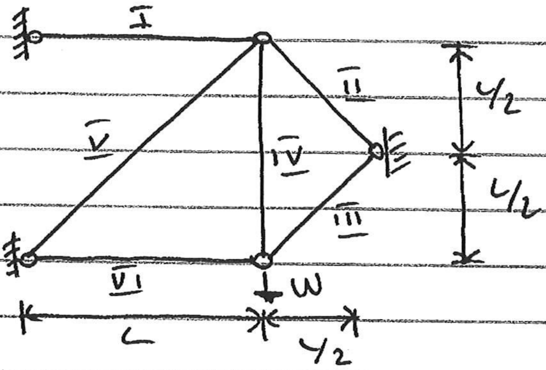
1B STRUCTURES EXAM 2010

SOLUTIONS

1)



BAR TENSIONS $\underset{\sim}{t} = \begin{bmatrix} t_I \\ t_{II} \\ t_{III} \\ t_{IV} \\ t_V \\ t_{VI} \end{bmatrix}$

a) $s - m = b + r - Dj$

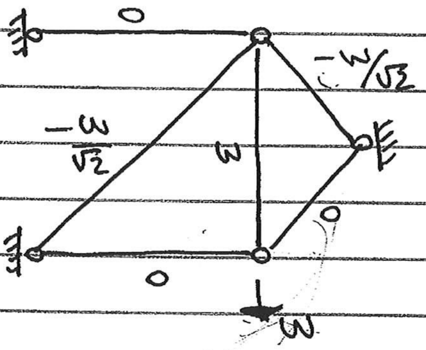
$s - 0 = 6 + (2 \times 3) - (2 \times 5)$

$\therefore s = 2$

ALTERNATIVELY: REMOVING L.H. SUPPORT OR REMOVING BARS II AND III PRODUCES STATICALLY DETERMINATE CANTILEVER

$\therefore s = 2$

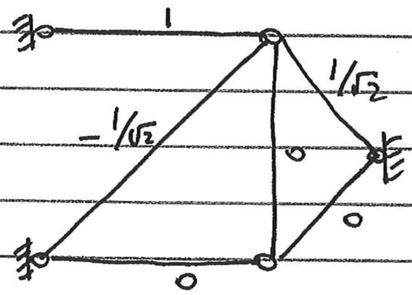
(b) i) PARTICULAR EQUILIBRIUM SOLUTION WITH $t_I = t_{III} = 0$



$\therefore \underset{\sim}{t}_0 = \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 0 \\ 1 \\ -1/\sqrt{2} \\ 0 \end{bmatrix} W$

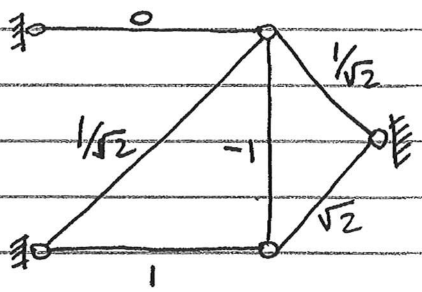
(ii) STATES OF SELF STRESS

$t_{III} = 0 \; ; \; t_I = 1$



$\therefore \underset{\sim}{s}_1 = \begin{bmatrix} 1 \\ 1/\sqrt{2} \\ 0 \\ 0 \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$

$t_{I} = 0 \; ; \; t_{III} = \sqrt{2}$



$$\underset{\sim}{S_2} = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ \sqrt{2} \\ -1 \\ 1/\sqrt{2} \\ 1 \end{bmatrix}$$

b) iii) ELASTIC SOLUTION $\underset{\sim}{t} = \underset{\sim}{t}_0 + x_1 \underset{\sim}{S}_1 + x_2 \underset{\sim}{S}_2$

FLEXIBILITY MATRIX $\underset{\sim}{F} = \dfrac{L}{EA} \begin{bmatrix} 1 & & & & & 0 \\ & 1/\sqrt{2} & & & & \\ & & 1/\sqrt{2} & & & \\ & & & 1 & & \\ & & & & \sqrt{2} & \\ 0 & & & & & 1 \end{bmatrix}$

BAR EXTENSIONS $\underset{\sim}{e} = \underset{\sim}{F}\underset{\sim}{t} = \begin{bmatrix} 0 \\ -\frac{1}{2} \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \dfrac{WL}{EA} + \begin{bmatrix} 1 \\ 1/2 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dfrac{x_1 L}{EA} + \begin{bmatrix} 0 \\ 1/2 \\ 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \dfrac{x_2 L}{EA}$

USING VIRTUAL WORK $\sum F d = \sum S e$

COMPATIBILITY:

$$\underset{\sim}{S}_1 \underset{\sim}{e} = 0 \Rightarrow \left(-\frac{1}{2\sqrt{2}} + \frac{1}{\sqrt{2}}\right) W + \left(1 + \frac{1}{2\sqrt{2}} + \frac{1}{\sqrt{2}}\right) x_1 + \left(\frac{1}{2\sqrt{2}} - \frac{1}{\sqrt{2}}\right) x_2 = 0$$

$$\underset{\sim}{S}_2 \underset{\sim}{e} = 0 \Rightarrow \left(-\frac{1}{2\sqrt{2}} - 1 - \frac{1}{\sqrt{2}}\right) W + \left(\frac{1}{2\sqrt{2}} - \frac{1}{\sqrt{2}}\right) x_1$$

$$+ \left(\frac{1}{2\sqrt{2}} + \frac{2}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} + 1\right) x_2 = 0$$

$$\therefore \underset{\sim}{S}_1 \underset{\sim}{e} = \frac{1}{2\sqrt{2}} W + \left(1 + \frac{3}{2\sqrt{2}}\right) x_1 - \frac{1}{2\sqrt{2}} x_2 = 0$$

$$\underset{\sim}{S}_2 \underset{\sim}{e} = \left(-1 - \frac{3}{2\sqrt{2}}\right) W - \frac{1}{2\sqrt{2}} x_1 + \left(2 + \frac{7}{2\sqrt{2}}\right) x_2 = 0$$

