CRIBS
SECTION A
Question 1
(a) (i) The analysis consists of developing the linear differential equation relating the overall stress and strain in the model and then solving the equation for particular inputs of stress or strain, as required.

Equilibrium and compatibility give the following relationships:

$$\sigma = \sigma_1 = \sigma_2 + \sigma_3 \tag{1}$$
$$\varepsilon = \varepsilon_1 + \varepsilon_2 \tag{2}$$

The subscripted terms can be expressed in terms of σ and ε as follows:

$$\sigma = \sigma_1 \qquad \varepsilon_1 = \sigma/E_1 \qquad \varepsilon_2 = \varepsilon - \sigma/E_1 \tag{3}$$

$$\sigma_2 = \varepsilon_2 E_2 = E_2 \varepsilon - E_2 \sigma/E_1 \tag{4}$$

$$\sigma_3 = \sigma - \sigma_2 = \sigma - E_2 \varepsilon + E_2 \sigma/E_1 \tag{5}$$

$$\frac{d\varepsilon_3}{dt} = \frac{\sigma\left(1 + E_2/E_1\right) - \varepsilon E_2}{\eta_3} \tag{6}$$

If the Voigt sub-system is to keep together, $\dot{\varepsilon}_3 = \dot{\varepsilon}_2$, i.e., from eqn (3)

$$\frac{d\varepsilon_3}{dt} = \frac{d\varepsilon_2}{dt} = \frac{d\varepsilon}{dt} - \frac{1}{E_1}\frac{d\sigma}{dt} \tag{7}$$

Equating (6) and (7) and re-arranging, gives the general relationship between applied stress and overall strain:

$$\frac{d\varepsilon}{dt} + \frac{E_2}{\eta_3}\varepsilon = \frac{E_1 + E_2}{\eta_3 E_1}\sigma + \frac{1}{E_1}\frac{d\sigma}{dt} \tag{8}$$

This equation can now be solved for particular inputs of stress or strain as the following example shows for creep.

(ii) Creep compliance

Putting $\sigma = \sigma_o$ = constant, $d\sigma/dt = 0$ so (8) becomes

$$\frac{d\varepsilon}{dt} + \frac{E_2}{\eta}\varepsilon = \frac{E_1 + E_2}{\eta E_1}\sigma_o \qquad (9)$$

This can be solved to give:

$$\varepsilon(t) = \frac{\sigma_o}{E_1} + \frac{\sigma_o}{E_2}\left[1 - \exp\left(-\frac{E_2 t}{\eta}\right)\right] \qquad (10)$$

In terms of creep compliance $C(t) = \varepsilon(t)/\sigma_o$ we get:

$$C(t) = \frac{1}{E_1} + \frac{1}{E_2}\left[1 - \exp\left(\frac{E_2 t}{\eta}\right)\right] \qquad (11)$$

At very short time (t = 0), the exponential term tends to unity and hence $C(t)$ tends to 1/E. At the other end of the time scale, as t tends in infinity, the exponential term disappears and hence $C(t) = \frac{1}{E_1} + \frac{1}{E_2}$.

(iii) This kind of model is inappropriate to account for yield stress because there is no built-in physical mechanism(s) and the yield behaviour of polymers is complex.

(b) The tensile hoop stress σ in the wall of a thin-walled pipe of radius R is given by pR/t where p is pressure and t is wall thickness. A safety factor of 2 leads to a minimum wall thickness of about **4mm.**

Why is this value roughly one-half that demanded by BS 3505? The point is that the yield strength depends upon temperature and on the timescale of loading. The British Standard describes behaviour of the polymer in a simple tensile test lasting a few minutes at ambient temperature. Over 50 years, the tensile strength will fall and hence the increase in wall thickness requirement. (Had the pipe been made from HDPE, the difference between the short and long-term strength would be about 3 orders of magnitude, not the several orders as for the PVC pipe).

