SECTION A *Introductory Business Economics*

1. (a) In the worldwide market for coffee, what effect would the following events have on the equilibrium price and quantity? Illustrate using a supply and demand diagram.

   (i) An advertising campaign to promote the drinking of coffee [3]
   (ii) A period of bad weather, resulting in a lower coffee crop [3]

   (b) How does monopoly differ from perfect competition in terms of:

   (i) the number of firms in the market [1]
   (ii) the profits of firms in the long run [3]
   (iii) consumer welfare [4]

   (c) Disequilibrium unemployment occurs when the labour market fails to clear, so that labour supply does not equal labour demand.

   (i) Explain why the wage rate can be "sticky" in the short run, and how this can prevent the labour market from clearing [3]
   (ii) What policies can the government use to reduce unemployment [3]

2. (a) Describe the model of perfect competition [6]

   (b) Consider a duopoly with two firms, X and Y, both of which produce a similar product, and charge a price of £2 per unit of their product. At this price both firms make an annual profit of £10 million. However, both firms are considering lowering the price of their product to £1.80, in order to increase their sales. The payoff matrix of the firms is shown below, where the first payoff in each cell is for firm Y, and the second payoff is for firm X:

		Firm X's price	
		£2	£1.80
Firm Y's price	£2	£10m, £10m	£5m, £12m
	£1.80	£12m, £5m	£8m, £8m

   (i) Explain why each firm needs to consider the actions of the other firm when setting its price [2]
   (ii) What is the Nash equilibrium in this game? Explain. [6]

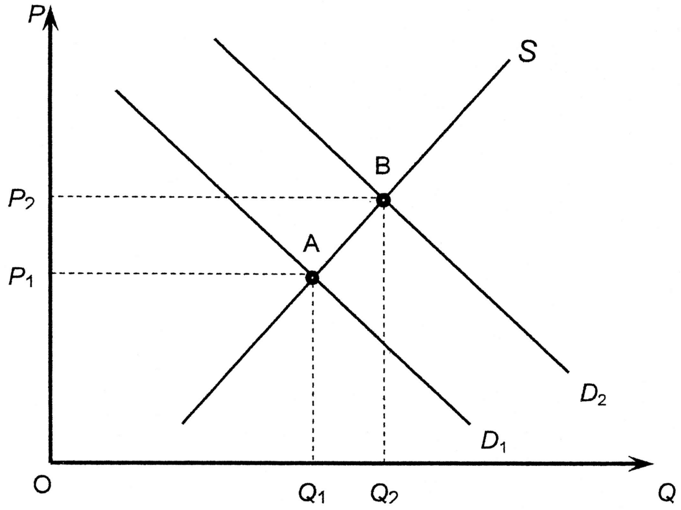
   (c) What policies can government implement to increase long-run economic growth? [6]

SECTION A *Introductory Business Economics*

1. (a) In the worldwide market for coffee, what effect would the following events have on the equilibrium price and quantity? Illustrate using a supply and demand diagram.

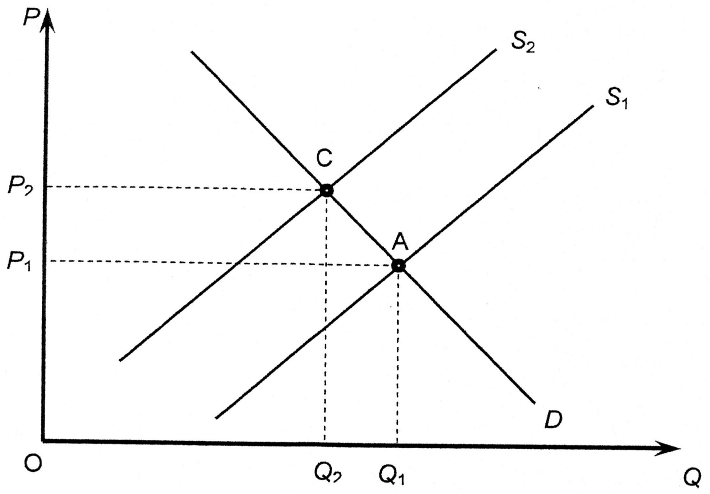
(i) An advertising campaign to promote the drinking of coffee [3]

An advertising campaign to promote drinking coffee would lead to an increase in the demand for coffee at all prices. As shown below, this is equivalent to a shift in the demand curve for coffee to the right. The new equilibrium is given by the intersection of the supply curve and the new demand curve, point B. In the new equilibrium, the quantity and price are both higher than in the previous equilibrium.



(ii) A period of bad weather, resulting in a lower coffee crop [3]

A lower coffee crop would lead to a fall in the supply of coffee at all prices, or a shift in the supply curve to the left. As shown in the diagram, this results in a new equilibrium at point C. In the new equilibrium the quantity is lower, but the price is higher.



