ENGINEERING TRIPOS PART IB

Wednesday 2 June 2010 2 to 4

Paper 5 - SOLUTIONS

ELECTRICAL ENGINEERING

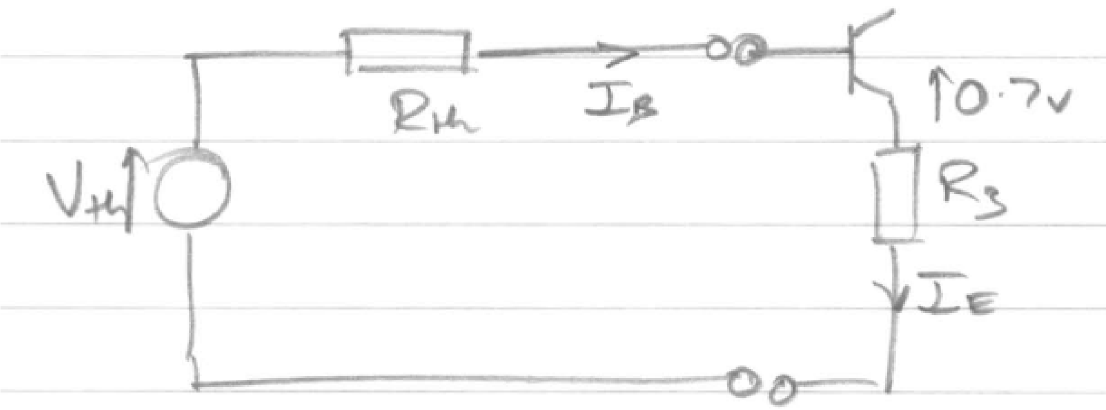
1 (a) A Class A amplifier is characterised by the drawing of a constant current from the supply, irrespective of the signal level. As a consequence, the static dissipation in the transistor is high and the efficiency is low.

A Class B amplifier is one where each transistor conducts for only half the cycle resulting in a high efficiency as there is no power dissipation when there is no input signal. However, as the transition between transistors is not smooth, crossover distortion occurs.

In Class AB amplifiers, the transistors are biased such that they are on for more than half the cycle, which improves linearity by removing crossover distortion, but reduces efficiency.

The circuit in Fig. 1 is a Class A amplifier.

(b) As the current in R_1 and R_2 are not equal, we must convert the 20 V and R_1 and R_2 into a Thevenin equivalent:



$$V_{th} = V_{CC} \frac{R_2}{R_1 + R_2} = 20.\frac{15}{15+20} = 12 \text{ V}$$

$$R_{th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 15}{10 + 15} = 6 \text{ k}\Omega$$

Now we apply a mesh current analysis to the whole circuit shown:

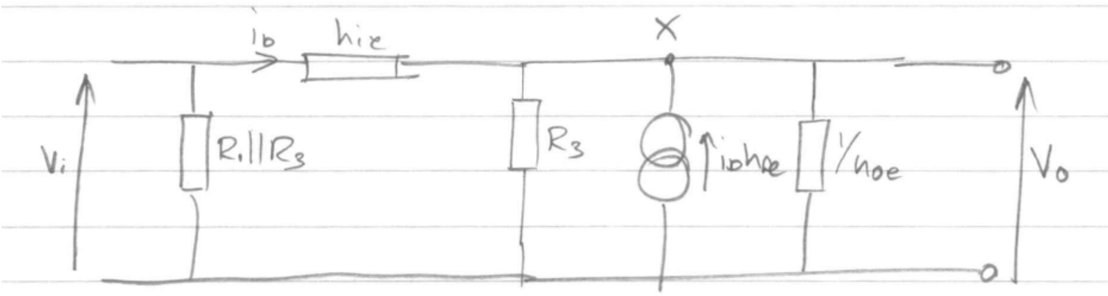
$$\sum V = 0 = V_{th} - I_B R_{th} - 0.7 \text{ V} - I_E R_3$$

However, we know that $I_E = h_{FE} I_B R_3$, so

$$0 = V_{th} - I_B R_{th} - 0.7 \text{ V} - h_{FE} I_B R_3$$

$$
\begin{aligned}
I_B &= \frac{V_{th} - 0.7}{R_{th} + h_{FE} R_3} \\
&= \frac{12 - 0.7}{6\,\mathrm{k} + 50.2\,\mathrm{k}} \\
&= \frac{11.3}{106000} \\
I_B &= 107\,\mu\mathrm{A}
\end{aligned}
$$

(c) The small-signal equivalent circuit is:



Summing currents at X (NVA):

$$\sum I_0 = 0 = \frac{V_o - V_i}{h_{ie}} + \frac{V_o}{R_3 \| \frac{1}{h_{oe}}} - i_b h_{FE}$$

