IB 2010 P4 Prof N Collings / Dr S Barrett

1B Paper 4 2010 Solutions

Q 1

a) For a spherical shell, the rate of heat transfer is given by

$$Q = -4\pi r^2 \lambda \frac{dT}{dr}, \quad or \quad \frac{Q}{4\pi\lambda}\frac{dr}{r^2} = -dT$$

As Q and λ are constant, the latter expression can be integrated between the inner and outer radii to give

$$R_{th} = \frac{T_1 - T_2}{Q} = \frac{1}{4\pi\lambda}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

b) i) For a convective boundary, $Q = hA(T_{wall} - T_{environment})$, so the thermal resistance is given by

$$R_{th} = \frac{T_{wall} - T_{environment}}{Q} = \frac{1}{hA} \text{ or, for this situation, } R_{th} = \frac{1}{4\pi r_2^2 h}$$

The total thermal resistance is

$$\sum R_{th} = \frac{T_1 - T_2}{Q} = \frac{1}{4\pi\lambda}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi r_2^2 h}$$

The total thermal resistance is thus

$$\sum R_{th} = \frac{1}{4\pi 0.1}\left(\frac{1}{0.25/2} - \frac{1}{0.3/2}\right) + \frac{1}{4\pi(0.3/2)^2\, 10.0} = 1.061 + 0.354 = 1.415 \text{ K/W}$$

ii) From tables we find that for saturated R134a, a pressure of 1.06 bar corresponds to a temperature of -25°C. The rate of heat transfer is thus $(-25 - 20)/1.415$ = -31.8 W.

c) i) As the insulation thickness increases, the area available for convective heat transfer also increases, and this could cause the combined thermal resistance to decrease.

ii) If we differentiate the expression above for the overall heat transfer with respect to the outer radius we find

$$\frac{d\sum R_{th}}{r_2} = \frac{1}{4\pi\lambda}\left(\frac{1}{r_2^2}\right) - \frac{1}{2\pi r_2^3 h}$$

And we find that $\frac{d\sum R_{th}}{dr_2} = 0$ when $r_2 = \frac{2\lambda}{h}$. This will be the condition giving the maximum heat transfer. For the data given, $r_2 = \frac{2\lambda}{h} = \frac{2*0.1}{10} = 0.02$ m, which is smaller than r_1 , so any insulation added decreases the heat transfer.

d) As the fluid boils off, the temperature remains at -25°C as the pressure remains constant in the vessel. One approach is to write the First Law between the end states for a control volume which encloses the *initial* mass in the vessel, we have

$$Q-0=U_{end}-U_{start}+m_{out}h_{out}-m_{in}h_{in}=(m_{end}u_g-m_{start}u_f)+(m_{start}-m_{end})h_g$$

Alternatively, and this is the approach taken here, we could take a system approach (fixed mass), and imagine a "membrane" that expands into the atmosphere (see figure below), with the pressure within, and outside the membrane constant at 1.06 bar, in which case

$$Q-W=U_{end}-U_{start} \text{ or, } Q-pV_m=m_{start}u_g-m_{start}u_f$$

Where V_m is the volume by which the membrane has expanded. Now total fluid mass is conserved, so $m_{start}=V_c/v_f=V_c/v_g + V_m/v_g$, where V_c is the volume of the container; eliminating V_c, we have $V_m=m_{start}(v_g-v_f)$.