Comments:
Straightforward manipulation of equations that describe the extension of elastic springs and viscous dashpots under stress arranged to show the time-dependent behaviour of a standard linear elastic solid under stress. Most students could not say why this model could not account for the yield behaviour of polymers. Calculation of minimum wall thickness of a plastic pipe under stress was done without difficulty. In explaining why British Standards is conservative in its specification of a minimum thickness, some students ignored the fact their calculation did not take into account fluctuations in temperature on the one hand and 50 years service on the other. In other words, creep of the pipe and a corresponding decrease in yield stress of the polymer over time demands the pipe's wall thickness be doubled over that determined from consideration of simple stress-strain behaviour of the polymer in monotonic tensile loading.
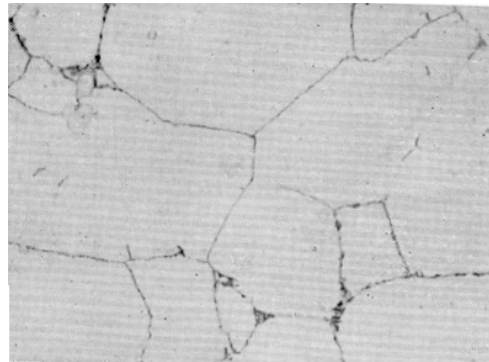
Question 2

(a) **(i)** Casting has a microstructure consisting of grains of primary ferrite (a) and grains of pearlite (b) (lamellae of ferrite and cementite Fe_3C platelets) in about 50:50 proportion.


(a)

(b)
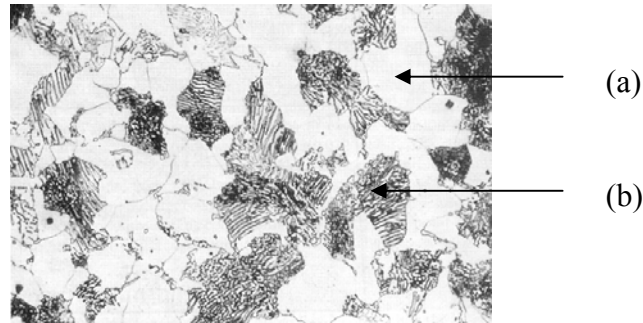
("as-cast") 0.4 wt% carbon steel

(ii) The casting is heated so that the steel becomes entirely austenitic.

100% austenite at 850 ^{o}C

It is then quenched thereby preventing the carbon from forming cementite and precipitating within the austenite grains; likewise for the primary ferrite. The supersaturated austenite undergoes an athermal transformation process (diffusionless) to produce a supersaturated solution of martensite, a metastable non-equilibrium phase in which the carbon atoms are contained in solution. Thus, this non-equilibrium structure (body-centred tetragonal) does not exist on the equilibrium phase diagram.

(iii) Subsequent heat-treatment allows the carbon to diffuse out of solution in the form of a fine precipitate of iron carbide (Fe_3C) within grains of ferrite transformed from the body-centred tetragonal structure.
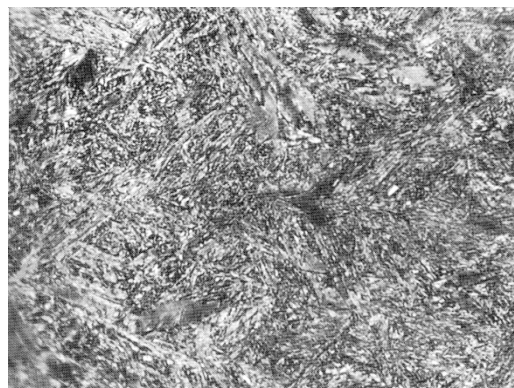
Very fine precipitate of Fe_3C particles dispersed uniformed throughout the structure. Too fine to be distinguished at this modest magnification.

(b) From an initial modest hardness and toughness, on quenching the former property increases 3-fold whilst the toughness falls by 2 orders of magnitude or more. Re-heating the casting imparts a combination of good toughness and hardness.