We need to eliminate i_b using the fact that

$$i_b = \frac{V_i - V_o}{h_{ie}}$$

Hence,

$$0 = \frac{V_o - V_i}{h_{ie}} + \frac{V_o}{R_3 \| \frac{1}{h_{oe}}} + \left(\frac{V_o - V_i}{h_{ie}} \right) \cdot h_{FE}$$

$$V_0 \left(\frac{1}{h_{ie}} + \frac{R_3 + \frac{1}{h_{oe}}}{R_3 / h_{oe}} + \frac{h_{fe}}{h_{ie}} \right) = V_i \left(\frac{1}{h_{ie}} + \frac{h_{fe}}{h_{ie}} \right)$$

$$V_0 \left(\frac{1 + h_{oe} h_{ie} R_3 + h_{ie}}{R_3} + h_{FE} \right) = V_i (1 + h_{FE})$$

$$\therefore \text{Gain} = \frac{V_0}{V_i} = \frac{1 + h_{FE}}{1 + h_{oe} h_{ie} + \frac{h_{ie}}{R_3} + h_{FE}}$$

Substituting values gives

$$\text{Gain} = \frac{1+50}{1+0.5\times10^{-3}.10^{3}+10^{3}/2\times10^{3}+50}$$

$$= \frac{51}{1+0.5+0.5+50}$$

$$\text{Gain} = 0.981$$

To get the output resistance, we apply a test input current, i.e. to the output and set $V_i = 0$. Summing current at X again gives

$$\sum I_0 = 0 = \frac{V_0}{h_{ie}} + \frac{V_0}{R_3 \| 1/h_{oe}} - i_b h_{FE} - i_0$$

As before

$$i_b = \frac{V_i - V_0}{h_{ie}} = -\frac{V_0}{h_{ie}}$$

and so

$$0 = V_0\left(\frac{1}{h_{ie}} + \frac{R_3 + 1/h_{oe}}{R_3/h_{oe}} + \frac{h_{FE}}{h_{ie}}\right)^{-1}$$

$$R_o = \frac{V_o}{i_0}$$
$$R_o = \left(\frac{1}{h_{ie}} + h_{oe} + \frac{1}{R_3} + \frac{h_{fe}}{h_{ie}}\right)^{-1}$$

Substituting values gives

$$R_0 = \left(\frac{1}{10^3} + 0.5\times10^{-3} + 1/2\times10^3 + \frac{50}{10^3}\right)^{-1}$$
$$R_0 = 19.2\ \Omega$$

(d) For maximum power transfer $R_L = R_o$ and so

$$R_L = 19.2\Omega$$

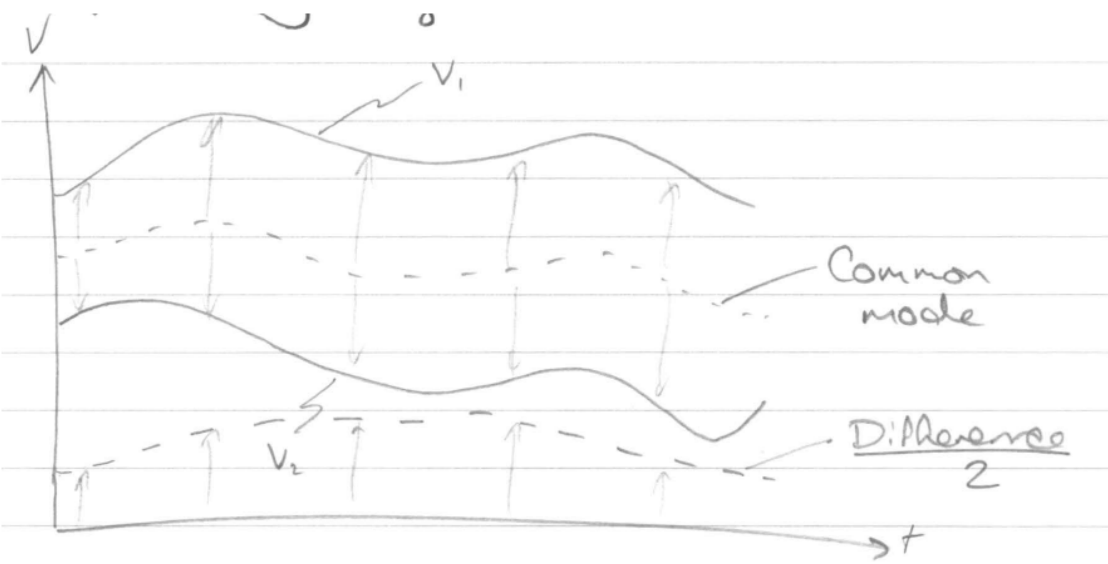
At the operating point the voltage at the emitter is

$$\begin{aligned} V_E &= R_3.I_E \\ &= R_3.I_B.h_{FE} \\ &= 2000 \times 107 \times 10^{-6} \times 50 \\ V_E &= 10.7\,\mathrm{V} \end{aligned}$$

However, as the supply rail is at +20 V, the maximum sinusoidal voltage amplitude is (20 – 10.7) = 9.3 V. This is equivalent to an rms voltage of 9.3/√2 = 6.58 V. Only half of this voltage (3.29 V) appears across the load resistor, so

$$P_{out} = V^2 / R_L = 3.29^2 / 19.2 = 0.56\ \mathrm{W}$$

2 (a) In the difference amplifier, we are interested in two input signals, in this case v_1 and v_2. For the purposes of analysis, these are decomposed into two components, as shown in the following diagram.



