5 \mathrm{~kg} \mathrm{~h}^{-1} \mathrm{~kg}^{-1}$.
(f) Give one assumption that is made in the Breguet range equation.

Take $\gamma=1.4, R=287.3 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$ and $g=9.81 \mathrm{~m} \mathrm{~s}^{-2}$.

## SECTION E Electrical Engineering

Answer no more than two questions from this section.

12 (a) With the aid of appropriate diagrams, outline the mode of operation of a passive matrix addressed liquid crystal display.
(b) Describe the principle of operation of a 3:1 dynamic addressing scheme for a simple 3 row and 4 column display. Show that for such a scheme a RMS voltage which is $91 \%$ greater than the "off" voltage is applied to each picture element which is selected to be "on".

Explain why this led to the use of active matrix addressing for large area displays.
(c) Provide a cross-sectional diagram of an amorphous silicon based thin film transistor which can be used in the active matrix addressing of liquid crystal displays and with the aid of diagrams explain how such displays are addressed.

13 (a) Describe what is meant by 'scaling' in CMOS integrated circuits. In simple scaling, if a feature length scales by a factor $k$, how much do the following properties scale by; transistor density, transistor speed and power per chip. Make clear any assumptions needed for your answer. Do any other devices obey scaling type rules?
(b) Derive the relationship between the conductivity and mobility in a material.
(c) Silver has the face centred cubic structure in which there are 4 atoms per unit cell of lattice constant $4.09 \AA$, and it has one conduction electron per atom. Its conductivity is $6.3 \times 10^{7} \Omega^{-1} \mathrm{~m}^{-1}$. What is the mobility of the electrons?
(d) A designer desires to make a power depletion mode field effect transistor (FET) from a film of n-type semiconductor. The FET should have a minimum transit time of 5 ps and a power dissipation of 2 W . The semiconductor has a fieldindependent mobility of $0.15 \mathrm{~m}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$, a donor density of $10^{22} \mathrm{~m}^{-3}$, a film thickness of $5 \mu \mathrm{~m}$, and design rules which allow a maximum width to length ( $W / L$ ) ratio of 50 . Evaluate the FET dimensions, the drain voltage and the drain current.

## 14 (a) Give evidence for the quantum and particulate nature of light.

(b) Calculate the energy of a photon of wavelength 200 nm in joules and in electron volts.
(c) Using the time independent Schrödinger equation, derive an expression for the kinetic energy of the series of electron standing waves in a one-dimensional box with infinitely high sides.
(d) A photon forms a standing wave in a one-dimensional box of length 200 nm . It resonates at the fundamental frequency with an electron wave of the same wavelength. What is the kinetic energy of the electron?

The electron mass is $9.11 \times 10^{-31} \mathrm{~kg}$, Planck's constant, $h$, is $6.62 \times 10^{-34} \mathrm{Js}$.

## SECTION F Information Engineering

Answer not more than two questions from this section.

15 (a) Interpolation between pixels is an operation used in most photo-editing packages. Explain why interpolation is useful, and discuss what sort of image processing operations would use it.
(b) Linear interpolation between a pair of one-dimensional data points, $x_{a}$ and $x_{b}$, sampled at locations $a$ and $b$ respectively, is of the following form

$$
x_{p}=x_{a}(1-f(p))+x_{b} f(p)
$$

where the interpolated point $x_{p}$ is at location $p$ and $a \leq p \leq b$. Obtain an expression for the linear function $f(p)$ and hence express $x_{p}$ in terms of $x_{a}, x_{b}, a, b$ and $p$.
(c) For two dimensional data, bi-linear interpolation is used. Derive an expression for the bi-linearly interpolated point $x_{p, q}$ in terms of the four points surrounding it:

$$
\begin{array}{ll}
x_{a, c} & x_{a, d} \\
x_{b, c} & x_{b, d}
\end{array}
$$

where $a \leq p \leq b$ and $c \leq q \leq d$.
(d) It is required to rotate an image anti-clockwise by an angle $\theta$. Derive expressions for the location $(p, q)$ from which a pixel at $(u, v)$ in the rotated output image should be interpolated. (Assume $p, q$ and $u, v$ are measured relative to the exact centre of the image of size $M \times N$ pixels, and that $p$ and $u$ increase from top to bottom of the image, while $q$ and $v$ increase from left to right).