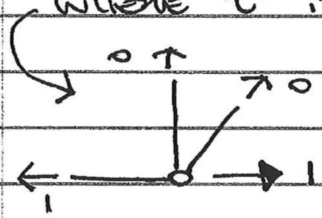
$$\therefore 2.061\, x_1 - 0.354\, x_2 = -0.354\, W$$
$$-0.354\, x_1 + 4.475\, x_2 = 2.061\, W$$

SOLVING SIMULTANEOUS EQNS : $x_1 = -0.093$
$x_2 = 0.453$

$$\underset{\sim}{t} = \underset{\sim}{t}_0 + x_1 \underset{\sim}{S}_1 + x_2 \underset{\sim}{S}_2$$

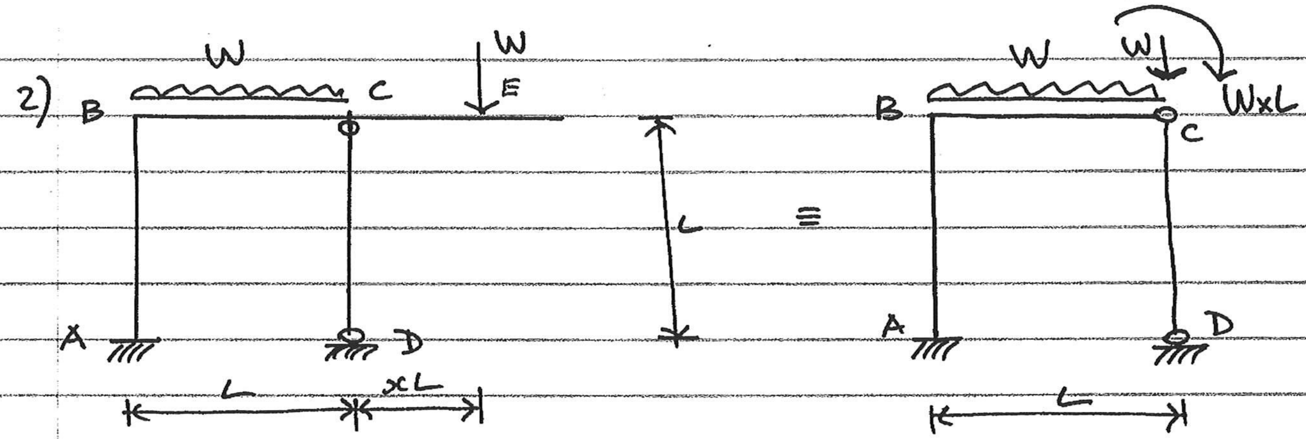
$\therefore t_{I} = -0.093\, W$
$t_{II} = -0.453\, W$
$t_{III} = 0.641\, W$
$t_{IV} = 0.547\, W$
$t_{V} = -0.321\, W$
$t_{VI} = 0.453\, W$

b) iv) VIRTUAL WORK $\delta_{\text{REQUIRED}} = t^* e$
WHERE t^* IS A VIRTUAL EQUILIBRIUM SYSTEM.



$\therefore \delta =$ HORIZONTAL EXTENSION OF BAR $\overline{VI}$
$= 0.453\, WL/EA$

This was a very popular and generally well answered question. All candidates were able to show that the structure had two redundancies as required in part (a). Most candidates successfully derived the particular equilibrium solution (bi) and the corresponding states of self stress in the structure (bii), but more candidates than expected committed errors in resolving the forces at the joints. Most candidates used the appropriate force method for finding the elastic solution (biii), and most were successful in assembling the flexibility matrix and applying virtual work to derive the two simultaneous equations. Several candidates lost some marks due to algebraic errors in assembling and solving the simultaneous equations. Some candidates were unable to determine the horizontal displacement required in (biv). The style and format of this question was one that the candidates were very familiar with. The most successful candidates were the ones that were proficient in resolving the forces in parts (bi) and (bii) thereby giving themselves sufficient time to assemble and resolve the simultaneous equations correctly and complete the question in good time.



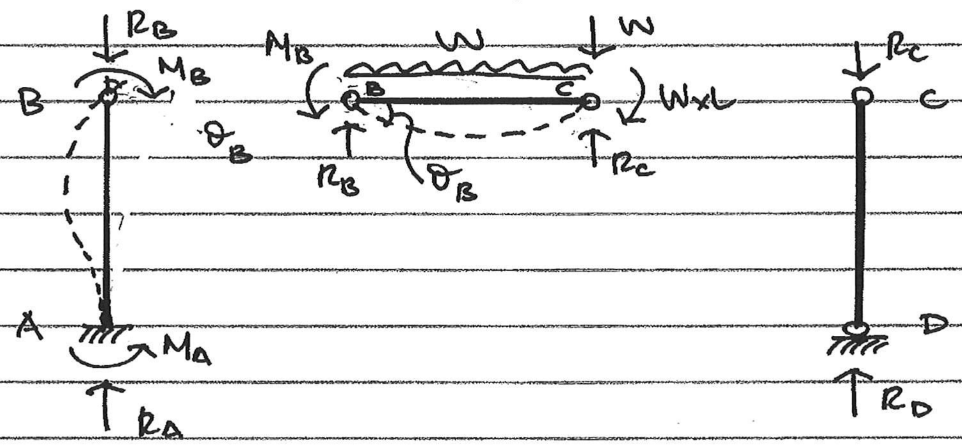
2)a) INTRODUCING ONE HINGE AT B RELEASES ONE D.O.F. (ROTATION) MAKING THE FRAME STATICALLY DETERMINATE

(2)

$\therefore$ ONE REDUNDANCY

2)b)i) DETERMINE x FOR $R_A = 0$

(4)