The common mode signal is the average of the two signals – i.e. $\left(\frac{v_1+v_2}{2}\right)$. Normally this component will be suppressed by a difference amplifier. Therefore, any signal that is common to both v_1 and v_2 will not be amplified. The difference signal is then the difference between the two signals – strictly speaking it is v_2–v_1. Both v_1 and v_2 can be calculated from the common mode and difference signal as

$$v_1 = \frac{v_2+v_1}{2} - \frac{v_2-v_1}{2}; v_2 = \frac{v_2+v_1}{2} + \frac{v_2-v_1}{2}$$

In this case, the differential amplifier as ideal as the 50 Hz signal (common-mode) will be suppressed.

(b) Take the voltage at the inverting and non-inverting terminals to be v_- and v_+ respectively. Perform NVA at the inverting terminal:

$$\sum I = 0 = \frac{v_- - v_1}{R_1} + \frac{v_- - v_0}{R_3}$$

$$V_-\left(\frac{1}{R_1} + \frac{1}{R_3}\right) = \frac{V_1}{R_1} + \frac{V_0}{R_3}$$

Also at non-inverting terminal:

$$\sum I = 0 = \frac{v_+ - v_2}{R_2} + \frac{v_+}{R_4}$$

$$v_-\left(\frac{1}{R_2} - \frac{1}{R_9}\right) = \frac{v_2}{R_2}$$

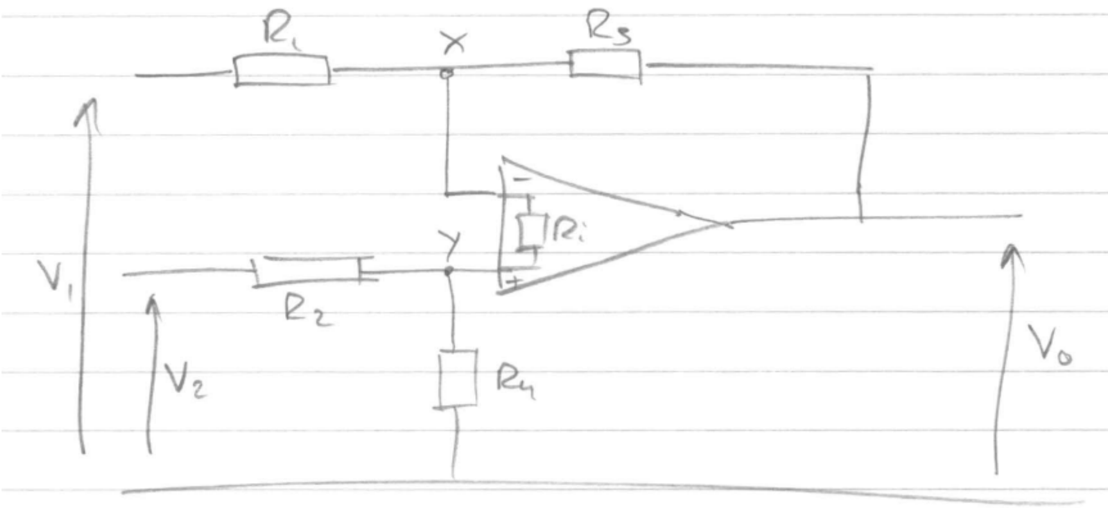
But $R_1 = R_2$ and $R_3 = R_4$, so

$$\frac{v_2}{R_1} = \frac{v_1}{R_1} + \frac{v_0}{R_3}$$

$$\frac{v_2 - v_1}{R_1} = \frac{v_0}{R_3}$$

$$\frac{v_0}{v_2 - v_1} = \frac{R_3}{R_1}$$

(c) R_i appears as a resistance between the two terminals, so we have the following circuit:



Performing as NVA at X:

$$\sum I_0 = 0 = \frac{v_- - v_1}{R_1} + \frac{v_- - v_0}{R_3} + \frac{v_- - v_+}{R_i}$$

$$v_-\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_i}\right) = \frac{v_1}{R_1} + \frac{v_0}{R_3} + \frac{v_+}{R_i} \qquad (1)$$

Also at Y:

$$\sum I = 0 = \frac{v_+ - v_2}{R_2} + \frac{v_+}{R_4} + \frac{v_+ - v_-}{R_i}$$

$$V_+\left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_i}\right) = \frac{v_2}{R_2} + \frac{v_-}{R_i} \qquad (2)$$

