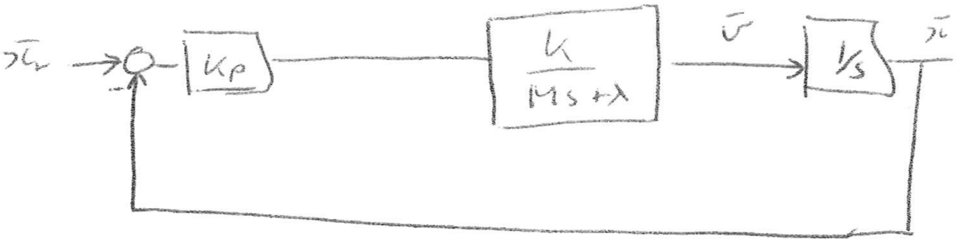
IB 2010 Crib Paper 6 Dr Vinnicombe / Prof Godsill

1) a) $M\dot{v} = T - \lambda v = ku - \lambda s$

$$\Rightarrow \bar{v}(s) = \underline{\underline{\frac{k}{Ms+\lambda}}}\,\bar{u}(s)$$

[5]

b)



$$\Rightarrow \bar{x}(s) = \frac{\frac{k_p k}{s(Ms+\lambda)}}{1 + \frac{k_p k}{s(Ms+\lambda)}}\,\bar{x}_r(s)$$

$$= \frac{k_p k}{Ms^2 + \lambda s + k_p k}\,\bar{x}_r(s)$$

$$\Rightarrow \bar{e} = \bar{x}_r - \bar{x} = \underline{\underline{\frac{Ms^2 + \lambda s}{Ms^2 + \lambda s + k_p k}}}\,\bar{x}_r(s)$$

[5]

c) $\bar{x}_r(s) = \frac{C}{s^2} \Rightarrow \bar{e} = \frac{Ms+\lambda}{(Ms^2+\lambda s+k_p k)} \times \frac{C}{s}$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\bar{e}(s) = \underline{\underline{\frac{C\lambda}{k_p k}}}$$

[5]

d) at $s = 1.5j$, $T_{e\,x_r\to x}(j\omega) = \frac{\cancel{-2.25M + \lambda + 1.5j}\; k_p k}{-2.25M + 1.5j\lambda + k_p k} = \frac{\cancel{2250 + 1800j}\; 2250}{1800j} = -1.25j = 1.25\angle -\frac{\pi}{2}$

$\Rightarrow$ resp $= 30t - \left(\frac{30 \times 1200}{2250}\right)^{16} + 1.25 \sin(1.5)t$

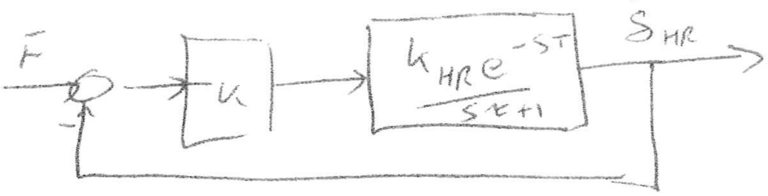
next car $= 30t - 32 - 1.5625 \cos(1.5)t$ etc

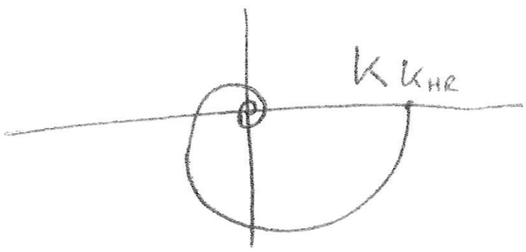
[5]

2) a)

[5]

b)





[6]

c) $L(s) = \frac{2e^{-12.1s}}{20s+1} \Rightarrow |L(j\omega)| = \frac{2}{\sqrt{20^2\omega^2+1}}$