$\underline{AB}$: $\theta_B = M_B L / EI$

$\underline{BC}$: $\theta_B = -\dfrac{M_B L}{3EI} + \dfrac{WL^2}{24EI} - \dfrac{WxL^2}{6EI}$

COMPATIBILITY OF ROTATIONS AT B:

$$\frac{4M_B L}{3EI} = \frac{WL^2}{24EI} - \frac{WxL^2}{6EI}$$

$$M_B = \frac{3WL}{4}\left(\frac{1}{24} - \frac{x}{6}\right)$$

(2)

$$\therefore \quad M_B = \frac{WL}{32}(1 - 4x) \qquad -(1)$$

$\circlearrowleft$ ABOUT C FOR BC:

$$R_B L - M_B - \frac{WL}{2} + WxL = 0$$

BUT $R_B = R_A = 0$ (ZERO VERTICAL REACTION AT A).

$$\therefore \; M_B = WxL - WL/2$$

SUBSTITUTE FOR M_B FROM (1):

$$\frac{WL}{32}(1-4x) = WxL - \frac{WL}{2}$$

$$\frac{WL}{32} + \frac{WL}{2} = WxL + \frac{WxL}{8}$$

(2)
$$\frac{17WL}{32} = \frac{9WxL}{8}$$

$$\therefore \; \underline{\underline{x = \frac{17}{36}}}$$

2)b)ii) REACTION FORCES AT A AND D:

SUBSTITUTE $x = 17/36$ INTO (1):

$$M_B = \frac{WL}{32}\left(1 - 4\cdot\frac{17}{36}\right) = M_A$$

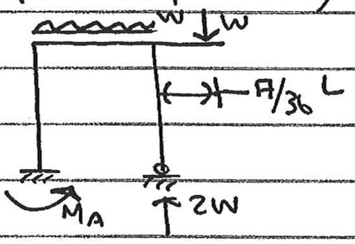
$$= \frac{WL}{32}\left(-\frac{32}{36}\right)$$

(2) $\therefore \; \underline{\underline{M_A = -\frac{WL}{36}}}$

(2) $R_A = 0$

$\underline{\underline{R_D = 2W \text{ (VERTICAL EQUILIBRIUM.)}}}$

ALTERNATIVE SOLUTION (FREE BODY DIAGRAM):



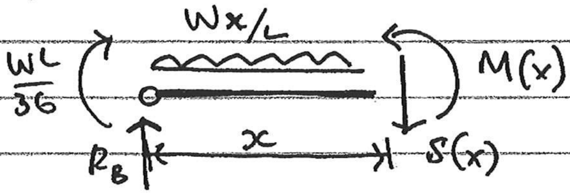
MTS ABOUT A:

$$-M_A + \frac{WL}{2} + \frac{53WL}{36} - 2WL = 0$$

$$\therefore \; M_A = \frac{WL}{36}(53-54)$$

$$= -\frac{WL}{36}$$

2)b)iii) CONSIDER SHEAR FORCE $S(x)$ ALONG BC:



$$S(x) = R_B - Wx/L$$

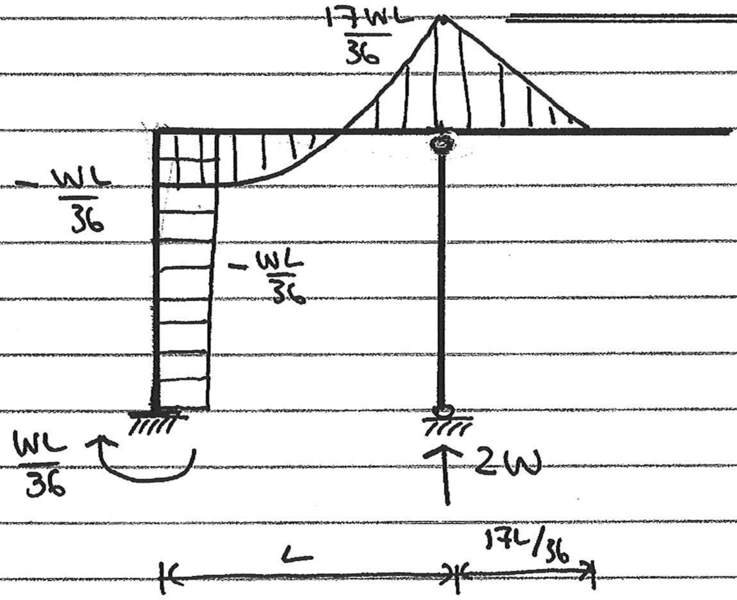
BUT $R_B = 0$

$$\therefore S(x) = -Wx/L$$

$M(x)$ is MAX. WHEN $\frac{dM}{dx} = S = 0$

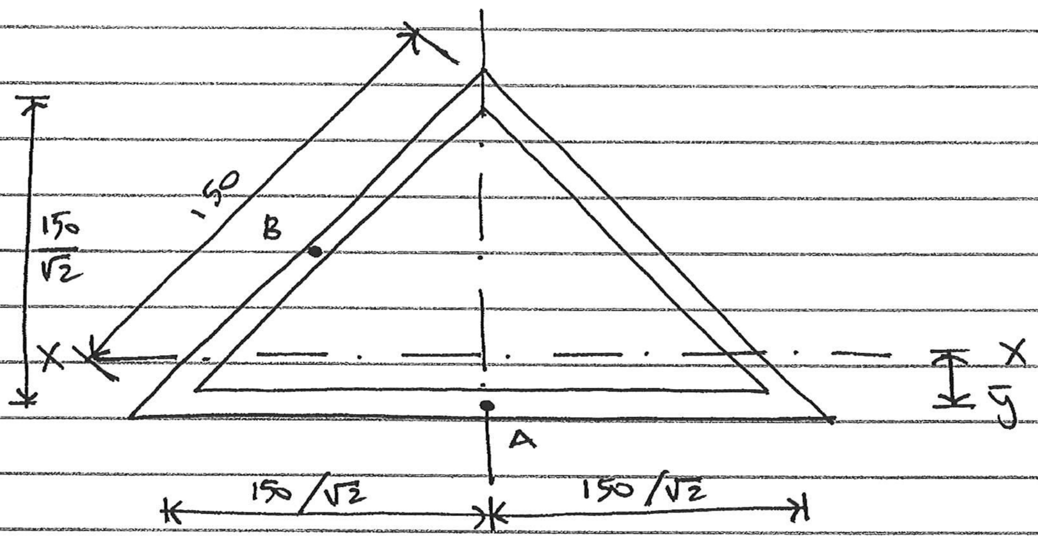
(2) $\therefore$ WHEN $x = 0 \Rightarrow M(x) = M_A = M_B = -WL/36$