Now $R_1 = R_2$ and $R_3 = R_4$. Therefore, we can subtract (1) from (2) to give

$$\left(v_+ - v_-\right)\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_i}\right) = -\frac{v_1}{R_1} - \frac{v_0}{R_3} + \frac{v_2}{R_2} - \left(\frac{v_+ - v_-}{R_i}\right)$$

Now, for a finite gain, we know that

$$v_0 = A\left(v_+ - v_-\right)$$
$$\therefore v_+ - v_- = \frac{v_0}{A}$$

Substituting gives

$$\frac{v_0}{A}\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_i}\right) = \frac{v_2 - v_1}{R_1} - \frac{v_0}{R_3} - \frac{v_0}{AR:}$$

$$v_0\left[\frac{1}{A}\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{2}{R_i}\right) + \frac{1}{R_3}\right] = \frac{v_2 - v_1}{R_1}$$

$$v_0 = \frac{v_2 - v_1}{\frac{R_1}{A}\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{2}{R_i}\right) + \frac{R_1}{R_3}}$$

(d) For a gain of 100, we require

$$\frac{R_1}{A}\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{2}{R_i}\right) + \frac{R_1}{R_3} = \frac{1}{100}$$

R_1 and R_3 should be less than R_i, so no more than $100\mathrm{k}\Omega$. Therefore, for A~10^7 the term in the brackets becomes negligible, so

$$\frac{R_1}{R_3} = \frac{1}{100}$$
$$\therefore R_3 = 100R_1$$

R_1 and R_3 should also be much greater than R_0 for R_0 to be negligible, so greater than $1\mathrm{k}\Omega$. Therefore, we select

R_3=100kΩ
R_1=1kΩ

3 (a) High voltage 3-phase as power transmission is favoured as 3 phases can be easily produced by a generator, and the voltage stepped up and down easily, to minimise transmission line losses using transformers. In particular, the cost of increasing the number of phases increases with the number of wires, while the efficiency improvement gained beyond three phases is almost negligible. Therefore, three phases are favoured. It is necessary to consider a town as being balanced so that the three phases are each connected to similar loads, and so the single-phase analysis can be applied.

(b) We know that

$$\begin{aligned}
P &= \sqrt{3} V_L I_L \cos\phi \\
I_L &= \frac{P}{\sqrt{3}.V_L \cos\phi} \\
&= \frac{100\times10^6}{\sqrt{3}.33\times10^3.0.85} \\
I_L &= 2058\,\text{A}
\end{aligned}$$

(c) Feeder line impedance = $Z_L = R_L + jX_L = 2 + 5j$. Therefore, the power dissipated in the transmission line , P_L, is

$$\begin{aligned}
P_L &= 3{I_L}^2.R_L \\
&= 3.2058^2.2 \\
P_L &= 25.4\,\text{MW}
\end{aligned}$$

To determine the voltage at the feeder, we need to know P and Q for the whole system. First, we calculate the reactive power for the town, Q_t

$$\begin{aligned}
Q_t &= P_t \tan\phi \\
&= 100\times10^6 \tan\left(\cos^{-1} 0.85\right) \\
Q_t &= 62.0\,\text{MVAR}
\end{aligned}$$

and the reactive power for the load, Q_L

$$\begin{aligned}
Q_L &= 3{I_L}^2 X_L \\
&= 3.2058^2.5 \\
Q_L &= 63.5\ \text{MVAR}
\end{aligned}$$

Hence, for the total system,

$$P_{tot} = P_t + P_L = 100\times10^6 + 25.4\times10^6 = 125.4\,\text{MW}$$
$$Q_{tot} = Q_t + Q_L = 62\times10^6 + 63.5\times10^6 = 125.5\,\text{MVAR}$$
$$S_{tot} = \sqrt{P_{tot}{}^2 + Q_{tot}{}^2}$$
$$= \left(125.4^2 + 125.5^2\right)^{1/2}$$
$$S_{tot} = 177.4\,\text{MVA}$$

We can now work out the line voltage at the feeder, V_{Lf}, from

$$S_{tot} = \sqrt{3}.V_{Lf}.I_L$$
$$\therefore V_{Lf} = \frac{S}{\sqrt{3}.I_L}$$
$$= \frac{177.4\times10^6}{\sqrt{3}.2058}$$
$$V_{Lf} = 49.8\,\text{kV}$$