(c) Thermal stresses can be set up where there are variations in cooling rate from a thin section to a thick section of the casting. At stress concentrators like corners, thermal (quench) cracking can occur. If the hardenability of the steel is low, fast cooling is required to produce 100% martensite. However, for a casting of complicated shape fast cooling causes the outside of the part to transform to hard martensite before the inside of the part has a chance to to cool down. Expansion of the interior when martensite eventually forms acts against an already hardened layer and residual stresses set up can cause distortion, even cracking. Therefore parts must be cooled as slowly as possible from the austenitic range e.g., by oil quenching and this requires additions of alloying elements to impart hardenability. Another consequence of variable thickness of component is that the thicker section to be hardened which cools more slowly requires a greater hardenability (ie alloy content) to achieve the desired hardness for a given cooling treatment. Where martensite forms at sections of faster cooling rate compared to other thicker parts, volumetric differences can cause cracking.

Comments:
All students attempting this question recognised it as being directly related to an experiment in the second year Materials Laboratory. Consequently, without exception (almost) the effect of quenching 0.4% carbon steel (austenitic from above 850C) produced a hard, brittle martensitic structure; and tempering changed that structure to a precipitation-strengthened steel of improved yield stress and high toughness. Some (incorrect) answers included a description of the formation of bainite by tempering martensite. Where things went wrong was accounting for possible cracking in a cast component of complex shape and variable thickness of section. Few students recognised that changing the hardenability of the steel by alloying enabled a reduction in cooling (quenching) rate in order to initially produce martensite. Very few students acknowledged that corners and change in section were stress concentrators with the possible initiation of cracks due to volumetric changes during transformation of austenite to martensite and the setting up of thermal stresses.

Question 3

Assumptions made: Continuity of material flow through the rolls and approximating contact between roll and strip as a flat horizontal surface.

For continuity: $h_1v_1 = h_2v_2$ and velocity of each roll is ωR.
At entry to rolls friction-hill is positive: v_1 is less than ωR.
At exit, friction-hill is negative and v_2 is greater than ωR.
At the peak of the friction-hill, roll force is maximum and $v_{strip} = \omega R$.

a) If the friction is high enough, the strip is dragged into the rolls. As the strip is reduced in thickness it elongates and increases its linear speed until at the exit it travels faster than the rolls. The neutral plane relates to the direction of friction and magnitude of velocity of the strip during rolling. Initially, the roll surface is moving faster than the strip entering the rolls, pulled in by friction. At the exit, the reverse is true because due to continuity the strip must be moving faster. Direction of friction must have reversed at some stage in the operation. There is a neutral point within the roll gap at which the surface velocity of the strip equals the peripheral velocity of the rolls, there is no slip and the direction of the friction force reverses. This is an important feature of rolling. Where friction becomes zero and surface velocity of the rolls equals that of the strip, that point is called the neutral plane. Here, pressure is a maximum and the normal stress on the strip resembles a friction-hill (force per unit length for a wide strip).

(b) The pressure distribution shows a "friction hill" (fig. below).
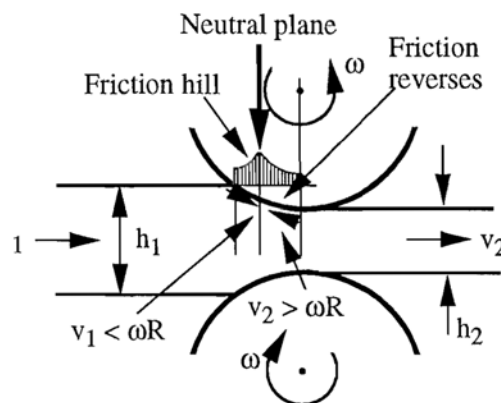
This relationship between friction and roll load arises because the frictional contribution to the longitudinal stress increases with distance inwards from entry and exit points, and this provides increasing resistance to the expansion of vertical sections under the vertical loads. The roll force required to produce a given deformation is thus increased by the presence of a longitudinal frictional hill.

There are 2 zones of slipping friction, near the entry and exit of the roll gap. These are separated by a region of sticking friction in which the strip surface moves with the roll surface.