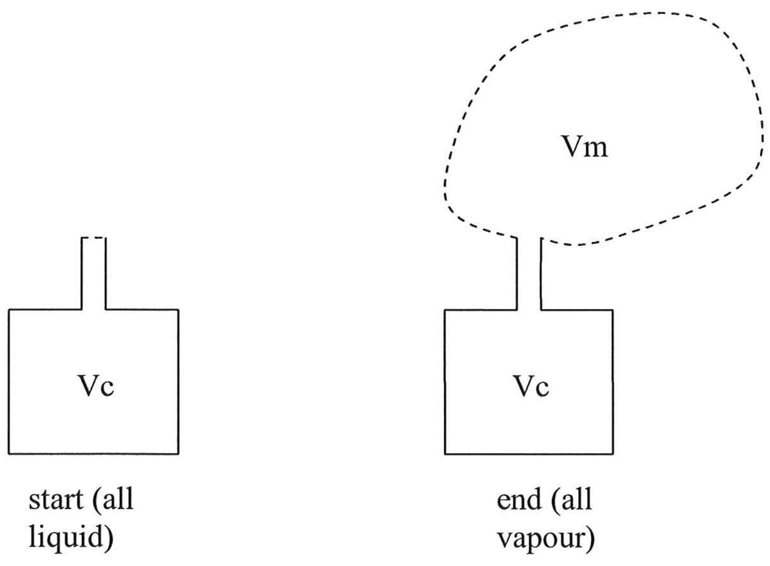
From tables we find that the specific volume of the contents at wet and dry saturated conditions are v_f = 0.00073 m^3/kg. The initial mass is thus

$$m_{start}=\frac{(4/3)*\pi*(0.25/2)^3}{0.00073}=11.21\text{kg}$$

Substitution into $Q-pV_m=m_{start}u_g-m_{start}u_f$ gives

$$Q=m_{start}(u_g+pv_g)-m_{start}(u_f+pv_f)=m_{start}h_{fg}=11.21*(383.4-167.2)=2423.6\text{ kJ}$$

(You may wish to show, by manipulation, that the expression based on a control volume approach is equivalent to that obtained by the system approach.)



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Q2.

a) Carbon balance $C_n + nO_2 = nCO_2$

Hydrogen balance $H_m + \frac{m}{4}O_2 = \frac{m}{2}H_2O$

For stoichiometric combustion, with an oxygen balance

$$C_nH_m + \left(n+\frac{m}{4}\right)(O_2 + kN_2) = nCO_2 + \frac{m}{2}H_2O + \left(n+\frac{m}{4}\right)kN_2$$

For $\lambda > 1$, the same quantity of oxygen will take part in combustion per unit fuel quantity, so the final chemical balance is

$$C_nH_m + \lambda\left(n+\frac{m}{4}\right)(O_2 + kN_2) = nCO_2 + \frac{m}{2}H_2O + (\lambda-1)\left(n+\frac{m}{4}\right)O_2 + \lambda\left(n+\frac{m}{4}\right)kN_2$$

b) (i). We are told that the fuel is CH_4, and that the dry concentration of O_2 is 14%. Thus, noting that this is a dry basis measurement, i.e. we ignore the water vapour component:-

$$\frac{(\lambda-1)\left(n+\frac{m}{4}\right)}{n+(\lambda-1)\left(n+\frac{m}{4}\right)+\lambda\left(1+\frac{m}{4}\right)k} = 0.14$$

Substituting for *n*, *m* and *k*, we have

$$\frac{(\lambda-1)(2)}{1+(\lambda-1)(2)+\lambda(2)3.762} = 0.14, \text{ giving } \lambda = 2.79$$

ii) Condensation will occur if the exhaust gas temperature drops below the dew point. The water vapour mol fraction in the exhaust is given by

$$\frac{\frac{m}{2}}{n+\frac{m}{2}+(\lambda-1)\left(n+\frac{m}{4}\right)+\lambda\left(1+\frac{m}{4}\right)k} = \frac{\frac{4}{2}}{1+\frac{4}{2}+(2.79-1)\left(1+\frac{4}{4}\right)+2.79\left(1+\frac{4}{4}\right)3.762}$$

$$= 0.0725$$

The partial pressure of the water vapour is thus 0.0725*1 bar (or 7.25 kPa), which the saturation tables for water show is equivalent to a temperature of close to 40°C. Below this temperature, condensation will occur.