(d) For a new power factor of 0.9,

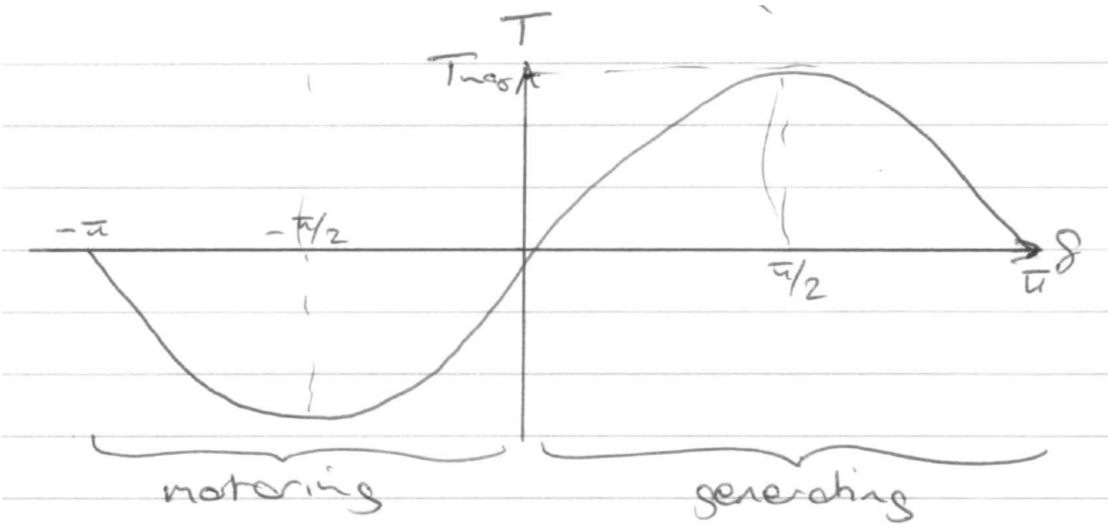
$$I_L = \frac{P}{\sqrt{3}.V_L\cos\phi}$$
$$= \frac{100\times10^6}{\sqrt{3}.33\times10^3.0.9}$$
$$I_L = 1944\,\text{A}$$

Hence

$$P_L = 3I_L{}^2R_L$$
$$= 3.1944^2.2$$
$$P_L = 22.6\,\text{MW}$$

Therefore, the reduction is 25.4–22.6=2.8 MW. This can be achieved by connecting a capacitance in parallel with the load across each of the three phases.

4 (a)



Synchronicity is lost when $\delta = \pm\pi/2$.

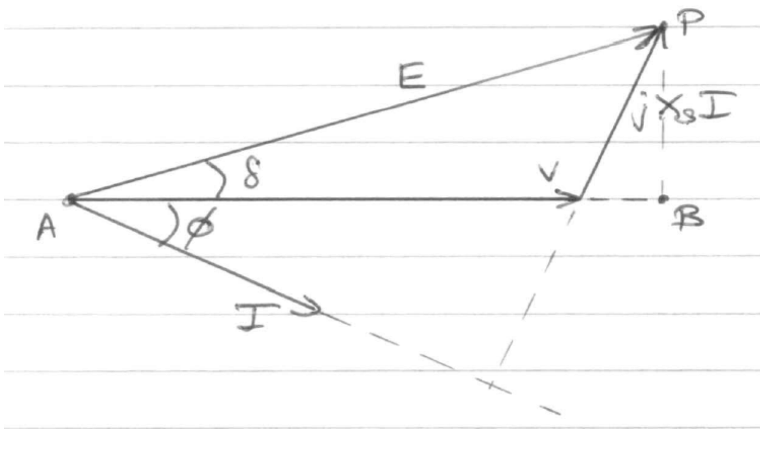
(b) (i) First, we need the speed of rotation, ω_s. For a 4 pole system, the number of pole pairs, p, is 2, and so

$$\omega_s = \frac{2\pi f}{p} = \frac{100\pi}{2} = 157.1\,\mathrm{rad\,s^{-1}}$$

Hence

$$T = \frac{P}{\omega_s} = \frac{400\times10^6}{157.1} = 2546\ \mathrm{kNm}$$

(ii) To get the excitation voltage, we need the phasor diagram.



For a star-connected system, $I_L = I_{ph}$, so

$$
\begin{aligned}
P &= \sqrt{3} V_L I_L \cos\phi \\
\therefore I_{ph} &= \frac{P}{\sqrt{3} V_L \cos\phi} \\
&= \frac{400 \times 10^6}{\sqrt{3} . 22 \times 10^3 . 0.85} \\
I_{ph} &= 12350 \, \mathrm{A}
\end{aligned}
$$

We now need the phase voltage

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{22 \times 10^3}{\sqrt{3}} = 12.7 \text{ kV}$$

Therefore,

$$
\begin{aligned}
E &= \sqrt{V_{ph}^2 + (X_s I)^2 - 2 V_{ph} X_s I \cos(90 + \phi)} \\
&= \sqrt{(12.7 \times 10^3)^2 + (1.2 \times 12350)^2 - 2 \times 12.7 \times 10^3 \times 1.2 \times 12350 \cos(90 + \cos^{-1} 0.85)} \\
E &= 24.06 \, \mathrm{kV}
\end{aligned}
$$

(iii) Finally,

$$
\begin{aligned}
E \sin\delta &= X_s I \cos\phi \\
\delta &= \sin^{-1}\left(\frac{X_s I \cos\phi}{E}\right) \\
&= \sin^{-1}\left(\frac{1.2 \times 12350 \times 0.85}{24060}\right) \\
\delta &= 31.6^\circ
\end{aligned}
$$