(c)   In well-lubricated rolling the friction is low and the process can be considered as essentially a compression process. Although strain and therefore yield stress of the material increases from entry to exit of the roll gap, as a first approximation ignore work hardening effects and assume an average yield stress σ_y and assume friction is zero. Now consider the relationships between the geometrical variables indicated in Fig.2. The rolls are usually

sufficiently large that their curvature can be ignored. From the geometry shown in Fig. 2, this gives a simple relationship between roll radius R, distance L, and "roll bite" χ where $\chi = (h_1 - h_2)/2$. Hence:

$$R^{2=} L^2 + (R - \chi)^2$$

Since $\chi = (h_1 - h_2)/2$ is nearly always small, then

$$L = [R(h_1 - h_2)]^{1/2}$$

Now the rolling force P must be sufficient to cause the metal to yield over the length L and width w (normal to fig. 2). Thus:

$$P = \sigma_y L w$$

$$P = \sigma_y w [R(h_1 - h_2)]^{1/2}$$

$$P/w = \sigma_y [R(h_1 - h_2)]^{1/2} \qquad \textit{(for plane stress).}$$

(d) The roll force and hence roll torque to drive the rolls increases with yield strength of the metal strip so hot-rolling takes less power than cold-rolling. Obviously, it increases with increasing reduction in thickness per pass, and it increases with roll diameter.

More often than not, metals do not work harden rapidly after annealing and in second and third passes. Since surface finish can change, friction conditions may alter. However, in cold rolling work hardening does occur and this will result in the roll force increasing.

Rolls and their housing do deform elastically thereby increasing the roll gap. This gap increases linearly as the roll force increases. This has to be allowed for in calculating rolling loads. There is a minimum thickness gauge that can be rolled. Any attempt to further reduce the thickness increases the roll load and deforms the rolls and housing.

Comments:
Accounting for the neutral plane in the rolling of strip steel was understood. The effect of variation in friction at the roll-strip interface upon the friction hill was equally well explained. Likewise, explanation of the effect of work hardening of steel in repeated passes and changes in operating temperature upon the roll force was straightforward to account for. The problem encountered was accepting that in well-lubricated rolling where friction is low the process can be considered as essentially a compression process. Although strain and hence yield stress of the steel increases from entry to exit within the roll gap, work hardening has to be ignored. And it is simpler to assume an average yield stress of the metal and to consider friction not to exist (even though we know the process is not possible without some friction). Furthermore, ignore curvature of large rolls and then consider geometrical variables only in obtaining an expression for rolling force in terms of yield stress, roll radius and roll gap. The final expression shows roll force and hence roll torque to drive the rolls increases with yield strength of the metal strip so hot rolling takes less power than cold rolling. Also, it increases with increasing reduction in thickness per pass, and it increases with roll diameter.