(4)



This was the least popular question and was poorly answered by around 75% of the candidates that attempted it, but very well answered by the remaining 25% of the candidates. Most candidates were able to determine that the structure had one redundancy. Several candidates did not proceed to resolve the rest of the question which explains the several low marks recorded. Some candidates who attempted parts (bi) and (bii) determined the reactions incorrectly by considering a free body diagram of the entire frame and in doing so implicitly ignored the effect of the redundancy on the distribution of forces within the frame structure (i.e. assumed node B to be a pin connection). The remaining candidates realised that the structure could be split into free body diagrams of its constituent parts that could be described by standard data book cases and resolved by applying compatibility of rotations or displacements. Most candidates that attempted b(iii) gained some marks, this included some students who failed to solve the previous sections correctly but were able to sketch the basic shape of the bending moment diagram.
Despite appearing in some past exam and examples papers, the style and format of this question was less familiar to the candidates. This question was less laborious than the other elasticity questions. The main challenge was to identify a correct approach for solving the question.

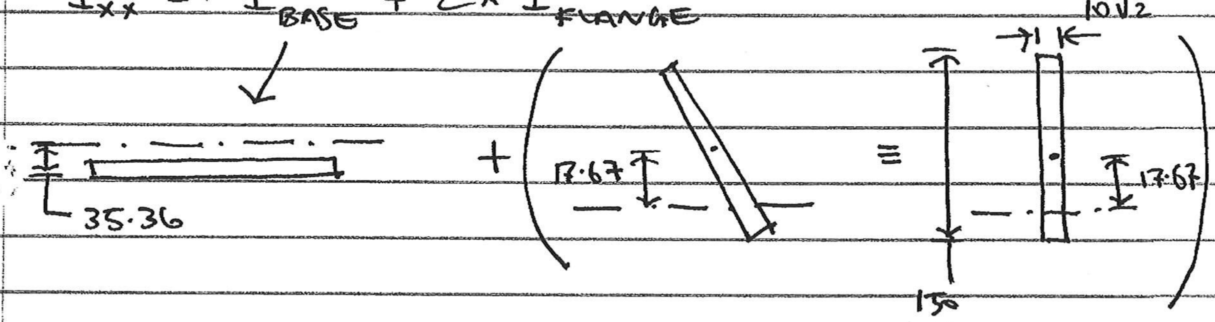
3)



IGNORING THICKNESS OF SECTION FOR $\bar{y}$ AND I_{xx}:

$$\bar{y} = \frac{h}{3} = \frac{150}{\sqrt{2}} \cdot \frac{1}{3} = 35.36\,\text{mm}.$$

$$I_{xx} = I_{BASE} + 2 \times I_{FLANGE}$$



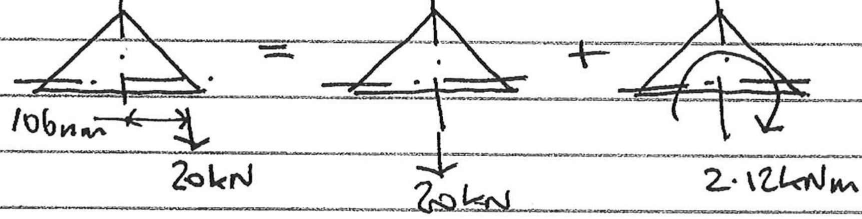
$$= \overbrace{150\sqrt{2} \times 10 \times (35.36)^2}^{I_{BASE}} + \overbrace{2\left[10\sqrt{2} \times \left(\frac{150}{\sqrt{2}}\right)^3 \times \frac{1}{12} + 10\sqrt{2} \times \frac{150}{\sqrt{2}} \times (17.67)^2\right]}^{I_{FLANGES}}$$

$$\therefore I_{xx} = 2.652 \times 10^6\,\text{mm}^4 + 3.749 \times 10^6\,\text{mm}^4$$
$$= 6.401 \times 10^6\,\text{mm}^4$$

$$\sigma = My/I$$

$$\therefore \sigma_A = -20 \times 10^6\,\text{Nmm} \times 35.36\,\text{mm} / 6.4 \times 10^6\,\text{mm}^4 = \underline{-110.5\,\text{N/mm}^2}$$
$$\sigma_B = -20 \times 10^6\,\text{Nmm} \times -17.67\,\text{mm} / 6.4 \times 10^6\,\text{mm}^4 = \underline{\underline{55.2\,\text{N/mm}^2}}$$

a) ii)