(c) If the excitation is increased whilst the prime mover is held constant, then the reactive power will change. In this case, the real power is 400 MW and the generator rating (due to the stator heating limit) is 750 MVA.

$$
\begin{aligned}
\therefore Q_{\max} &= \sqrt{{S_{\max}}^2 - P^2} \\
&= \left(750^2 - 400^2\right)^{1/2} \\
Q_{\max} &= 634 \, \mathrm{MVAR}
\end{aligned}
$$

5 (a) (i) First we need the phase voltage and phase power:

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{3} = 240\,\mathrm{V}$$
$$P_{ph} = \frac{P}{3} = \frac{1100}{3} = 366.7\,\mathrm{W}$$

Now, the iron loss resistance, R_{i}, is

$$R_i = \frac{V_{ph}{}^2}{P_{ph}} = \frac{240^2}{366 \cdot 7} = 157\,\Omega$$

(ii) First, we need the apparent power per phase,

$$S_{ph} = V_{ph} I_{ph} = 240 \times 4.2 = 1008\ \mathrm{VA}$$

The reactive power per phase is then

$$Q_{ph} = \left(S_{ph}{}^2 - P_{ph}{}^2\right)^{1/2}$$
$$= \left(1008^2 - 366.7^2\right)^{1/2}$$
$$Q_{ph} = 939\,\mathrm{VAR}$$

The magnetising reactance is therefore,

$$X_m = \frac{V_{ph}{}^2}{Q_{ph}}$$
$$= \frac{240^2}{939}$$
$$X_m = 61.3\Omega$$

(iii) In the locked rotor test, real power is dissipated in the stator and rotor resistances, so

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 51.96\,\mathrm{V} \quad P_{ph} = \frac{P}{3} = \frac{4500}{3} = 1500\,\mathrm{W}$$

Therefore,

$$R_1 + R_2' = \frac{P_{ph}}{I_{ph}{}^2} = \frac{1500}{65^2} = 0.355\Omega$$

However, $R_1 = 0.2\ \Omega$, so

$$R_2' = 0.16\Omega$$

(iv) For the stator leakage reactance,

$$S_{ph} = V_{ph} I_{ph} = 52.65 = 3380 \text{ VA}$$
$$Q_{ph} = \left(S_{ph}{}^2 - P_{ph}{}^2\right)^{1/2} = \left(3380^2 - 1500^2\right)^{1/2} = 3029 \text{ VAR}$$

Therefore,

$$X_1 + X_2' = \frac{Q_{ph}}{I_{ph}^2} = \frac{3029}{65^2} = 0.72\ \Omega$$

Now X_1 is half X_2', so

$$X_1 = 0.24\Omega$$

(b) As the parallel loss components are much greater than those in series, the former can be ignored as almost no current flows through them. Hence, we can consider the Thevenin equivalent impedance for the circuit to be

$$Z_{th} = R_1 + j\left(X_1 + X_2'\right)$$
$$Z_{th} = 0.20 + j.0.72$$
$$\therefore \left|Z_{th}\right| = 0.75\Omega$$

Therefore, at maximum torque,

$$\frac{R_2'}{s} = \left|Z_{th}\right|$$
$$\therefore s = \frac{R_2'}{\left|Z_{th}\right|} = \frac{0.16}{0.75} = 0.213$$

The speed that this corresponds to is

$$N_r = (1 - s)N_s = (1 - 0.213).750 = 590 \text{ rpm}$$

The equation for torque is from the Data Book:

$$T = \frac{3I_2'^2}{\omega_s} \cdot \frac{R_2'}{s}$$

The Thevanin voltage for the circuit remains the supply voltage, 240 V. Hence, at maximum torque, when $R_2'/s = |Z_{th}|$,

$$\begin{aligned} I_2' &= \frac{V_{th}}{Z_{th} + R_2'/s} \\ &= \frac{240}{0.20 + 0.72j + 0.75} \\ I_2' &= (160 - 121j)\ \text{A} \\ |I_2'| &= 200\ \text{A} \end{aligned}$$

Therefore,

$$\begin{aligned} T &= \frac{3{I_2'}^2}{\omega_S} \cdot \frac{R_2'}{s} \\ &= \frac{3.200^2}{2\pi.50/4}.0.75 \\ T &= 1145\ \text{Nm} \end{aligned}$$