iii) Per kmol of fuel, 2 kmol of water vapour is produced, and the exhaust consists of (1 + 2 + (2.79-1) * 2 + 2.79*2*3.762) = 27.57 kmol. At 25°C, from tables, the water vapour partial pressure is 3.17 kPa, so if the mols of condensed water are n_w we can write

$$\frac{2-n_w}{27.572-n_w}=\frac{3.17}{100}\text{, hence } n_w = 1.163 \text{ kmol} = 20.934 \text{ kg}$$

Applying the SFEE between the locations where the gas is at 40°C and 25°C
We have

$$Q - 0 = H_{out} - H_{in} = m_x c_{px}(25-40) + (m_{H_2O,out} h_{g,25} - m_{H_2O,in} h_{g,40}) + m_w h_{f,25},$$

Where m_x is the mass of dry exhaust gas (per kmol of fuel)

On the basis of 1kmol of fuel, we have (first term in brackets is the dry exhaust kmols/kmol fuel)

$$\begin{aligned} Q &= (27.57-2) * 29.7 * 1.01 * (25-40) \\ &+ ((2-1.163) * 18 * 2547.3 - 2 * 18 * 2574.4) \\ &+ 1.163 * 18 * 104.8 \end{aligned}$$

Thus

$Q = -11506 - 54301 + 2194$ (Note that the condensation term is dominant.)

= -63613 kJ, or -63613/16 = -3976 kJ/kg fuel

As a percentage of the fuel energy, this is

$3976 / 50010 = 7.95\%$

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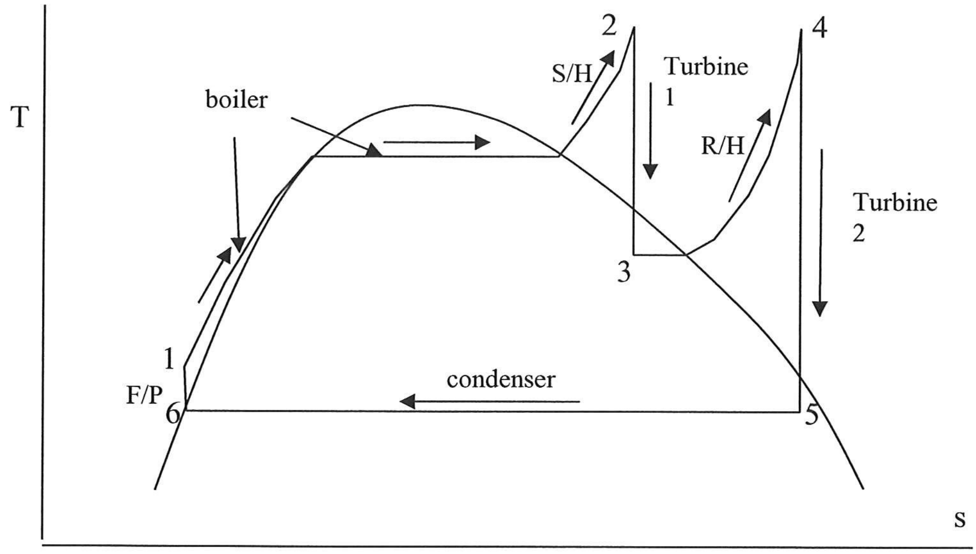
Q 3 (a)



b) (i, ii, iii) Specific enthalpy values in kJ/kg are

from tables: h_6 = 0.5*(151.5+121.4) = 136.5, (= h_1 as feed pump work negligible)

from the chart: h_2 = 3500, h_3 = 2720, h_4 = 3590, h_5 = 2500. (Many students who attempted this question used interpolation in the tables – this is very time consuming.)