$|L(j\omega)| = 1$ at $20^2\omega^2 = 3 \Rightarrow \omega = \frac{\sqrt{3}}{20}$

at this ω, $\angle L(j\omega) = -12.1\frac{\sqrt{3}}{20} - \tan^{-1}\sqrt{3}$

$= -120° \Rightarrow PM = 60°$

[4]

d) $L(j\omega) = \frac{3e^{-10j\omega}}{15j\omega+1}$

$\Rightarrow |L(j\omega)| = 1$ at $15^2\omega^2 = 8 \Rightarrow \omega = \frac{\sqrt{8}}{15}$

$\Rightarrow \angle L(j\omega) = -10 \cdot \frac{\sqrt{8}}{15} - \tan^{-1} \cancel{15} \frac{\sqrt{8}}{15} = \cancel{128.3°}\ -179.65°$

$\Rightarrow PM = 0.35° \approx 0$

$\Rightarrow$ System will oscillate at freq of $\frac{\sqrt{8}}{15}$ rad/s $= 0.03$ Hz
(33 sec cycle)

[5]

3) a) by varying the torque sinusoidally at a variety of frequencies and measuring the magnitude and phase of the S.S response.

as $s \to 0$ $\quad G(s) \to 1/(J+J_0)s^2$

magnitude of low freq asymptote $= 1$ at $\omega \approx 0.022$

$$\Rightarrow J + J_0 \approx \frac{1}{0.025^2} \approx \underline{\underline{1600}}$$

$s \to \infty$ $\quad G(s) \to 1/J_0 s^2$, mag of HF asymptote $= 1$ at $\omega \approx 0.031$

$$\Rightarrow J_0 \approx \frac{1}{0.031^2} \approx \underline{\underline{1000}}$$

[6]

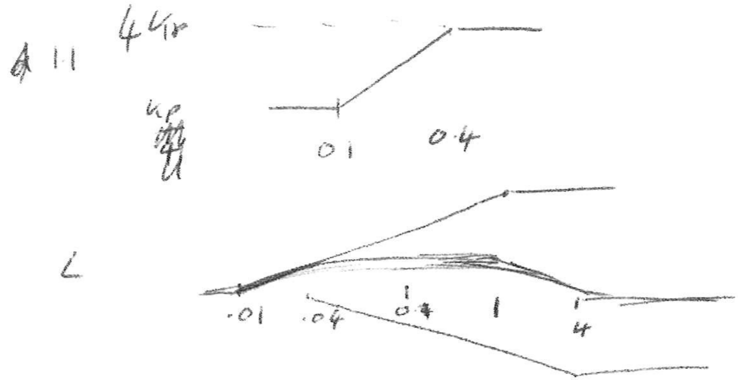
b) $\angle$ is max at $\omega \approx 0.082$, $|G(j0.082)| \approx -17dB$

$\Rightarrow$ require $k_p = 17dB = \frac{10}{\sqrt{2}} \approx \underline{\underline{7}}$

$PM \approx \underline{\underline{34^\circ}}$

[6]

c)