(c) For maximum starting torque, when *s*=1, we require

$$R_2'' = |Z_{th}| = 0.75\Omega$$

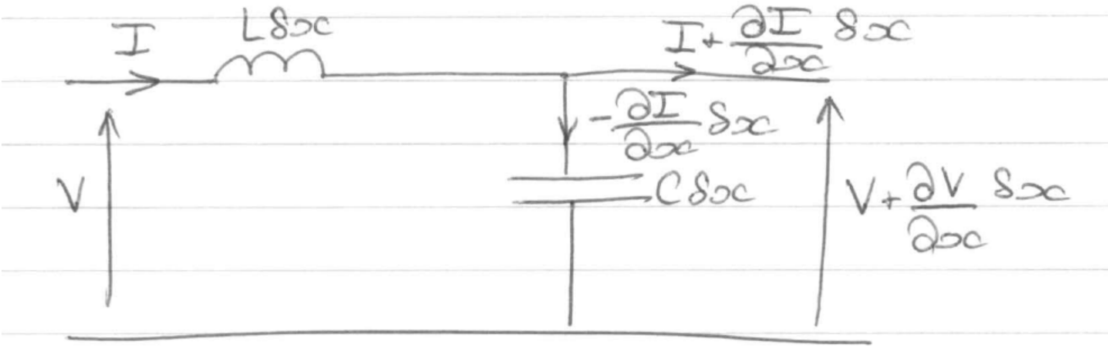
However $R_2' = 0.16\,\Omega$. Hence, the extra load required is

$$R_L' = R_2'' - R_2' = 0.75 - 0.16 = 0.59\ \Omega$$

Referring back to the rotor,

$$\begin{aligned} R_L &= \frac{R_L'}{k^2} \\ &= \frac{0.59}{2^2} \\ R_L &= 0.15\ \Omega \end{aligned}$$

6 (a) Starting from the circuit diagram:



$$\text{V across L}: \frac{\partial V}{\partial x}\cdot\delta x = -L\delta x\frac{\partial I}{\partial t}$$

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t} \qquad (1)$$

$$\text{I through C}: \frac{-\partial \mathrm{I}}{\partial \mathrm{x}}.\delta x = C\delta x\frac{\partial V}{\partial t}$$

$$\therefore \frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t} \qquad (2)$$

Differentiating (1) with respect to x gives

$$\frac{\partial^2 V}{\partial x^2} = -L\frac{\partial}{\partial x}\left(\frac{\partial I}{\partial t}\right)$$

$$\frac{\partial^2 V}{\partial x^2} = -L\frac{\partial}{\partial t}\left(\frac{\partial I}{\partial x}\right)$$

Substituting from (2)

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

$$\therefore \frac{\partial^2 V}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 V}{\partial x^2}$$

Also differentiating (2) with respect to x gives

$$\frac{\partial^2 I}{\partial x^2} = -C\frac{\partial}{\partial x}\left(\frac{\partial V}{\partial t}\right)$$
$$= -C\frac{\partial}{\partial t}\left(\frac{\partial V}{\partial x}\right)$$
$$\frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2}$$
$$\therefore \frac{\partial^2 I}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 I}{\partial x^2}$$

(b) From the Telegrapher's equations,

$$c = \frac{1}{\sqrt{LC}}$$

However,

$$LC = \frac{\mu_0\mu_r \ln\left(r_2/r_1\right)}{2\pi}\cdot\frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(r_2/r_1\right)} = \mu_0\mu_r\varepsilon_0\varepsilon_r$$
$$\therefore c = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$$

The wave velocity is entirely dependent upon the physical properties of the dielectric. This is because the actual electromagnetic wave exists between the conductors in the dielectric. The conductors simply support and guide the wave.

(c) From the data book,

$$Z_0 = \sqrt{\frac{L}{C}}$$
$$\frac{L}{C} = \frac{\mu_0\mu_r \ln\left(r_2/r_1\right)}{2\pi}\cdot\frac{\ln\left(r_2/r_1\right)}{2\pi\varepsilon_0\varepsilon_r}$$
$$\frac{L}{C} = \frac{\mu_0}{4\pi^2\varepsilon_0\varepsilon_r}\ \ln^2\left(\frac{r_2}{r_1}\right) = Z_0{}^2$$

$$\therefore \ln^2\left(\frac{r_2}{r_1}\right) = \frac{4\pi^2\varepsilon_0\varepsilon_r Z_0{}^2}{\mu_0}$$

$$= \frac{4\pi^2 8.854\times10^{-12}.2.1.75^2}{4\pi\times10^{-7}}$$

$$\ln^2\left(r_2/r_1\right) = 1.813$$

$$\therefore \frac{r_2}{r_1} = 6.13$$

(d) From the data book, we know that the input impedance to a cable of length l with a load Z_L is

$$Z_{in} = Z_0 \frac{Z_L + Z_0 j\tan(\beta l)}{Z_0 + Z_2 j\tan(\beta l)}$$

If $l = \lambda/4$, then $\beta l = 2\pi/\lambda . \lambda/4 = \pi/2$. Also tan(π/2) = ∞. Hence, for $l = \lambda/4$

$$Z_{in} = \frac{Z_0{}^2}{Z_L}$$

$$\therefore Z_0 = \sqrt{Z_{in}Z_L}$$

We want Z_{in} = 75 Ω for a Z_L = 50 Ω, so

$$Z_0 = \sqrt{75.50} = 61.2\Omega$$

Now,

$$C = \frac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r}}$$

$$= \frac{1}{\sqrt{4\pi\times10^{-7}}.8.854\times10^{-12}.2.1}$$

$$C = 2.069\times10^8\,\mathrm{ms}^{-1}$$

Therefore,

$$\lambda = \frac{c}{f}$$
$$= \frac{2.069 \times 10^8}{10 \times 10^6}$$
$$\lambda = 20.7 \text{ m}$$
$$\therefore l = \lambda/4 = 517 \text{ cm}$$