SFEE $\dot{Q} - \dot{W}_x = h_2 - h_1$. Thus per unit mass flow, (W and Q in kJ/kg)

Turbine 1	W_{23} = (3500 – 2720) = 780
Turbine 2	W_{45} = (3590 – 2500) = 1090
Boiler and superheater:	Q_{62} = (3500 – 136.5) = 3363.5
Reheater:	Q_{34} = (3590 – 2720) = 870
Condenser:	Q_{56} = (136.5 – 2500) = -2363.5

The net work output is thus (3363.5 + 870 -2363.5) = 1870 (= (780 + 1090))

The thermal efficiency is 1870/(3363.5 + 870) = 44.2%

c) The throttling process is isenthalpic (SFEE with Q = W = 0, with no KE effects)

So h_2 remains unchanged, but the final h_3 becomes (chart) 2850 kJ/kg. All other values remain unchanged. Thus :-

Turbine 1	W_{23} = (3500 – 2850) = 650
Turbine 2	W_{45} = (3590 – 2500) = 1090

Boiler and superheater:	$Q_{62} = (3500 - 136.5) = 3363.5$
Reheater:	$Q_{34} = (3590 - 2850) = 740$
Condenser:	$Q_{56} = (136.5 - 2500) = -2363.5$

The work output is thus (3363.5 + 740 - 2363.5) = 1740 (= (650 + 1090))

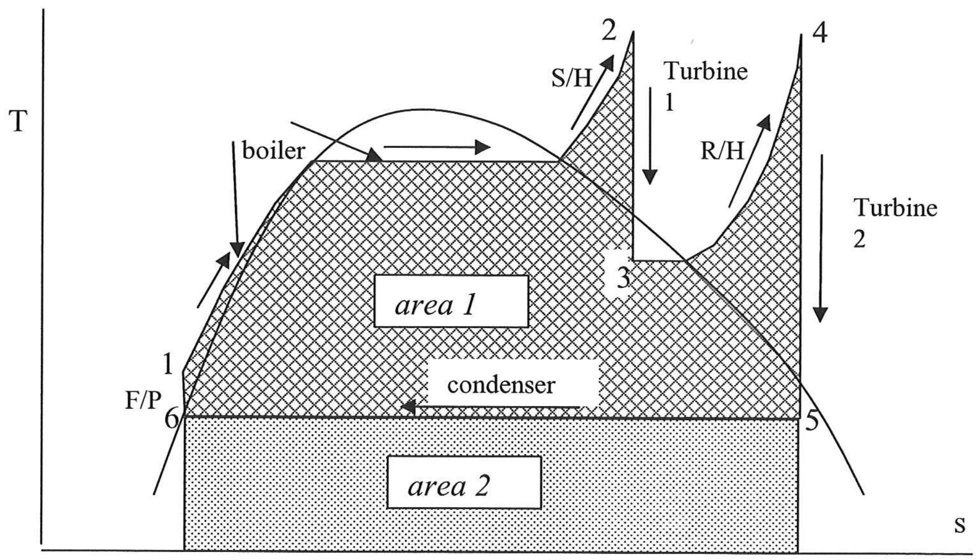
This represents a reduction in the output of (1870-1740)/1870 = 7%,

The new thermal efficiency is 1740/(3363.5 + 740) = 42.4%

[It is interesting that though the throttle reduces the output by 7%, the plant efficiency only falls by 4% points]

d) The work done in a steady flow process with no KE changes is $dQ-dW_x=dh$. Now we know that $Tds = dh - vdp$, so for the reversible, adiabatic feed pump, we can write $-dW_x=vdp$. As water may be regarded as incompressible, we have $-W_x=v_f \int dp=v_f(p_6-p_1)$. $v_f=0.001 m^3/kg$, and the pressure change is ~ 100 bar. Thus $-W_x= 0.001*100*\frac{1e5}{1e3}$ = 10 kJ/kg (1e5/1e3 to give kJ/kg). This is negligible compared to the 1815 kJ/kg from the turbines (0.55%).

e)



For a reversible process, $Q = \int Tds$, therefore *area 1* +*area 2* represents the boiler, superheater and reheater heat input. *Area 2* represents the condenser heat rejection. The net work must therefore be represented by area 1, and the thermal efficiency by *(area 1)/(area 1 + area 2)*.