SECTION B
Question 4
All calculations are based on assuming equilibrium conditions and therefore the lever rule is applicable.
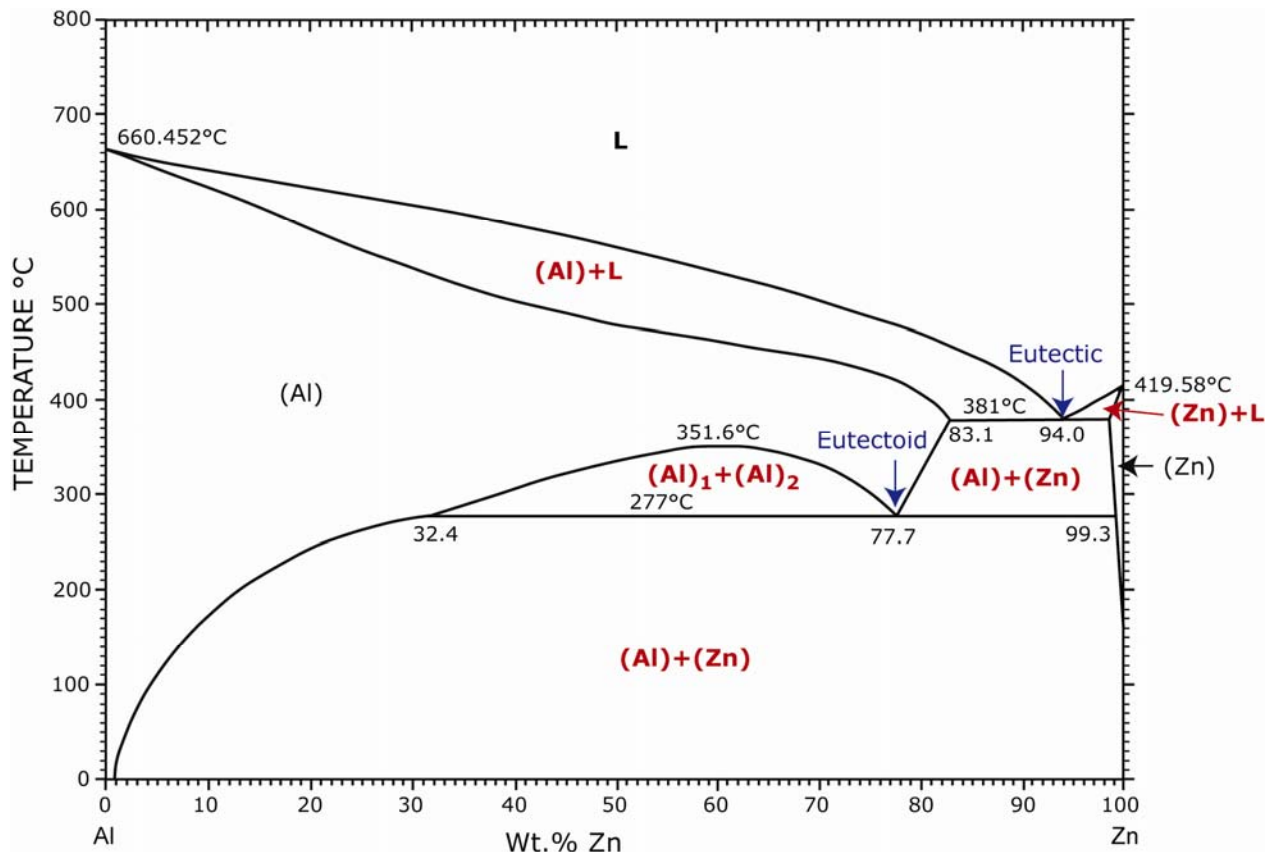**(a)**
(i) A: (Al) + L
    B: (Zn) + L
    C: (Al) + (Zn)
    D: $(Al)_1 + (Al)_2$ (both based on Al, but with different Zn content dissolved in solid solution)
    E: (Al) + (Zn)
The two-phase regions are also labelled red in the phase diagram below

(ii) *Eutectic* reaction at 381°C

L (94% Zn)→(Al) (83.1%Zn)+(Zn) (98%Zn)

Eutectoid reaction at 277°C

(Al) (77.7% Zn)→(Al) (32.4%Zn)+(Zn) (99.3%Zn)

(iii) Al-Zn alloys are "non-heat treatable" alloys ie they do not precipitate harden. This is due to the fact that Zn diffuses very slowly in Al (and vice versa) that precipitation does not occur for practical ageing and times. The two hardening mechanisms for this alloy are : solid-solution hardening and work hardening.

(b) 20 g of Al and 30 g of Zn, thus the alloy contains 40 wt% Al, **60** wt% Zn

(i)   500°C

| Phases	Composition (wt% Zn)	Proportion (wt%)
(Al)	40	$\dfrac{69-60}{69-40}\times100\approx21$
L	69	79

(ii)   300°C

2 phases, both based on Al, but with different Zn content dissolved in solid solution.

Phases	Composition (wt% Zn)	Proportion (wt%)
$(Al)_1$	38	$\dfrac{75-60}{75-38}\times100\approx40$
$(Al)_2$	75	60

(iii) 250°C

Phases	Composition (wt% Zn)	Proportion (wt%)
(Al)	21	$\dfrac{99.5-60}{99.5-21}\times100\approx50$
(Zn)	99.5	50