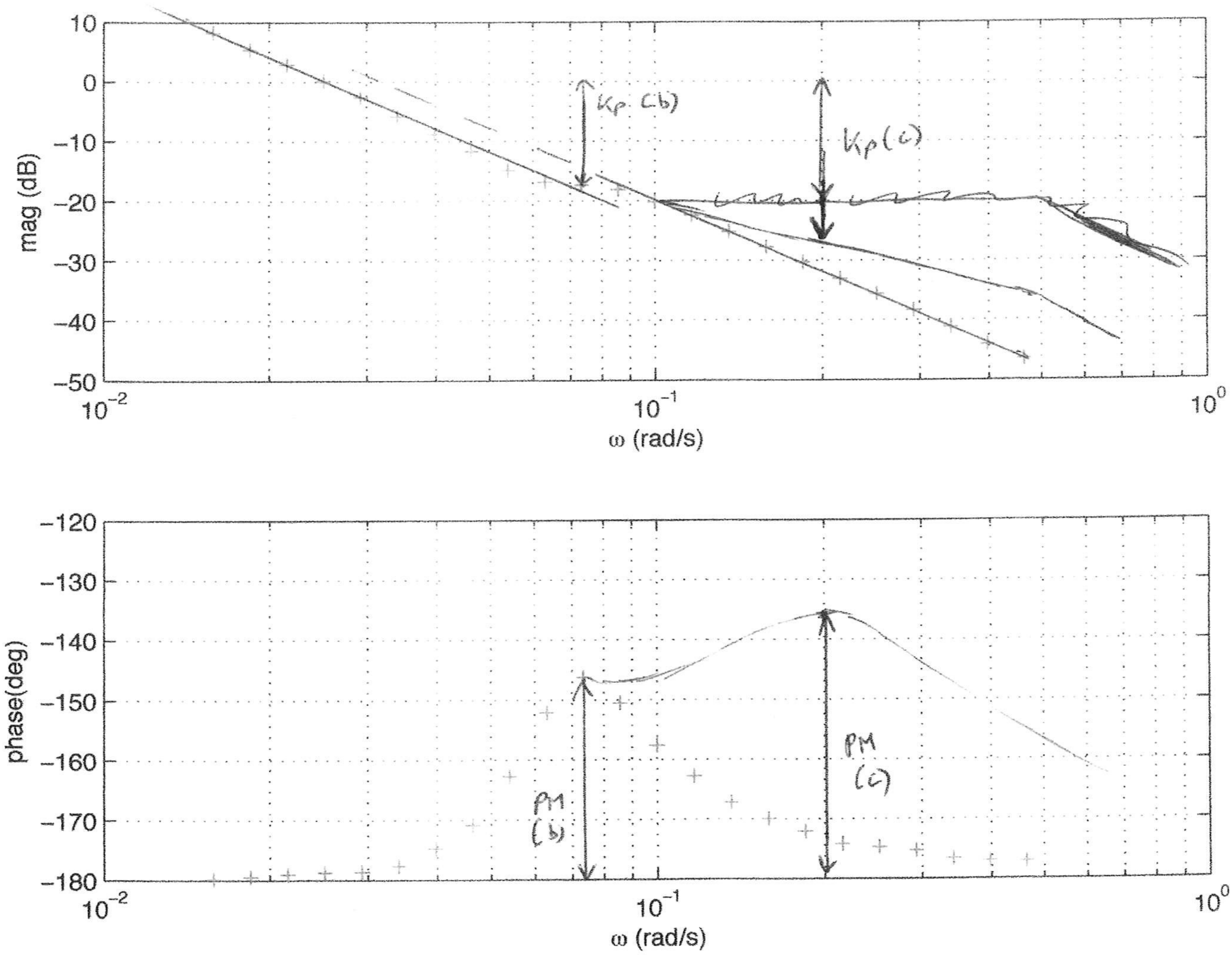
max advance at $\omega \approx 0.2$ $= \tan^{-1} 2 - \tan^{-1} 1/2$
$= 37^\circ$

To get max PM subject to constraint need $|L(j0.2)| = 1$ (as phase contribution from bad load term on $G(s)$ is more negative at higher crossover frequencies)

$\Rightarrow PM \approx 44^\circ$ and $k_p \approx 37dB = \frac{100}{\sqrt{2}} \approx 70$

The requirement ensures that the control bandwidth ≥ 0.2

[8]



mag (dB)
10
0
−10
−20
−30
−40
−50
10^{-2}
10^{-1}
10^{0}
ω (rad/s)
Kp (b)
Kp (c)
phase(deg)
−120
−130
−140
−150
−160
−170
−180
10^{-2}
10^{-1}
10^{0}
ω (rad/s)
PM (b)
PM (c)

Fig. 1

4) a) Energy $= \int_{-\infty}^{\infty} |f(t)|^2 \, dt$

Power $= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |f(t)|^2 \, dt$ [2]

b) ~~$F(\omega) =$~~ $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, d\omega = \frac{1}{2\pi} \int_{-B}^{B} e^{j\omega t} h \, d\omega$