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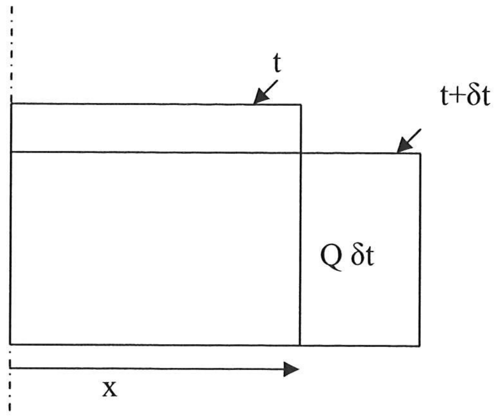
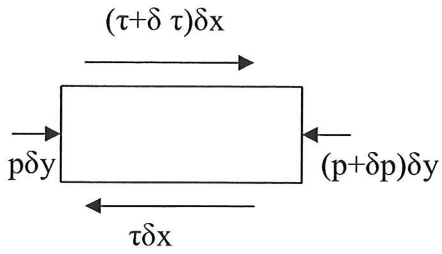


4(a) Consider x>0. By symmetry, $Q = |x|$

(b) Force balance ($\rightarrow$): $\delta\tau\,\delta x \;=\; \delta p\,\delta y$

$$\delta\tau = \frac{d\tau}{dy}\delta y,\ \ \delta p = \frac{dp}{dx}\delta x \Rightarrow \frac{dp}{dx} = \frac{d\tau}{dy} = \frac{d}{dy}\left(\mu\frac{du}{dy}\right) = \frac{d^2u}{dy^2}$$



[Alternatively derive from Navier-Stokes.]

(c) $\quad u = 0 \ @\ y = 0,\ H.$

Integrate: $$\mu\frac{du}{dy} + A = \frac{dp}{dx}y$$

And again $$\mu u + Ay + B = \frac{dp}{dx}\frac{y^2}{2}$$

B = 0; A = $$\frac{dp}{dx}\frac{H}{2} \Rightarrow A = \frac{dp}{dx}\frac{H}{2} \Rightarrow u = \frac{y}{2\mu}\frac{dp}{dx}(y-H)$$

$$Q = \int_0^H u\,dy = \frac{dp}{dx}\frac{1}{2\mu}\left[\frac{y^3}{3} - \frac{Hy^2}{2}\right]_o^H$$

$$= -\frac{1}{12\mu}\frac{dp}{dx}H^3 = Vx \leftarrow \text{from (a)}$$

$$\Rightarrow \frac{dp}{dx} = A\frac{\mu Vx}{H^3},\quad \text{A} = -12$$

[or $\dfrac{dp}{dx} = \text{-sign}\,(x)\,\dfrac{12\mu V\,|\,x\,|}{\mu^3}$ etc.]

(d) $$p = \int\frac{-12\mu V}{H^3}x\,dx = \frac{6\mu V}{H^3}x^2 + \text{const.}$$

p = 0 at l: $$\text{p} = \frac{6\mu V}{H^3}\left(L^2 - X^2\right)$$

$$\text{F} = 2\int_0^L p\,dx = \frac{12\mu V}{H^3}\left(L^2X - \frac{X^3}{3}\right)\Bigg|_o^L = \frac{8\mu VL^3}{H^3}$$

(e) Power = F.V = $8\mu V^2L^3/H^3$ (goes into heating the oil.)