(b) At 600°C, the Gibbs free energies G of (Al) and liquid overlap. At low Zn content (<15 wt%), $G_{(Al)} < G_{liquid}$ (ie (Al) is stable), whereas at high Zn content (>34 wt%), the liquid phase is stable. The lowest possible free energy is achieved when the mixture is made of (Al) containing 15 wt% Zn and a liquid containing 34 wt% Zn (see schematic below)
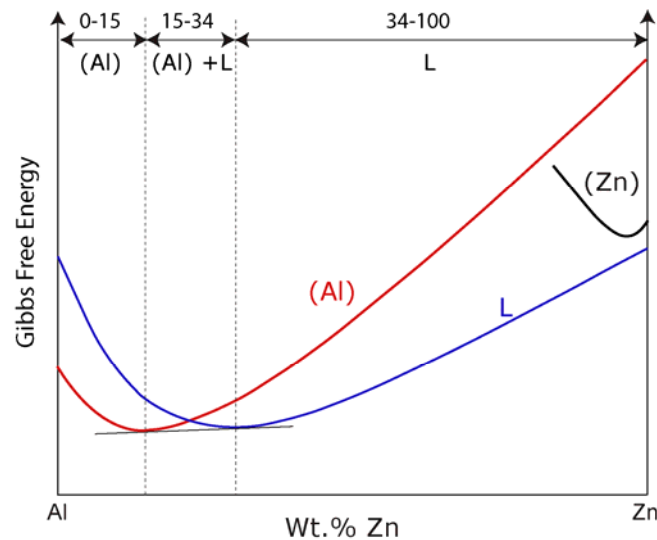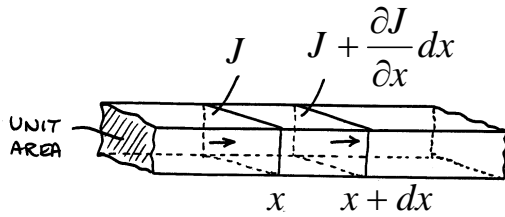

Comments:

This question was generally very well answered. In part (a), quite a few candidates failed to identify that region D contained 2 solid solutions, both based on Al but with different Zn content. Parts (b) and (d) were answered well. A few candidates incorrectly converted grams to moles, which led to incorrect weight fractions. The main shortcoming of this question was part (c). A significant number of candidates had difficulty in plotting correctly the Gibbs free energy as a function of composition.

Question 5

<table>
<tr><td align="center">mass diffusion</td><td align="center">thermal diffusion</td></tr>
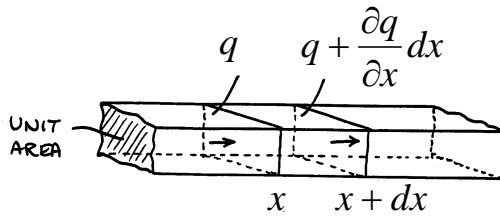</table>

Unsteady 1D diffusion: $C(x,t)$ | Unsteady 1D diffusion: $T(x,t)$

$$J \quad J + \frac{\partial J}{\partial x} dx$$

$$x \quad x + dx$$

$$q \quad q + \frac{\partial q}{\partial x} dx$$

$$x \quad x + dx$$

Conservation of mass: | Conservation of energy:

net flow of atoms in = change in concentration x volume

net flow of energy in = mass x specific heat capacity x change in temp.

$$J dt - \left(J + \frac{\partial J}{\partial x} dx \right) dt = dC dx$$

$$q dt - \left(q + \frac{\partial q}{\partial x} dx \right) dt = c \left(\rho dx \right) dT$$

$$\frac{\partial C}{\partial t} = -\frac{\partial J}{\partial x} = D \frac{\partial^2 C}{\partial x^2}$$

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c} \frac{\partial q}{\partial x} = \frac{\lambda}{\rho c} \frac{\partial^2 T}{\partial x^2}$$

$$\lambda / \rho c = a = \text{thermal diffusivity}$$

$$(\text{m}^2 \text{s}^{-1}, \text{same units as } D)$$

It can be seen that the equations for mass and thermal diffusion have the same form. The solutions will therefore also have the same form, exchanging concentration C for temperature T, and diffusion coefficient D for the thermal diffusivity a.