16 (a) A grey scale image, $I(x, y)$, is to be smoothed and differentiated as part of the edge detection process.
(i) Why is smoothing required? Describe the smoothing filter used in practice, and explain the effects of increasing the size of the filter.
(ii) Give an expression for computing the intensity of a smoothed pixel in terms of two discrete 1D convolutions.
(iii) Give expressions for computing spatial derivatives after smoothing.
(b) Image features such as blob-like shapes can be localized in an image by filtering with a band-pass filter.
(i) How is the band-pass filtering implemented in practice and give details of the isotropic filter used.
(ii) Explain how the image blob/feature is localized in the image and how an appropriate scale and orientation are selected.
(iii) Give simple examples of applications that require the detection and description of these image features.

17 Consider a set of $N$ images $S=\left\{\mathbf{x}_{1}, \ldots, \mathbf{x}_{N}\right\}$ where each image is represented as a vector of $M$ binary features, e.g. $\mathbf{x}_{n}=\left(x_{n 1}, \ldots, x_{n M}\right)$ and $x_{n m} \in\{0,1\}$.
(a) Assume a probabilistic model of images where each feature is independent of the other features, and each feature $x_{n m}$ has probability $p_{m}$ of taking the value 1 ,

$$
P\left(x_{n m}=1\right)=p_{m} \text { for all } n, m .
$$

Let $\mathbf{p}=\left(p_{1}, p_{2}, \ldots, p_{M}\right)$ be the vector of parameters of this model. Write down the likelihood function for this model given $S$ and derive the maximum likelihood estimate for $\mathbf{p}$.
(b) Assume uniform priors on all the parameters $\mathbf{p}$. Derive an expression for the marginal probability of the data $P(S)$ and give an interpretation of this. You may wish to use the following identity:

$$
\int_{0}^{1} x^{a}(1-x)^{b} d x=\frac{a!b!}{(a+b+1)!}
$$

for integers $a$ and $b$.
(c) Given two data sets of images, $S$ and $S^{\prime \prime}$, for example representing images of two concepts (e.g. "cats" and "dogs"), describe an automatic method (including algorithm and equations if needed) for determining whether an image $\mathbf{x}$ fits better with $S$ or $S^{\prime}$.

## SECTION G Engineering for the Life Sciences

Answer not more than two questions from this section.

18 Write brief notes on each of the following.
(a) How the theory of evolution has made Darwin the "father" of biology as a coherent subject.
(b) The mode of operation of a virus when it attacks a cell such as bacterium.
(c) How the DNA base-sequence prescribes the sequence of amino acids on the polypeptide chain, and hence the conformation of a protein molecule.
(d) The role of phases of smooth swimming and tumbling in the navigation of bacteria such as $E$. coli towards a source of nutrition.
(e) How nature has solved the problem of building a curved flagellar filament from a single type of building-block, which self-assembles into a cylindrical surface lattice.
(a) The following relates to synaptic learning.
(i) What is anti-hebbian learning?
(ii) Describe, with the aid of a sketch, the circuit and mechanism of learning in the cerebellar-like structure of the weakly-electric fish.
(iii) What are the benefits of anti-hebbian learning for sensory processing in the weakly-electric fish?
(b) The following relates to reinforcement learning.
(i) Sketch the firing response of dopaminergic neurons in the basal ganglia to: an unexpected reward; a reward that is expected based on an earlier sensory stimulus; a reward that is expected but does not occur.
(ii) Write down the temporal-difference learning rule and explain what each term represents.
(iii) Explain the firing response in (i) based on temporal-difference learning.
(c) The following relates to motor commands carrying appreciable amounts of noise.
(i) Describe the characteristic relation between the properties of the noise and the motor command.
(ii) Explain how this noise can determine the particular trajectory chosen for a movement.
(d) The following relates to dealing with sensory uncertainty.
(i) Define the relation between the likelihood and the posterior estimate of a parameter.
(ii) Give a biological example where the maximum likelihood estimate and the maximum a posteriori estimate are different.
(iii) Under what type of loss function would the maximum of the posterior be optimal and under what type of loss function would the median of the posterior be optimal?