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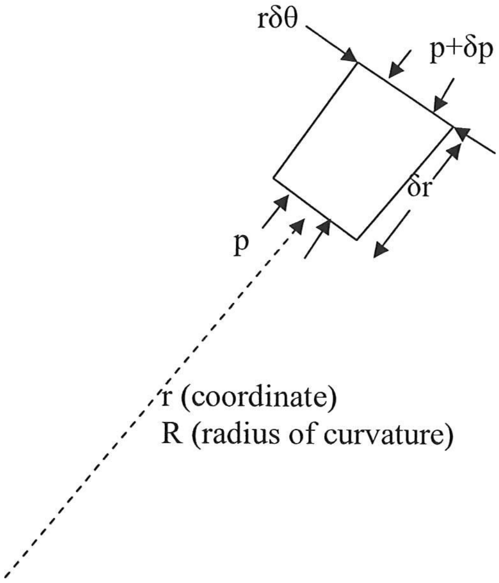
Q5 (a)



$$-\rho.\delta r\, r\delta\theta\delta z\frac{u^2}{R} = [p-(p+\delta p)]r\delta\theta\delta z$$

$$\Rightarrow \rho\delta r\frac{u^2}{R} = \delta p$$

$$\Rightarrow \rho\delta r\frac{u^2}{R} = \frac{\partial p}{\partial r}\delta r$$

Since streamlines circular $p = p(r)$ only, thus

$$\frac{dp}{dr} = \frac{\rho u^2}{R}$$ as required.

(b) Start with Bernoulli:

$$\frac{1}{2}\rho u_0^2 + p_A = \frac{1}{2}\rho u^2 + p \quad (= const).$$

where: $u = u(r),\ p = p(r)$

Differentiate w.r.t. r:

$$0 = \frac{dp}{dr} + \frac{1}{2}u^2\frac{d\rho}{dr} + \rho u\frac{du}{dr}, \text{ and } \frac{d\rho}{dr} = 0$$

Sub in result from part (a)

$$0 = u + R\frac{du}{dr}$$

R and r are equivalent because the coordinate r is the same as the radius of curvature R .

$$\Rightarrow 0 = \int\frac{du}{u} + \int\frac{dr}{r}$$

Integrate

$$\Rightarrow C_1 = \ell n\, u + \ell n\, r = ln(ur)$$

$$\Rightarrow e^{C_1} = C_2 = ur$$

$\Rightarrow \quad ur \quad = \quad M$ as required.

The outer edge of jet has const p (i.e. ambient) so the velocity is constant (Bernoulli), let this velocity be u_0, then

$$r\,u \quad = \quad \left(\frac{D}{2} + w\right) u_0 , \qquad \text{where } w \text{ is a constant.}$$

(c) Apply continuity:

$$w_o\, u_o \quad = \quad \int_{D/2}^{D/2+w} \frac{(D/2 + w)u_0}{r} dr$$

$$\Rightarrow \quad \frac{w_0}{(D/2 + w)} = \ell n \left(\frac{D/2 + w}{D/2} \right)$$

Or, using the approximation given, for $w << D/2$

$$\frac{w_0}{(D/2 + w)} \quad \cong \quad \left(\frac{2w}{D} - \frac{1}{2}\left(\frac{2w}{D}\right)^2 \right)$$

Again noting $w << D/2$, we obtain

$$w = \frac{1}{2}\left[\sqrt{D^2 + 4w_0 D} - D\right]$$

(d) Easiest by <u>SFME</u>

The momentum in each jet is $\rho u_0^{\ 2}\, w_0$, so applying the SFME in the vertical direction,

$$F = 2\rho u_o^2\, w_o \; sin\left(\frac{\alpha}{2}\right)$$

Which acts to create an upward force on the cylinder.