(b) How does monopoly differ from perfect competition in terms of:

(i) the number of firms in the market [1]

In monopoly there is only one firm in the market, while under perfect competition there are many firms.

(ii) the profits of firms in the long run [3]

Under perfect competition firms can make a profit in the short run, but in the long run other firms may enter the market, and profits will be eroded. This occurs because under perfect competition there is free entry and exit of firms. Firms will enter the market until profits are reduced to zero.

Under monopoly there are barriers to entry, so that other firms are prevented from entering the market. Non-zero profits can therefore be maintained in the long run.

(iii) consumer welfare [4]

Perfect competition is the optimal market structure in terms of consumer welfare, because firms are producing in the most efficient way, prices are low, and the quantity produced and consumed is at its highest, satisfying consumer demand.

Under monopoly consumer welfare is lower because the monopolist has market power, and can set its price above the perfectly competitive level. At this higher price consumers are only able to buy a lower quantity at a higher price, and consumer welfare is reduced.

(c) Disequilibrium unemployment occurs when the labour market fails to clear, so that labour supply does not equal labour demand.

(i) Explain why the wage rate can be "sticky" in the short run, and how this can prevent the labour market from clearing [3]

Wage rates can be "sticky" in the short run because it is difficult for firms to reduce them, particularly during recessions. This occurs because workers resist wage cuts, or because firms are be reluctant to lower wages since this could result in low morale. In addition trade unions may negotiate wages that are above the equilibrium price, or the government may set a minimum wage rate that is too high. If the wage rate is above the equilibrium level, labour supply is greater than labour demand, and unemployment persists.

(ii) What policies can the government use to reduce unemployment [3]

Unemployment can be reduced by reducing real wages, or by stimulating aggregate demand during a recession. The government can also encourage unemployed workers to find new jobs by providing training opportunities and encouraging workers to move to other regions with lower unemployment rates. More interventionist policies are also possible, for instance, the government could provide grants to firms in areas of high unemployment.

2. (a) Describe the model of perfect competition [6]

The model of perfect competition makes a number of assumptions regarding the firms in the market:

- There are many firms
- Each firm produces a small amount
- The products are identical across firms, and there is no branding or advertising.
- There is perfect knowledge among all market participants

Because there are many firms, and each firm is relatively small, firms are price-takers in their markets, that is, they take the price of their product as given. Because the firms are assumed to be identical, each firm produces the same amount of the good, and the market is divided equally between the firms. Since there are many firms, there is no strategic decision making. While in the short run firms can make some profits, in the long run other firms enter the market, and profits are eroded. Under perfect competition there are therefore zero profits in the long run.

The model of perfect competition is useful as a benchmark against which to measure other types of market structure, because it is optimal in terms of consumer welfare. However, it is not often observed in practice, except in the case of commodity markets, or some forms of e-commerce.

This question can also be illustrated using a diagram.

(b) Consider a duopoly with two firms, X and Y, both of which produce a similar product, and charge a price of £2 per unit of their product. At this price both firms make an annual profit of £10 million. However, both firms are considering lowering the price of their product to £1.80, in order to increase their sales. The payoff matrix of the firms is shown below, where the first payoff in each cell is for firm Y, and the second payoff is for firm X:

		Firm X's price	
		£2	£1.80
Firm Y's price	£2	£10m, £10m	£5m, £12m
	£1.80	£12m, £5m	£8m, £8m

(i) Explain why each firm needs to consider the actions of the other firm when setting its price. [2]

In a duopoly, which is a form of oligopoly with only two firms, any action by one firm will affect the market share and profits of the other firm, and the other firm would therefore be expected to respond in some form. Given this situation of strategic thinking, each firm needs to take into account the reaction of the other firm when making decisions about pricing.

(ii) What is the Nash equilibrium in this game? Explain. [6]

The Nash equilibrium is the outcome from which neither player wants to deviate, given the actions of the other player. In this game, each firm will be better off reducing its price to £1.80, regardless of the actions of the other firm. This is the dominant strategy of each firm. The Nash equilibrium is the outcome where both firms reduce their price to £1.80, and both firms earn £8m in profits.

(c) What policies can the government follow to encourage long-run economic growth? [6]

In the short run, the government can increase the growth rate by encouraging the population to save more, although the government can only hope to increase the saving rate up to a point. Good students may point out the paradox of thrift – that increases in savings reduce effective demand and may deter investment, thus inhibiting medium to long term growth. In the long run, government can increase the growth rate by improving the productivity of the workforce. This can be achieved by promoting research and development (R&D), improving the education and training of the workforce, promoting efficiency in the organisation and management of firms, encouraging new firms to enter the market (entrepreneurship) and providing a competitive business environment.