20 (a) Describe the primary differences between the mechanical behaviour of engineering metals and ceramics and soft biological tissues. Describe the microstructural differences that give rise to these mechanical differences.
(b) Describe the composition, structure and mechanical response of articular cartilage and of bone, highlighting the key differences.
(c) A cylindrical titanium medullary screw with diameter 5 mm is implanted into the full-length of the marrow space of a cylindrical (tube-like) bone with a constant diameter of 15 mm . Given that the elastic modulus of the implant is eight times that of bone, what proportion of the axial force is carried in the implant and what proportion is in the bone? State clearly any assumptions made in the calculation. Comment on the effectiveness of this implant design and suggest possible improvements, including how these improvements would influence the load partitioning.

## SECTION H Manufacturing, Management and Design

Answer not more than two questions from this section.

21 Your company has invented a new type of hairdryer, in which hot air is blown at high speed from a linear row of small nozzles, simultaneously drying and combing the hair. In the prototype there are ten nozzles, 1 mm in diameter, spaced evenly along a line 25 mm long, and the jets of air emerge at $20 \mathrm{~m} \mathrm{~s}^{-1}$. Tests have shown that the combing effect is insignificant at air speeds below $5 \mathrm{~m} \mathrm{~s}^{-1}$. The body of the hairdryer is moulded in bright blue plastic with a distinctive shape, and it is proposed to market it under the name 'Blu-jet'.
(a) Identify the tests that an invention must satisfy in order to be patentable.
(b) Describe the structure of a typical patent document and explain why it is desirable to use more than one claim. Suggest drafts of one main claim and one dependent claim which could be used in a patent for the invention described above.
(c) Describe any other types of intellectual property (IP), apart from patents, which would be relevant to this product. In each case identify issues that should be considered in protecting that IP.

22 (a) List and describe the typical stages of the evolution of an industry.
(b) Explain what is meant by the following types of innovation:
(i) radical innovation;
(ii) incremental innovation;
(iii) sustaining innovation;
(iv) disruptive innovation.

Illustrate your answer with examples of each of the four types of innovation.
(c) Discuss, using examples, why large, long-established firms often find it difficult to deal with disruptive innovations. Describe possible strategies that such firms can use for managing disruptive innovations.

23 (a) Describe the difference between:
(i) market pull, and
(ii) technology push.

List three ways in which inventions may arise from each of (i) and (ii) above. Give examples for each one listed.
(b) It has been said that: "In order to develop a new product which is commercially successful, it is essential to have a clear view of exactly who it is aimed at and why they will want to buy it". Explain how you could effectively assess potential markets for a new product.

## Data sheet - Soil Mechanics

## General definitions



Soil structure

## considered

 as

Volumes


Specific gravity of solid

## Voids ratio

Specific volume
Water content
Degree of saturation
Unit weight of water

Unit weight of soil
$\mathrm{G}_{\mathrm{s}}$
$\mathrm{e}=\mathrm{V}_{\mathrm{V}} / \mathrm{V}_{\mathrm{S}}$
$\mathrm{v}=\mathrm{V}_{\mathrm{t}} / \mathrm{V}_{\mathrm{s}}=1+\mathrm{e}$
$\mathrm{w}=\left(\mathrm{W}_{\mathrm{w}} / \mathrm{W}_{\mathrm{s}}\right)$
$S_{r}=V_{w} / V_{v}=\left(w_{\mathrm{s}} / e\right)$
$\gamma_{\mathrm{w}}=9.81 \mathrm{kN} / \mathrm{m}^{3}$ (although we assume $10 \mathrm{kN} / \mathrm{m}^{3}$ )
$\gamma=\mathrm{W}_{\mathrm{t}} / \mathrm{V}_{\mathrm{t}}=\left(\frac{\mathrm{G}_{\mathrm{s}}+\mathrm{S}_{\mathrm{r}} \mathrm{e}}{1+\mathrm{e}}\right) \gamma_{\mathrm{w}}$
Buoyant (effective or submerged) unit weight $\quad \gamma^{\prime}=\gamma-\gamma_{\mathrm{w}}=\left(\frac{\mathrm{G}_{\mathrm{S}}-1}{1+\mathrm{e}}\right) \gamma_{\mathrm{w}} \quad$ (soil saturated)
Unit weight of dry soil

Relative density
$\gamma_{\mathrm{d}}=\mathrm{W}_{\mathrm{s}} / \mathrm{V}_{\mathrm{t}}=\left(\frac{\mathrm{G}_{\mathrm{s}}}{1+\mathrm{e}}\right) \gamma_{\mathrm{w}}$
$I_{d}=\frac{\left(e_{\text {max }}-e\right)}{\left(e_{\text {max }}-e_{\text {min }}\right)}$
where $e_{\max }$ is the maximum voids ratio achievable in the quick tilt test (for sands), and $e_{\min }$ is the minimum voids ratio achievable by vibratory compaction (for sands).