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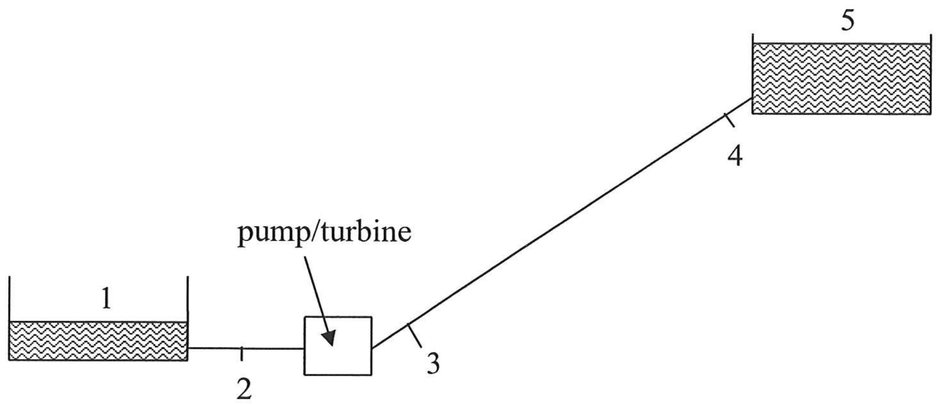
Q 6



a) We know (by use of the data book if necessary) $Tds = dh - \frac{dp}{\rho}$, so for reversible, adiabatic (isentropic) flow, $0 = dh - \frac{dp}{\rho}$. For constant density, integrating between stations 1 and 2, we have $h_2 - h_1 = (p_2 - p_1)/\rho$. Also we know from the SFEE that for an adiabatic steady flow process,

$$0 - \dot{W}_x = \left(h_2 + \frac{1}{2}v_2^{\,2} + \rho g z_2\right) - \left(h_1 + \frac{1}{2}v_1^{\,2} + \rho g z_1\right) \quad \text{per unit mass flow}$$

Or, for a mass flow of $\dot{m}$, at adiabatic conditions, (and using the definition for an incompressible fluid relating total and static pressures: $p_0 = p + 0.5\rho v^2 + \rho g z$)

$$-\dot{W}_x = \frac{\dot{m}}{\rho}\left[\left(p_2 + \frac{1}{2}\rho v_2^{\,2} + \rho g z_2\right) - \left(p_1 + \frac{1}{2}\rho v_1^{\,2} + \rho g z_1\right)\right] = Q(p_{0,2} - p_{0,1})$$

b) (i) As the pipes between the pump/turbine and the lower reservoir are very large, the static and total pressure at station 2 will be ambient (no height difference, and low velocities, so negligible frictional losses). Thus for pumping and generation, $p_2 = p_{0,2} = p_{amb}$, and $p_{2,gauge} = 0$. The area of one pipe is $A = \pi 2.8^2/4 = 6.16\, m^2$

Pumping.

Writing Bernoulli's equation between stations 3 and 4, between which the velocity will be constant, we have (assuming that 3 is at z = 0):-

$$p_3 + \frac{1}{2}\rho v^2 + \rho g 0 = p_4 + \frac{1}{2}\rho v^2 + \rho g z + 4c_f \frac{l}{d}\frac{1}{2}\rho v^2$$

The static pressure at 4 can be assumed to be ambient because the pipe there exits into the upper reservoir (parallel streamlines argument). If the total flow rate is Q, we can write this expression as

$$p_3 = p_{amb} + \rho gz + 4c_f \frac{l}{d}\frac{1}{2}\rho v^2 \text{ and } \frac{1}{2}\rho v^2 = 0.5 * 1000\left(\frac{80}{4*6.16}\right)^2 = 0.05E5Pa$$

And $\rho gh = 1000 * 9.81 * 320 = 31.39E5\,Pa$. Substituting in the data given, we have

$$p_3 = p_{amb} + 31.39E5 + 4 * 0.005 * (16000/2.8) * 0.05E5 = p_{amb} + 31.99E5 \text{ , so}$$

$p_{3,gauge}$ = <u>31.99 bar</u>

<u>Generating</u>

For the water moving between 5 and 4 we can write

$$p_5 + \frac{1}{2}\rho 0^2 + \rho g0 = p_4 + \frac{1}{2}\rho v^2 + \rho g0 = p_{amb},$$