VERTICAL SHEAR FORCE AT SUPPORT $S = 20\text{kN}$

TORQUE AT SUPPORT $T = 2.12\text{kNm}$

AT A

SHEAR DUE TO VERTICAL SHEAR FORCE $\tau_s \simeq 0$

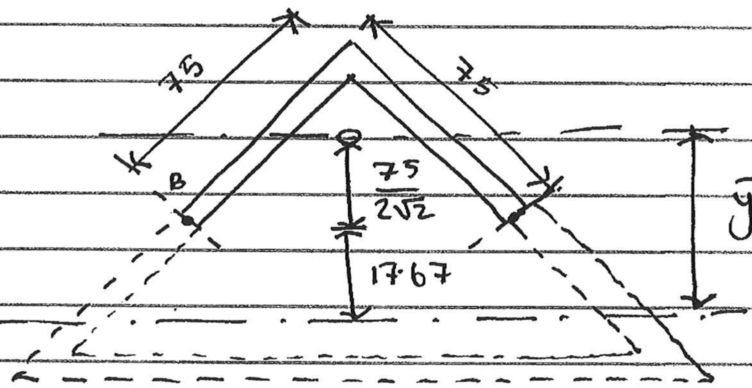
SHEAR DUE TO TORQUE $\tau_T = q/t = \dfrac{T}{2A_e t}$

$$= \frac{2.12 \times 10^6}{2 \times \left(150/\sqrt{2}\right)^2 \times 10}$$

$$= 9.4\ \text{N/mm}^2$$

AT B

$$\tau_s = \frac{SA\bar{y}}{It}$$



$A = 75 \times 10 \times 2 = 1500\ \text{mm}^2$

$\bar{y} = \dfrac{75}{2\sqrt{2}} + 17.67 = 44.2\ \text{mm}$

$$\therefore \tau_s = \frac{20 \times 10^3\text{N} \times 1500\text{mm}^2 \times 44.2\text{mm.}}{6.401 \times 10^6\text{mm}^4 \times 10\text{mm.}} = 20.7\ \text{N/mm}^2$$

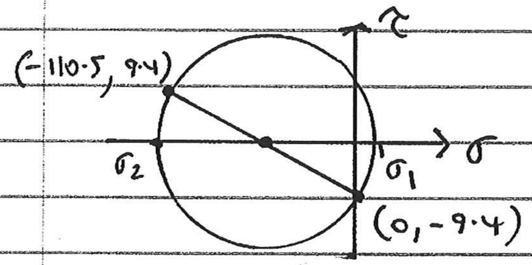
$\tau_T = 9.4\ \text{N/mm}^2$ (NOTE THAT THIS IS THE OPPOSITE DIRECTION w.r.t τ_s AT B!!)

$\therefore$ AT A, $\tau_A = 9.4\,N/mm^2$

AT B, $\tau_B = 20.7 - 9.4 = \underline{\underline{11.3\,N/mm^2}}$

b) BY INSPECTION (σ_A, τ_A VS. $\sigma_B\, \tau_B$) LARGEST PRINCIPAL STRESSES OCCUR AT A.

$\therefore$ MOHR'S CIRCLE AT A:



$$\sigma_2 = -\frac{110.5}{2} - \sqrt{\left(\frac{110.5}{2}\right)^2 + (9.4)^2}$$

$$= -111.3\,N/mm^2$$

$$\sigma_1 = -\frac{110.5}{2} + \sqrt{\left(\frac{110.5}{2}\right)^2 + (9.4)^2}$$

$$= 1.04\,N/mm^2$$

$$\sigma_3 = 0$$

TRESCA $\Rightarrow \lambda(\sigma_1 - \sigma_2) = Y$

$$112.34\lambda = 275$$

$$\therefore \underline{\underline{\lambda = 2.45}}$$

VON MISES $\Rightarrow (112.34^2 + 111.3^2 + 1.04^2)\lambda^2 = 2 \times 275^2$

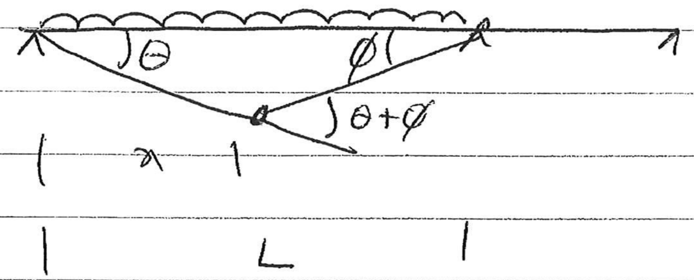
$$25009\,\lambda^2 = 151250$$

$$\therefore \underline{\underline{\lambda = 2.46}}$$

This was a moderately well answered question where most candidates were able to identify the correct approach, but there were several errors and omission sin some of the answers provided. There were several mathematical errors in the calculation of the second moment of area of the triangular hollow section required for part (ai) and most candidates seemed to spend a disproportionately large amount of time on this part of the question. Some candidates realised that the applied forces could be described by superimposing two simple cases in part (aii) i.e. a simple cantilever with a central vertical point load and a single torque applied at the free end of the cantilever. However several candidates ignored the flexural or the torsional contributions to the shear stresses at points A and B. Most candidates that attempted part (b) were able to plot Mohr's circle and apply the Tresca and von Mises yield criteria correctly.

The style and format of this question were very familiar. The principal variation was the triangular cross section. More candidates than expected ignored the flexural or the torsional components of the stress, but the application of Mohr's circle and yield criteria was generally good.

4 (a)



$$\theta x = \phi (L - x) \qquad \theta = \phi \frac{(L-x)}{x}$$

$$M_p (\theta + \phi) + M_p \phi = \frac{wL}{2} \cdot \frac{\theta x}{2}$$

$$M_p \cancel{\phi} \left(\frac{(L-x)}{x} + 2 \right) = wL \cancel{\phi} \frac{(L-x)}{2}$$

$$\Rightarrow \frac{wL}{2M_p} = \left(\frac{(L-x)}{x} + 2 \right) \cdot \frac{1}{(L-x)} = \frac{(L+x)}{x(L-x)} \quad \left(= \frac{u}{v} \right)$$