$= \frac{h}{2\pi} \left[\frac{e^{j\omega t}}{jt} \right]_{-B}^{B} = \frac{h}{\pi t} \sin \beta t$ [4]

c) $\int_{-\infty}^{\infty} f(t)^2 \, dt = \int_{-\infty}^{\infty} f(t) \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, d\omega \, dt$

$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2\pi} f(t) e^{j\omega t} F(\omega) \, dt \, d\omega$

$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \int_{-\infty}^{\infty} f(t) e^{j\omega t} \, dt \, d\omega$

$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) F(-\omega) \, d\omega$ [8]

d) Energy $= \int_{-\infty}^{\infty} \left(\frac{\sin \alpha t}{\pi t} \right)^2 dt = \frac{\alpha}{\pi} \to \infty$ as $\alpha \to \infty$

but area $= \int_{-\infty}^{\infty} \frac{\sin \alpha t}{\pi t} \, dt = 1 \to f(t) \to$ impulse as $\alpha \to \infty$

$\Rightarrow$ impulse has infinite energy

[6]

5)a) BW = max freq component, ie B here

need to sample > 2× this frequency to avoid aliasing as sampling, ie multiplying by impulse train, makes spectrum periodic

etc.

To recover, use low pass filter with cutoff between B and f_s-B. In practice need a guard band to allow for non-ideal filters, and also an anti-aliasing filter before sampling to remove any components >B in original signal.

[5]

b) $f_s(t) = f(t) \underbrace{\sum_n \delta(t-nT)}_{\delta_p(t)} = \sum_n f(nT)\delta(t-nT)$

[3]

c) $F_s(\omega) = \int_{-\infty}^{\infty} f(t) \sum_n \delta(t-nT)\, e^{-j\omega t}\, dt$

$= \sum_n \int f(t)\,\delta(t-nT)\, e^{-j\omega t}\, dt = \sum_n f(nT)\, e^{-j\omega nT}$ (i)

Clearly periodic, period $\frac{2\pi}{T}$ as $e^{-j(\omega+\frac{2\pi}{T})nT} = e^{-j(\omega nT+2\ldots)}$

[6]

d) Equation (i) above is a Fourier series in ω, $c_n \rightleftharpoons f(nT)$

$\Rightarrow c_n = \frac{T}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} F_s(\omega)\, e^{jn\omega T}\, d\omega$ from data book.

1 period

[6]

6) a) AM Cheap, simple demodulation but prone to noise and interference. SSB lower power, but more complex demodulation.

FM Greater bandwidth requirement, but less sensitive to interference & noise

Also PM, similar to FM, & QAM

[6]

b) Amplitude modulation, AM. $\alpha = 4$, $\omega_c = 2\pi \cdot 800 \cdot 10^3$

$$s(t) = 10 \cos \omega_c t + 2 f(t) \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$$

$$\Rightarrow S(\omega) = 5\delta(\omega - \omega_c) + 5\delta(\omega + \omega_c) + 2F(\omega - \omega_c) + 2F(\omega + \omega_c)$$

but $\delta(\omega + \omega_c)$ and $F(\omega + \omega_c) = 0$ for $\omega < 0$ (as BW of F $\ll 2\omega_c$)

BW of $f(t) \approx 5\,\text{kHz}$ $\Rightarrow$ BW of channel $\approx 2 \times 5 + 3 = 13\,\text{kHz}$ [8]

c) $$x(t)^2 = 1 + f(t)^2 + \cos^2(\omega_c t) + 2f(t) + 2\cos\omega_c t + 2f(t)\cos\omega_c t$$

$$\Rightarrow F\left(x(t)^2\right) = \frac{1}{2\pi} F(\omega) * F(\omega) + \frac{1}{4}\delta(\omega - 2\omega_c) + F(\omega) + \delta(\omega - \omega_c) + F(\omega - \omega_c) \quad (\omega > 0)$$

[6]

Filtering with a centre frequency at ω_c and bandwidth $\geq 2 \times$ BW of f picks out last two terms as required.

(note, highest component of $F(\omega) * F(\omega)$ is at $2\times$ bandwidth of $f(t) \ll \omega_c$)