Noting $\frac{1}{2}\rho v^2 = \frac{1}{2}1000\left(\frac{120}{4*6.16}\right)^2 = 0.12E5$, we have

$$p_4 = p_{amb} - \frac{1}{2}\rho v^2 = p_{amb} - 0.12E5\,\text{Pa}.$$

I.e $\underline{p_{4,gauge} = -0.12\,bar}$

Because the flow direction has reversed, we now have, between stations 4 and 3:-

$$p_3 + \frac{1}{2}\rho v^2 + \rho g0 = p_4 + \frac{1}{2}\rho v^2 + \rho gz - 4c_f \frac{l}{d}\frac{1}{2}\rho v^2, \text{ and noting } p_4 + \frac{1}{2}\rho v^2 = p_{amb}$$

$$p_3 = p_{amb} - \frac{1}{2}\rho v^2 + \rho gz - 4c_f \frac{l}{d}\frac{1}{2}\rho v^2 = p_{amb} + \rho gz - \frac{1}{2}\rho v^2\left(1 + 4c_f \frac{l}{d}\right)$$

Therefore

$$p_3 = p_{amb} + 1000 * 9.81 * 320 - 0.12E5\left(1 + 4 * 0.005\frac{1600}{2.8}\right) = p_{amb} + 31.39E5 - 1.48E5$$

$\underline{p_{3,gauge} = 29.91\ bar}$

ii)

The isentropic power required for <u>pumping</u> is $-\dot{W}_x = Q(p_{0,3} - p_{0,2})$

$-\dot{W}_x = Q(p_3 + \frac{1}{2}\rho v^2 - p_2) = 80(31.99E5 + 0.05E5 - 0) = -256.4\,MW$. Thus the electrical power required is 256.4/0.9 = 284.9 MW

The isentropic power required for generating is also $-\dot{W}_x = Q(p_{0,3} - p_{0,2})$, so

$-\dot{W}_x = 80(p_3 + \frac{1}{2}\rho\left(\frac{Q}{4A}\right)^2 - p_2) = 80(29.92E5 + 0.12E5 - 0) = -360.4\,MW$. Thus the electrical power produced is 360.4*0.9 = 324.4 MW.

The overall efficiency of the installation will be determined by the energy out divided by the energy in, when the start and end states are the same. So on the basis of V m^3 of water being pumped up, and generating energy on its return,

$$\text{Energy in} = \frac{V}{80*3600}*284.9 = V*9.89E\text{-}4 \text{ MWhr}$$

$$\text{Energy out} = \frac{V}{120*3600}*324.4 = V*7.51E\text{-}4 \text{ MWhr.}$$

The installation efficiency is thus $\frac{7.51}{9.89} = 75.9\%$

[Note that "fluid" losses are relatively unimportant – neglecting these, the maximum installation efficiency is 81% (0.9*0.9)]

iii) We want to know how the work output changes with a flow rate change:-

$$-\dot{W}_x = Q\left(\rho g z - 4c_f \frac{l}{d}\frac{1}{2}\rho\left(\frac{Q}{4A}\right)^2\right)$$

$$\frac{d\dot{W}_x}{dQ} = \left(\rho g z - 12c_f \frac{l}{d}\frac{1}{2}\rho\left(\frac{Q}{4A}\right)^2\right)$$

$$\frac{d\dot{W}_x/\dot{W}_x}{dQ/Q} = \left(31.39E5 - 12*0.005\frac{1600}{2.8}0.12E5\right)\frac{120}{360.4} = 0.91$$

I.e increasing the water flow (at 120 m^3/s) by (say) 1%, increases the output by 0.91%

[The question is based (loosely) on the Ffestiniog pump storage scheme in North Wales http://www.fhc.co.uk/ffestiniog.htm]