Need $\frac{dw}{dx} = 0 \Rightarrow \frac{u'v - uv'}{v^2} = \frac{x(L-x) - (L+x)(L-2x)}{(x(L-x))^2} = 0$

when $x(L-x) - (L+x)(L-2x) = 0$

$$\Rightarrow x^2 + 2xL \cancel{4} - L^2 = 0$$

$$\Rightarrow x = 0.4142\,L = \underline{\underline{4.142\,m}}$$

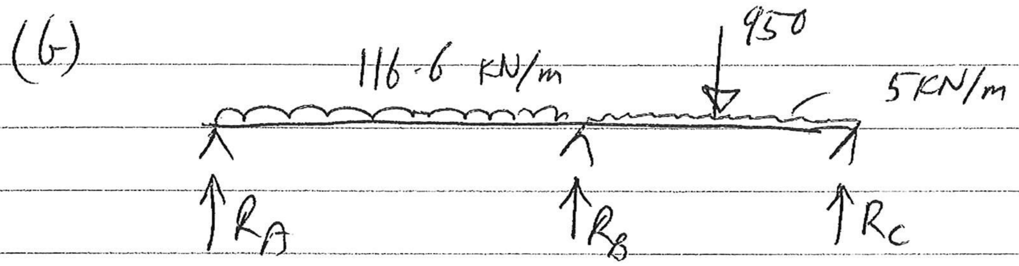
$$\Rightarrow \frac{wL}{2M_p} = \frac{(L+x)}{x(L-x)} = 0.5828$$

$$\Rightarrow w = \frac{2\,M_p.\,0.5828}{10} = 116.6\ kN/m$$

Self weight $= 5\ kN/m$ $\therefore$ $111.6\ kN/m$ for live load

$$\therefore \lambda = \frac{111.6}{50} = \underline{\underline{2.232}}$$

(b)



We know that M at B = 1000 kNm (hogging)

$\therefore\ 116.6 \cdot \frac{10^2}{2} - R_A \cdot 10 = 1000 \quad \Rightarrow \underline{\underline{R_A = 483\text{ kN}}}$

[Check at $x = 4.142$ M should be 1000 kNm (sagging)
$483 \cdot 4.142 - 116.6 \frac{(4.142)^2}{2} = 1000$ (OK)]

Similarly $\frac{5 \cdot 6^2}{2} + 950 \cdot 3 - R_c \cdot 6 = 1000$

$\Rightarrow \underline{\underline{R_c = 323\text{ kN}}}$

Total load $= R_A + R_B + R_C = 116.6 \cdot 10 + 950 + 5 \times 6$

$\Rightarrow \underline{\underline{R_B = 1340\text{ kN}}}$

(c) This is an upper bound. However, we can check the moment under the point load

$323 \cdot 3 - \frac{5 \cdot 3^2}{2} = 969 - 22.5 = \underline{\underline{946.5}}$

This is less than M_p

$\therefore$ The beam has a set of moments that is in equilibrium with the loads and nowhere exceeds M_p

$\therefore$ It also satisfies the lower bound theorem

$\therefore$ It is the exact collapse load. No need to check any other case.

4. This question was done surprisingly badly; what is even more worrying, most of the candidates probably thought they got it right.

Part (a) was a basic collapse analysis of a beam under ud load. Quite a lot of candidates got the basic mechanism right, but then didn't try to find the worst case, or if they did gave up because they thought the differentiation (of a fraction, which leads to a quadratic) was too hard, or if they tried it they got it wrong. Many guessed that the critical case was with the sagging hinge in the centre; others justified it by saying that the loading is symmetrical so the collapse mechanism is symmetrical (it isn't true because the support conditions at A and B are different – A is a simple support, B is a continuous support). Many others put a plastic hinge (or assumed that energy was being dissipated) at the simple support at A, which has the effect that the critical case IS with the hinge in the centre. Others assumed that the self weight could be taken as a point load half way along the beam. The result was that less than 10% of the candidates got this basic analysis right.

Part (b) was done even worse. The correct solution takes account of the fact that the bending moment must be M_p at the plastic hinges, in particular at B. So it is possible then to calculate the support reactions at A and C by simple moment equilibrium – a one-line calculation. The reaction at B can then easily be found from force equilibrium. There were two sorts of answers by most candidates. One group treated the beam (incorrectly) as a statically indeterminate elastic beam and tried to analyse it using data book coefficients; even if they did this correctly they of course ended up with the wrong answer. The other group simply tried to write down three equations of overall equilibrium. However, there are ONLY two equilibrium conditions that can be applied (vertical force and moment equilibrium) – taking moments about different places does NOT generate new information. They performed the miraculous achievement of being able to solve two equations for three unknowns – next time I have to feed 5000 people with 5 loaves and two fishes I will know who to go to for assistance.

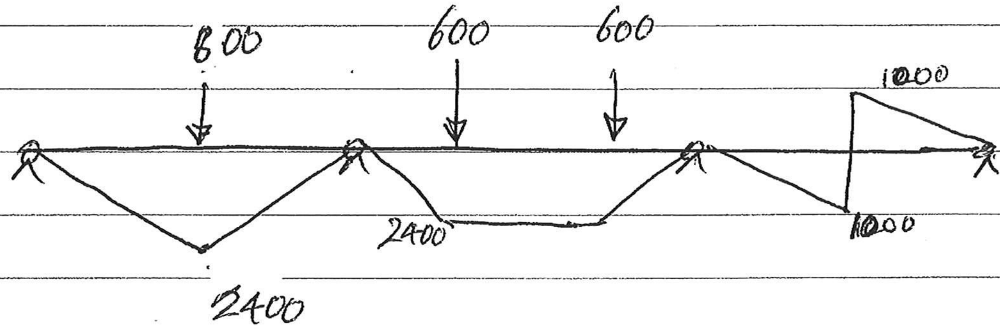
Part (c) was meant to sort out who really knew what they were talking about (answer, no one). I was expecting most people to say that they had calculated an upper bound, which is what happened. The better candidates said that they would need to do a lower-bound solution by checking the equilibrium state, but none realised that this is precisely what they had (or at least should have) done in section (b), so no one got the last couple of marks I was reserving for the best candidates.