Q3

~~P8, Q1~~

(a) Cut-and-cover construction.

- Advantages: relatively simple if tunnel is shallow, could be open excavation but more normally involves constructing retaining walls to support ground, and then excavating ground between them, propping as necessary, constructing tunnel structure and back-filling with soil. Especially advantageous if tunnel is shallow – depth of excavation not large, construction can be undertaken fairly quickly.
- Disadvantages: becomes more complex if tunnel is deeper, more propping required, deeper and higher strength walls necessary. Biggest disadvantage: highly disruptive, whether shallow or deep tunnel, involving closure of road, diversion of services, and noise – environmentally not attractive.
- Ground supported by retaining walls, which might be steel sheet piles driven or jacked into ground, walls comprising concrete bored piles (contiguous or secant pile walls), or reinforced concrete diaphragm walls.
- High water table would mean that open excavation problematic because dewatering pumps would be necessary to keep excavation dry. If retaining walls are constructed the walls msut be watertight to prevent leakage of water and potential loss of soil through leaking joints. [5]

(b) Bored tunnels.

- Shield tunnelling would be suitable, and if the ground is stable (but see below) tunnelling without a shield could be possible if sprayed concrete linings (SCL) are used. For the deeper tunnels, the maximum depth to bottom of tunnels = 20m, hence to axis of tunnels 17m (diameter = 6m). For open face tunnels entirely in the Gault Clay, stability ratio = (17 x 20)/150 = 2.3. This is very low which means that the deeper tunnels could safely be constructed with an open face.
- However, for the shallower tunnels, the minimum depth to the bottom of the tunnels is only 10m, which means that the crown of the tunnel would be 4m below ground level, and the top of the tunnel would be in the gravels, sands and silts with a high water table – this would mean that an open face is not possible (these soils would flow into the face). Hence a closed face tunnelling machine would be needed – either a slurry shield or an earth pressure balance (EPB) machine. [4]

(c) Tunnel linings

- For the deeper tunnel depths, sprayed concrete linings could be used as an alternative to shield tunnelling with pre-cast concrete or cast iron segmental linings.
- For the shallower tunnels sprayed concrete linings would not be suitable in such unstable ground conditions, and only shield tunnelling (closed face machine) with pre-cast concrete or cast iron segmental linings would be suitable. [2]

(d) Settlement damage

- Settlement can potentially cause damage to buildings if differential settlement occurs causing tensile strains in masonry, leading to cracking. Buildings tend to be more vulnerable to hogging deformation, with tensile strain (and hence cracking) occurring high up in the building. Sagging deformation with tensile strain occurring nearer the base of the building is generally less damaging, because the foundations restrain the tendency for cracking. Buildings directly above the tunnel tend to experience more sagging deformation, whereas buildings to one side (but still close to the tunnel) experience more hogging and are therefore more vulnerable to damage. [3]