(i) The diffusion coefficient D is given from

$D = D_o \exp \left(-\frac{Q}{RT} \right)$, where $D_o = 1 \times 10^{-6}$ m^2 s^{-1}, $Q = 87$ kJ mol^{-1}, $R = 8.3143$ kJ kmol^{-1} K^{-1},

$T = 1273$ K

Hence $D = 2.69 \times 10^{-10}$ m^2 s^{-1}

(ii) $C_s = 0.1$ wt% and $C_o = 0$ wt%

$$C(x,t) = C_s - (C_s - C_o) \text{erf} \left(\frac{x}{2\sqrt{Dt}} \right) \Rightarrow C(x,t) = 0.1 - (0.1 - 0) \text{erf} \left(\frac{1 \times 10^{-4}}{2\sqrt{2.69 \times 10^{-10} \cdot 60}} \right) =$$

$$= 0.1 \left[1 - \text{erf} \left(0.4 \right) \right]$$

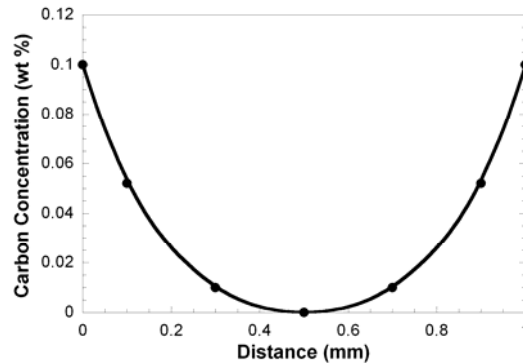From Page 8 of the Materials Data Book

$\text{erf}(0.4) = 0.428$

$C(x,t) = 0.0572 \text{ wt}\%$

(iii) $x=0$ mm, C=0.1%

$x=0.1$ mm, C=0.052%

$x=0.3$ mm, C=0.01%

$x=0.5$ mm, C=0%

Assuming that there is negligible overlap of the diffusion fields from the 2 opposite faces of the wafer, the carbon concentration profile after 1 minute is given below

A polycrystalline sample would contain grain boundaries and diffusion in those regions would be locally enhanced. However, bulk diffusion is already very high at this elevated temperature and so it is likely that any contribution from grain boundary diffusion would be small. However there might be a measurable increase if the grain size is extremely small.

Comments:

This question was generally less well answered than Question 4. The main shortcoming of this question was deriving the equations for transient 1-D mass and thermal diffusion, which was covered in the lecture notes. A lot of candidates resorted to various artifices in order to obtain the required answers. Part (a) was very well answered. However, quite a few candidates made errors in the calculations (incorrect units), which led to incorrect values for the diffusion coefficient, and the carbon concentration in part (b). Part (b) was generally well answered, a few candidates fail to identify that C_o was 0 wt%. In part (c), a few candidates didn't consider that diffusion was taking place from the 2 opposite faces of the wafer.

Question 6
(a) Metals deform plastically by slip (dislocation motion), a shear process that changes the shape of a material (through the action of shear stresses). Slip does not lead to any change in the volume of the material, so the yield stress of a metal does not depend on the magnitude of the hydrostatic (dilatational) component of stress imposed on the metal. Instead, it is sensitive to the components of the deviatoric stress.

Note: Metals deform also by twinning, which is also a shear process, but this is not covered in this course.

(b) The failure surface is shown below. When the Mohr's circle touches the failure surface the material deforms, with the failure plane given by the position where contact with the failure surface occurs.
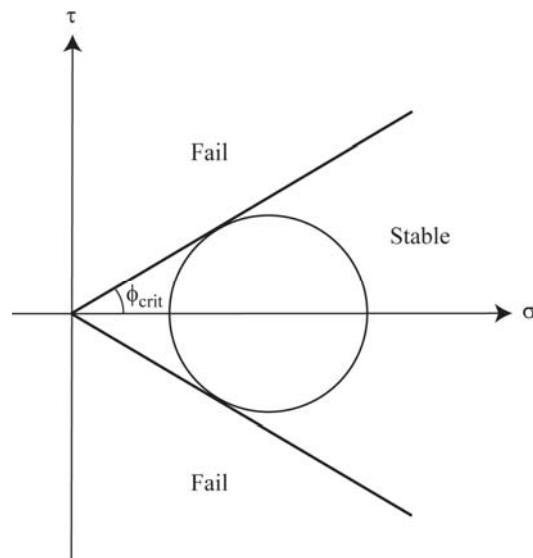
Dry soil is defined as a porous aggregate of grains. Consider a pair of grains with contact forces N and F and corresponding nominal normal and shear stresses σ and τ. For soils, "failure" is considered when sliding takes place. (Note that σ is a compressive stress)