Most of the marks awarded in this question were to odd things done correctly in the middle of a mass of nonsense.

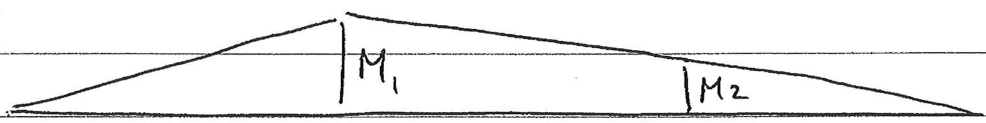
5 (a) Lower bound theorem states that if any set of internal forces (moments etc) can be found which nowhere exceeds the yield condition and ~~the~~ is in equilibrium with the applied loads, then the ~~[illegible]~~ loads are a lower bound on the true collapse load.

(b) Any equilibrium state can be expressed as a set of free B.M diagrams and a reactant diagram.

Fixed



Variable

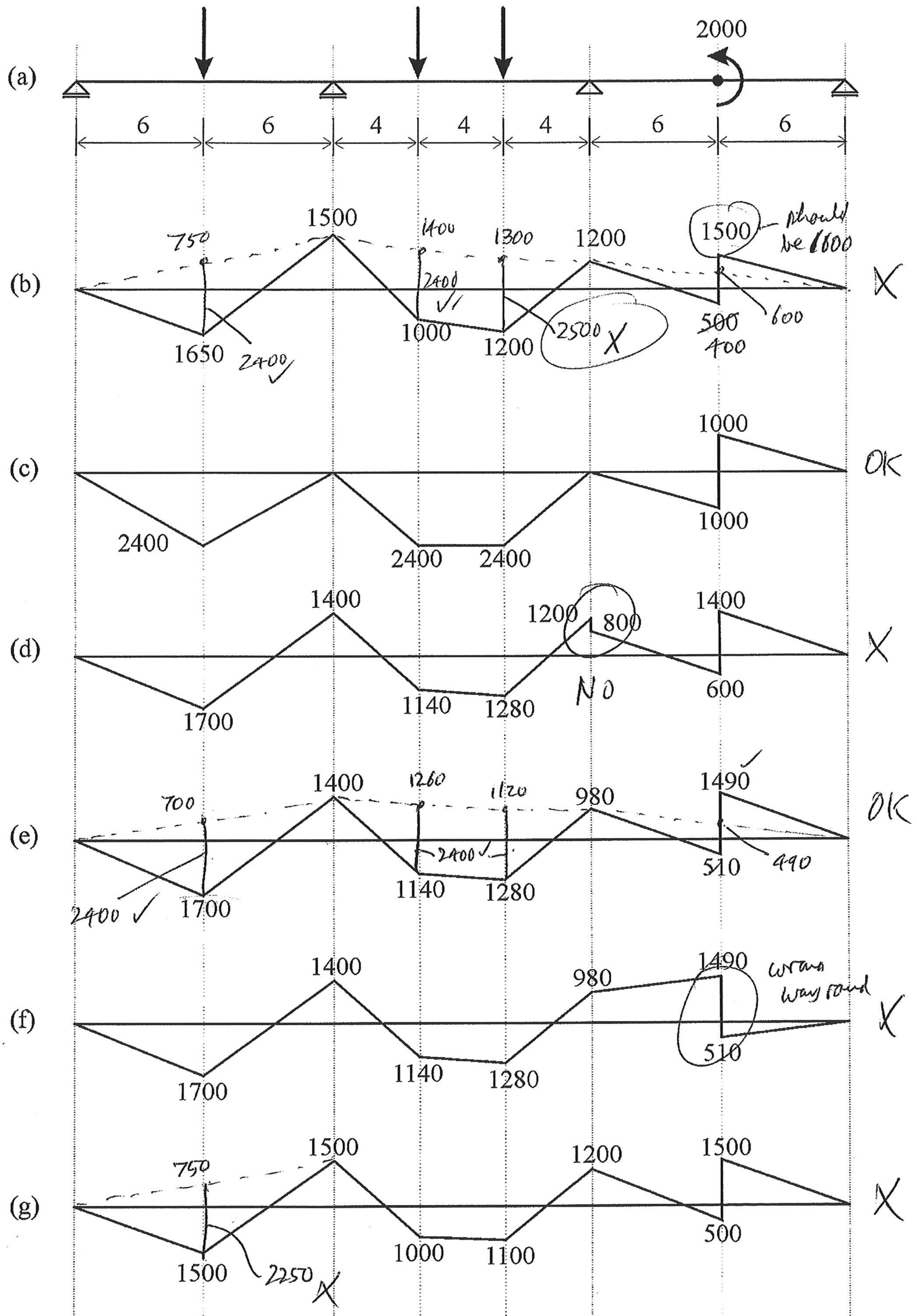


(c) & (e) OK — all others wrong (see next sheet).

(c) For load case (c) loads can be increased by $\frac{3000}{2400} = 1.25$

For load case (e) loads can be increased by $\frac{3000}{1700} = 1.76$

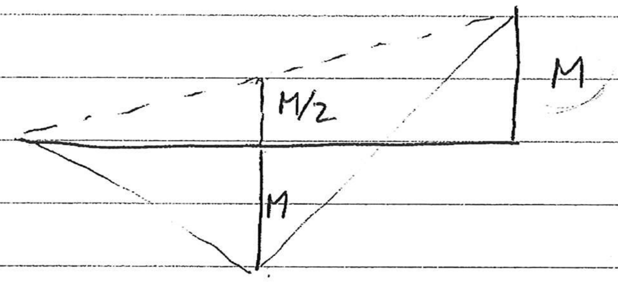
IB/2010/P2/5/2



(all dimensions in m, loads in kN, moments in kNm)

1B/2010/P2/5/3.