## Classification of particle sizes



## Stress components

Principle of effective stress (saturated soil):

$$
\begin{array}{lll}
\text { total stress } & \sigma=\text { effective stress } & \sigma^{\prime}+ \\
& & \text { pore water pressure } u \\
& \tau & =
\end{array}
$$

and

$$
\begin{aligned}
\sigma_{\mathrm{v}} & =\text { vertical stress } \\
\sigma_{\mathrm{h}} & =\text { horizontal stresss } \\
\tau & =\text { shear stress }
\end{aligned}
$$

## Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the undrained shear strength $\mathrm{c}_{\mathrm{u}}$ which is mobilized when the shear stress $\tau=\mathrm{c}_{\mathrm{u}}$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, $\sigma_{a}$ and $\sigma_{p}$ respectively, are given by
$\sigma_{\mathrm{a}}=\sigma_{\mathrm{v}}-2 \mathrm{c}_{\mathrm{u}}$
$\sigma_{p}=\sigma_{v}+2 \mathrm{c}_{\mathrm{u}}$
where $\sigma_{\mathrm{v}}$ is the total vertical stress.


## Strength of sands

Mobilised angle of shearing $\phi^{\prime}$
where

$$
\tau=\sigma^{\prime} \tan \phi^{\prime}
$$



$$
\begin{aligned}
\sin \phi^{\prime} & =\mathrm{TS} / \mathrm{OS} \\
& =\frac{\left(\sigma_{1}^{\prime}-\sigma_{3}^{\prime}\right) / 2}{\left(\sigma_{1}^{\prime}+\sigma_{3}^{\prime}\right) / 2} \\
\therefore \phi^{\prime} & =\sin ^{-1}\left[\frac{\left(\sigma_{1}^{\prime} / \sigma_{3}^{\prime}\right)-1}{\left(\sigma_{1}^{\prime} / \sigma_{3}^{\prime}\right)+1}\right]
\end{aligned}
$$

Earth pressure coefficient K:

$$
\sigma_{\mathrm{h}}^{\prime}=\mathrm{K} \sigma_{\mathrm{v}}^{\prime}
$$

Active pressure:

$$
\begin{aligned}
\sigma_{\mathrm{v}}^{\prime}>\sigma_{\mathrm{h}}^{\prime} \quad \therefore & \sigma_{1}^{\prime}=\sigma_{\mathrm{v}}^{\prime} \\
& \sigma_{3}^{\prime}=\sigma_{\mathrm{h}}^{\prime} \\
& \mathrm{K}_{\mathrm{a}}=\left(1-\sin \phi^{\prime} / 1+\sin \phi^{\prime}\right)
\end{aligned}
$$

Passive pressure: $\quad \sigma_{\mathrm{h}}^{\prime}>\sigma_{\mathrm{v}}^{\prime} \quad \therefore \sigma_{1}^{\prime}=\sigma_{\mathrm{h}}^{\prime}$
[We assume principal stresses

$$
\begin{aligned}
& \sigma_{3}^{\prime}=\sigma_{v}^{\prime} \\
& K_{p}=\left(1+\sin \phi^{\prime}\right) /\left(1-\sin \phi^{\prime}\right)=\frac{1}{K_{a}}
\end{aligned}
$$

are horizontal and vertical]

Angle of shearing resistance:
at peak strength $\phi_{\text {max }}^{\prime}$ at $\left(\sigma_{1}^{\prime} / \sigma_{3}^{\prime}\right)_{\text {max }}$
at critical state $\phi_{\text {crit }}^{\prime}$ after large strains.

## Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi^{\prime}=\phi_{\text {max }}^{\prime}$ and

$$
\phi_{\max }^{\prime}=\phi_{\text {crit }}^{\prime}+\phi_{\text {dilatancy }}^{\prime}
$$

where $\phi_{\text {crit }}^{\prime}$ is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero $\left(\phi_{\text {dilatancy }}^{\prime}->0\right)$ as $\phi_{\text {max }}^{\prime}->\phi_{\text {crit }}^{\prime}$.
$\phi_{\text {crit }}^{\prime}$ is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values $\left( \pm 2^{\circ}\right)$ :

feldspar $\quad 40^{\circ}$
quartz $33^{\circ}$
$53^{\circ} \quad\left(\mathrm{I}_{\mathrm{d}}=1\right.$, and mean effective stress $\left.\mathrm{OS}<150 \mathrm{kPa}\right)$ mica $25^{\circ}$

## Seepage

Excess pore water pressure

Head

$$
\mathrm{h}=\mathrm{u} / \gamma_{\mathrm{w}}
$$

Potential $\quad \overline{\mathrm{h}}=\mathrm{h}+\mathrm{z}$


Total pore water pressure head at $\mathrm{A}: \quad \mathrm{u}=\gamma_{\mathrm{w}} \mathrm{h}=\gamma_{\mathrm{w}}(\overline{\mathrm{h}}-(-\mathrm{z}))$

$$
\mathrm{B}: \quad \mathrm{u}+\Delta \mathrm{u}=\gamma_{\mathrm{w}}(\mathrm{~h}+\Delta \mathrm{h})=\gamma_{\mathrm{w}}(\overline{\mathrm{~h}}+\mathrm{z}+\Delta \overline{\mathrm{h}}+\Delta \mathrm{z})
$$

[Excess pore water pressure at

$$
\begin{array}{ll}
\text { A: } & \overline{\mathrm{u}}=\gamma_{\mathrm{w}} \overline{\mathrm{~h}} \\
\text { B: } & \left.\overline{\mathrm{u}}+\Delta \overline{\mathrm{u}}=\gamma_{\mathrm{w}}(\overline{\mathrm{~h}}+\Delta \overline{\mathrm{h}})\right]
\end{array}
$$

Hydraulic gradient $\mathrm{A}-\mathrm{B}$

$$
\mathrm{i}=-\frac{\Delta \overline{\mathrm{h}}}{\Delta \mathrm{~s}}=-\frac{\Delta \overline{\mathrm{u}}}{\gamma_{\mathrm{w}} \Delta \mathrm{~s}}
$$

Darcy's law $\quad \mathrm{v}=\mathrm{ki}$
$\mathrm{v}=$ average or superficial seepage velocity
$\mathrm{k}=$ coefficient of permeability
Typical permeabilities

$$
\begin{array}{lll}
\mathrm{D}_{10}>10 \mathrm{~mm} & : & \text { non-laminar flow } \\
10 \mathrm{~mm}>\mathrm{D}_{10}>1 \mu \mathrm{~m} & : & \mathrm{k} \cong 0.01\left(\mathrm{D}_{10} \text { in } \mathrm{mm}\right)^{2} \mathrm{~m} / \mathrm{s} \\
\text { clays } & : & \mathrm{k} \cong 10^{-9} \text { to } 10^{-11} \mathrm{~m} / \mathrm{s}
\end{array}
$$

## Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.


## Design Stresses

Cube strength for concrete $\mathrm{f}_{\mathrm{cu}}$. At failure in bending, stress in concrete $=0.4 \mathrm{f}_{\mathrm{cu}}$ over whole area of concrete in compression.

Tensile yield stress of steel $\mathrm{f}_{\mathrm{y}}$. At failure in bending, stress in bars in tension $=0.87 f_{y}$, stress in bars in compression $=0.75 \mathrm{f}_{\mathrm{y}}$.