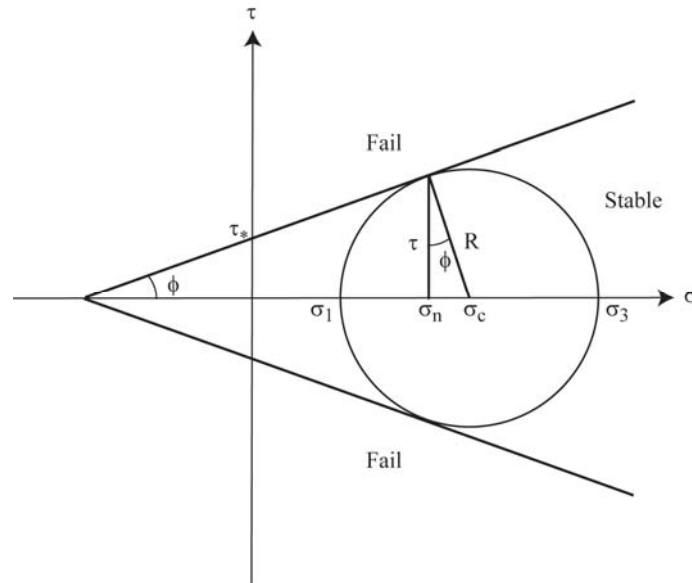$\tan \phi_\mu = \dfrac{F}{N} = \dfrac{\tau}{\sigma}$ where ϕ_μ is the friction angle.
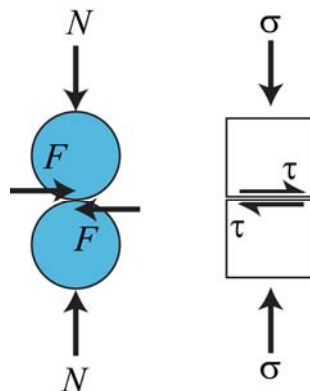
The maximum shear strength of soils can be defined as
$$\tau_{max} = \sigma \tan \phi_{crit}$$

(c)
(i)

where σ_c is the centre of the Mohr's circle and σ_1, σ and σ_3 are the principal stresses.

τ_* represents "cohesive" shear strength, the shear strength due to the interlocking of the soil particles.

(ii) From equation

$$\tau = \tau_* + \sigma_n \tan\phi$$

It follows that if $\tau = 50$ kPa, $\tau_* = 20$ kPa and $\phi = 35°$

$$\sigma_n = \frac{\tau - \tau_*}{\tan\phi} = \frac{50 - 20}{\tan 35°} = 42.8 \text{ kPa}$$

Note that σ_n is a compressive stress.
From simple geometry (see schematic above), it can be seen that there are two similar triangles.

$$\tan\phi = \frac{\sigma_c - \sigma_n}{\tau}$$

$$\sigma_c - \sigma_n = \tau \cdot \tan\phi$$

$$\sigma_c = \tau \cdot \tan\phi + \sigma_n$$

$$\therefore \sigma_c = 35 + 42.8 = 77.8 \text{ kPa}$$

Also, $\sigma_1 = \sigma_c - R = \sigma_c - \tau/\cos\phi = 77.8 - 61 = 16.8$ kPa and

$\sigma_3 = \sigma_c + R = 138.8$ kPa

Comments:
This was the least popular question on the paper. Very few students answered correctly Part (a). Parts (b) and (c) were answered reasonably well. Some candidates failed to explain with sketches the soil failure criterion. For some candidates, it was clear that this was either a desperate 4th question or they were running out of time.