(e) In practice an FM signal has a finite bandwidth around the carrier frequency. However, the linking cable will only be fully effective at 10 MHz. Therefore, reflections will occur at other frequencies, and the system should be designed to take into account the reflections that will occur across the full range of frequencies.

7 (a) Starting from the reduced form of the Maxwell equation given,

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\therefore \frac{\partial B_y}{\partial t} = -\frac{\partial}{\partial z}\left(\mathrm{E}_{x0}\exp\left[j(\omega t - \beta z)\right]\right)$$

$$\frac{\partial B_y}{\partial t} = j\beta \mathrm{E}_{x0}\exp\left[j(\omega t - \beta z)\right]$$

Integrating with respect to time gives

$$B_y = \frac{\beta}{\omega}\mathrm{E}_{x0}\exp\left[j(\omega t - \beta z)\right]$$

However, $\beta/\omega = 1/c = \sqrt{\mu_0 \varepsilon_0}$, and so

$$H_y = \frac{B_y}{\mu_0}$$

$$= \frac{\sqrt{\mu_0 \varepsilon_0}}{\mu_0}\mathrm{E}_{x0}\exp\left[j(\omega t - \beta z)\right]$$

$$H_y = \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E}_{x0}\exp\left[j(\omega t - \beta z)\right]$$

(b) First let us calculate $\omega\varepsilon_0\varepsilon_r$

$$\omega\varepsilon_0\varepsilon_r = 2\pi \times 2.4 \times 10^{-9} \times 8.854 \times 10^{-12}.3.8 = 0.508$$

Hence, for the propagation contant,

$$\gamma = \sqrt{j\omega\mu_0(\sigma + j\omega\varepsilon_0\varepsilon_r)}$$

$$= \sqrt{j.2\pi \times 2.4 \times 10^9 \times 4\pi \times 10^{-7}(10^5 + j.0.508)}$$

$$= \sqrt{1.89 \times 10^9 j - 9626}$$

$$\gamma = 30781(1 + j)$$

For the intrinsic impedance,

$$\eta = \left(\frac{j\omega\mu_0}{\sigma + j\omega\varepsilon_0\varepsilon_r} \right)^{1/2}$$

$$= \left(\frac{j.2\pi \times 2.4 \times 10^9 \times 4\pi \times 10^{-7}}{10^5 + 0.508j} \right)^{1/2}$$

$$\eta = 0.308(1 + j)$$

(c) If the components of **E** and **H** parallel to the boundary surface are continuous, then

$$E_i + E_r = E_t \qquad (1)$$

where i, r and t denote the incident, reflected and transmitted waves respectively, and

$$H_i + H_r = H_t$$

However, E and H are related by the characteristic impedance as

$$\frac{E_i}{H_i} = \eta_1; \frac{E_r}{H_r} = -\eta_1; \frac{E_t}{H_t} = \eta_2$$

Hence, substituting for H gives,

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

Substituting for E_r from equation (1) gives

$$\rho_T = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

(d) The transmitted power density is given by

$$\frac{P_t}{P_i} = \frac{E_t^2 / 2\eta_2}{E_i^2 / 2\eta_1} = \frac{E_t^2}{E_i^2} \cdot \frac{\eta_1}{\eta_2} = \frac{4\eta_1\eta_2}{(\eta_1 + \eta_2)^2}$$

Now,

$$\frac{P_t}{P_i} = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2}$$
$$= \frac{4.377.0.308(1+j)}{(377+0.308(1+j))^2}$$
$$\frac{P_t}{P_i} = (1+j)3.2\times10^{-3}$$
$$\left|\frac{P_t}{P_i}\right| = 4.53\times10^{-3}$$

Hence,

$$P_t = \rho_T{}^2.P_i = 4.53\times10^{-3}.10\times10^{-3} = 4.53\times10^{-5}$$

The power decays as $\exp(-2\alpha d)$ where α is the real part of the propagation constant and d is the thickness of material. We want a decay factor of $100\times10^{-9}/4.53\times10^{-5}$ = 2.21×10^{-2}.

$$\exp(-2\alpha d) = 2.21\times10^{-3}$$
$$-2\alpha d = -6.116$$
$$\therefore d = \frac{6.116}{2\alpha}$$
$$= \frac{6.116}{2\times30781}$$
$$d = 99\ \mu\text{m}$$