## Design Equations

Moment capacity of singly reinforced beam

$$
\begin{aligned}
& \mathrm{M} \leq 0.15 \mathrm{f}_{\mathrm{cu}} \mathrm{bd}{ }^{2} \\
& \mathrm{M}=0.87 \mathrm{f}_{\mathrm{y}} \mathrm{~A}_{\mathrm{s}} \mathrm{~d}(1-\mathrm{x} / 2) \\
& \mathrm{x}=2.175\left(\mathrm{f}_{\mathrm{y}} / \mathrm{f}_{\mathrm{cu}}\right) \cdot\left(\mathrm{A}_{\mathrm{s}} / \mathrm{bd}\right) \quad(\leq 0.5 \text { to avoid over reinforcement })
\end{aligned}
$$

Moment capacity of doubly reinforced beam

$$
\begin{aligned}
& M=0.15 f_{c u} b d^{2}+0.75 f_{y} A_{s}^{\prime}\left(d-d^{\prime}\right) \\
& 0.87 f_{y} A_{s}=0.75 f_{y} A_{s}^{\prime}+0.2 f_{c u} b d
\end{aligned}
$$

## Shear capacity of all beams

Total shear capacity $\mathrm{V}=\left(\mathrm{v}_{\mathrm{c}}+\mathrm{v}_{\mathrm{s}}\right) \mathrm{bd}$
Where, $\mathrm{v}_{\mathrm{c}}=0.68\left(100 \mathrm{~A}_{\mathrm{s}} / \mathrm{bd}\right)^{0.33} .(400 / \mathrm{d})^{0.25} \quad\left(\mathrm{~N} / \mathrm{mm}^{2}\right)$
and $\mathrm{v}_{\mathrm{s}}=0.87 \mathrm{f}_{\mathrm{y}} \mathrm{A}_{\mathrm{sq}} /(\mathrm{bs})$
in which $s=$ shear link spacing, $\mathrm{A}_{\mathrm{sq}}$ is total area of all shear bars in a link and $\mathrm{A}_{\mathrm{s}}$ is the total area of effective longitudinal tension steel at the section.

## Standard bar sizes

| Diameter (mm) | 6 | 8 | 10 | 12 | 16 | 20 | 25 | 32 | 40 | 50 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Area $\left(\mathrm{mm}^{2}\right)$ | 28 | 50 | 78 | 113 | 201 | 314 | 491 | 804 | 1256 | 1963 |

## Available steel types

Deformed high yield steel

$$
\begin{array}{ll}
\mathrm{f}_{\mathrm{y}}=460 & \mathrm{~N} / \mathrm{mm}^{2} \\
\mathrm{f}_{\mathrm{y}}=250 & \mathrm{~N} / \mathrm{mm}^{2}
\end{array}
$$

Plain mild steel

Lap and anchorage lengths 40 bar diameters
Density of reinforced concrete: $24 \mathrm{kN} / \mathrm{m}^{3}$
Reinforcement areas per metre width

|  | Spacing of bars (mm) |  |  |  |  |  |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 75 | 100 | 125 | 150 | 175 | 200 | 225 | 250 | 275 | 300 |
| Bar Dia. (mm) |  |  |  |  |  |  |  |  |  |  |
| 6 | 377 | 283 | 226 | 189 | 162 | 142 | 126 | 113 | 103 | 94.3 |
| 8 | 671 | 503 | 402 | 335 | 287 | 252 | 224 | 201 | 183 | 168 |
| 10 | 1050 | 785 | 628 | 523 | 449 | 393 | 349 | 314 | 285 | 262 |
| 12 | 1510 | 1130 | 905 | 754 | 646 | 566 | 503 | 452 | 411 | 377 |
| 16 | 2680 | 2010 | 1610 | 1340 | 1150 | 1010 | 894 | 804 | 731 | 670 |
| 20 | 4190 | 3140 | 2510 | 2090 | 1800 | 1570 | 1400 | 1260 | 1140 | 1050 |
| 25 | 6550 | 4910 | 3930 | 3270 | 2810 | 2450 | 2180 | 1960 | 1790 | 1640 |
| 32 | 10700 | 8040 | 6430 | 5360 | 4600 | 4020 | 3570 | 3220 | 2920 | 2680 |
| 40 | 16800 | 12600 | 10100 | 8380 | 7180 | 6280 | 5580 | 5030 | 4570 | 4190 |
| 50 | 26200 | 19600 | 15700 | 13100 | 11200 | 9820 | 8730 | 7850 | 7140 | 6540 |
|  |  |  |  |  |  |  |  |  |  |  |
| Areas calculated to 3 significant figures according to B.S.I recommendations |  |  |  |  |  |  |  |  |  |  |