(d) Moment range will be critical in L.H span



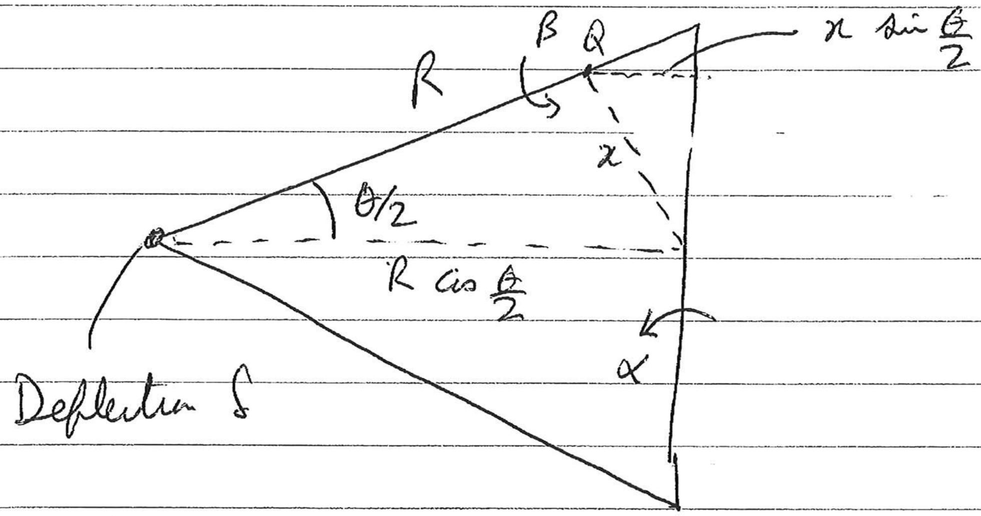
If $\frac{3M}{2} = 2400$ then we will have same moment in hogging and sagging

$\therefore M = 1600$

$\therefore$ Load factor at collapse would be $\frac{3000}{1600} = \underline{\underline{1.875}}$

Moment at R.H support will not be critical provided moment nowhere exceeds 1600.

5. Lower bound question done reasonably well by many candidates. In part (a) they were asked to quote the theorem, not to summarise how it should be employed. It can be expressed in various ways but they had to include "equilibrium everywhere" and "yield nowhere" to get full marks. They were given 6 potential bending moment diagrams to see if these satisfied the conditions of the lower bound theorem. The objective was to see which of the potential diagrams was in equilibrium with the loads, NOT to see which had moments less than M_p. As it happens, all did, but it would have been quite possible to suggest a set of moments for which this wasn't true (in which case the load factor would have been less than one). Several candidates talked about a set of bending moments being "compatible with the loads"; beware of using "compatible" in this way – it has a more specific meaning in structural engineering than in common parlance. Many were confused by bending moment diagram (c): it is the bending moment diagram you get if there are no reactant moments, and thus it IS in equilibrium with the applied loads, so can be used to get a lower bound estimate of the collapse load. It will give a fairly low estimate, since you would expect there to be some hogging moments there, but it is still valid. There also seemed much confusion about what the load factor was; for any BM diagram that satisfies equilibrium, it is simply the ratio between M_p and the maximum moment in the diagram. Many candidates did quite complicated calculations that I could not follow. The last part was done badly; most assumed that if you picked the reactant moments so that the peak moment was at M_p then you had the maximum load, but in reality you should pick the reactant moments such that the maximum sagging and maximum hogging moments were equal (and thus as small as possible). All the loads can then be multiplied by the biggest load factor before collapse would occur.



$$\delta = \alpha \, . \, R \cos \frac{\theta}{2}$$

Compatibility at Q requires

$$\alpha \, . \, \cancel{x} \sin \frac{\theta}{2} = \beta \, . \, \cancel{x}$$

$\therefore$ Work done by load $= P \, . \, \delta$

Work done in edge hinge

$$= 2 \, M \, . \, \alpha \, . \, R \sin \frac{\theta}{2}$$

Work done in edge hinges

$$= 2 \, . \, m \, . \, \beta \, . \, R \quad = \quad 2 m \alpha \sin \frac{\theta}{2} \, . \, R$$

$$P . \cancel{\alpha} \cancel{R} \cos \frac{\theta}{2} = n \left(4 m \cancel{\alpha} \cancel{R} \sin \frac{\theta}{2} \right) \cancel{\tfrac{1}{2}}$$

$$P = 4 \, m \, n \, \frac{\sin \frac{\theta}{2}}{\cos \theta/2}$$

But $\theta = \frac{2\pi}{n}$

$\therefore P = 4mn \tan \pi/n$

Calculate some values.

n	$n \tan \pi/n$
3	5.196
4	4
5	3.63
10	3.25

Reducing $\therefore$ Large n is best

∞ $\tan \pi/n \approx \pi/n$ $\therefore n\frac{\pi}{n} = \pi$.

$\therefore P = 4m\pi$.

6. Yield line question. Proves the old adage that it is impossible to set a question that is too easy! There was a worrying inability to do trigonometry; many candidates used Pythagoras or the cosine rule when trying to work out the length of the odd side in an isosceles triangle. It harks back to the old A level approach of asking "what is the formula for ..." instead of thinking. It isn't wrong but it leads to equations in terms of square roots and terms involving both cos(θ) and cos(θ/2) that just get complicated, and these lead to other errors. A simple sin(θ/2) is all that is needed anywhere. Many assumed that because their answer didn't involve R it must be wrong, when a moment's thought (which good candidates expressed) was that both the work done and the energy dissipated vary linearly with R, so it should cancel. Some said that the correct answer was dimensionally incorrect, probably forgetting that m_p is a moment per unit length, so has dimensions of force. Many tried to find the optimum value of n by differentiating with respect to it, but it is an integer and you can only differentiate continuous functions. One of the biggest problems however was the inability of candidates to say what they were doing. There were many random equations written down, often illegibly, using variables that weren't defined and involving principles that weren't stated.