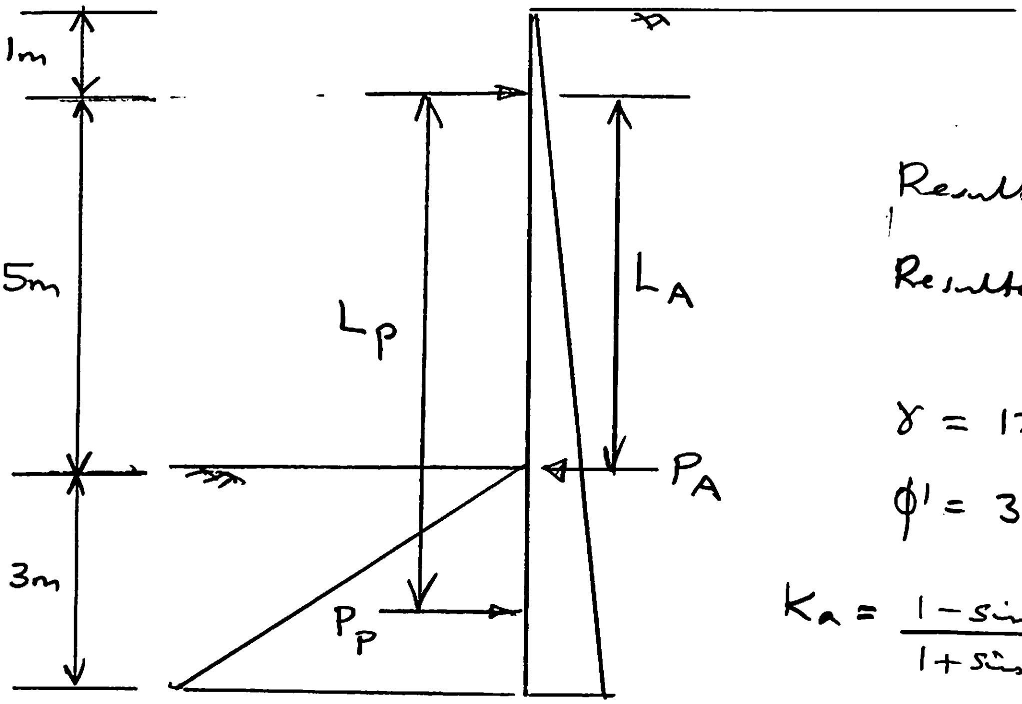
(e) Methods of preventing damage

- Steel ties can be used to prevent masonry from experiencing high tensile strains.
- The ground can be improved and stiffened beneath the building by grouting (injection of cement or chemicals) to reduce differential settlements
- Compensation grouting can be used. This involves the injection of grout (usually cement grout) into the ground between the building and the tunnel in a controlled manner. Grout is injected from horizontal steel tubes installed in the ground from a shaft (or perhaps from an existing tunnel). Control of the building differential settlement is achieved by injecting grout in carefully defined locations and in carefully specified small quantities – instrumentation is very important. [3]

(f) Role of instrumentation

- Instrumentation and monitoring provide forewarning of unexpected events, a means of facilitating adjustment of construction procedures in a safe and systematic manner, a means of controlling the quality of the work, records of movements of ground and buildings for repair purposes, and an important source of data for future projects. Typical examples of instrumentation are settlement monitoring stations, subsurface rod extensometers to measure vertical movements, subsurface inclinometers to measure horizontal movements, and piezometers to measure pore pressures. [3]

Question 4 solution



Resultant active force P_A

Resultant passive force P_p

$\gamma = 17 \text{ kN/m}^3$

$\phi' = 35°$

$K_a = \dfrac{1-\sin\phi'}{1+\sin\phi'} = 0.27$

$K_p = 1/K_a = 3.68$

(a) Factor of safety against rotation about prop

$$F = \frac{P_p . L_p}{P_A . L_A}$$

$P_A = \frac{1}{2} \times 0.27 \times 17 \times 9^2 = 186 \text{ kN/m}$

P_A acting 2/3 down length of wall $\Rightarrow L_A = 5\text{m}$

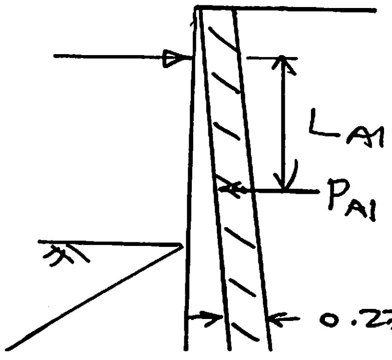
$P_p = \frac{1}{2} \times 3.68 \times 17 \times 9^2 = 281 \text{ kN/m}$

P_p acting 2/3 down from bottom of excavation

$\therefore L_p = 7\text{m}$

hence $F = \dfrac{281 \times 7}{186 \times 5} = \underline{\underline{2.1}}$ [7]

(b)(i) Effect of 1.5m surcharge



P_A, L_{A1}, P_p, L_p as before

Additional force P_{A1}, lever arm L_{A1}

$P_{A1} = 0.27 \times 17 \times 1.5 \times 9 = 62 \text{ kN/m}$

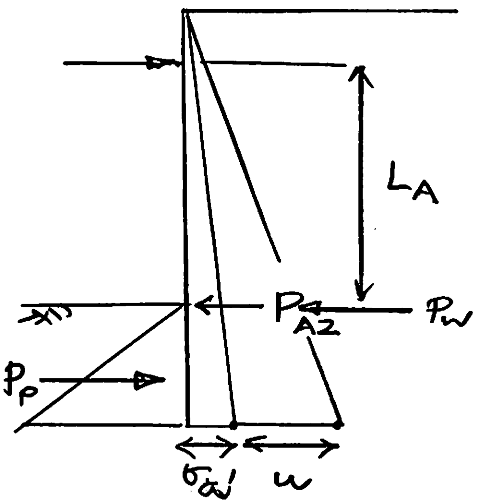
$L_{A1} = 3.5\text{m}$

$$\therefore \; F = \frac{281 \times 7}{(186 \times 5) + (62 \times 3.5)} = \frac{1967}{930 + 217} = \underline{\underline{1.71}}$$

[3]

(b)(ii) Effect of flash flood.

$\gamma = 20 \text{ kN/m}^3$ in sand behind wall and water pressure acting over full depth of wall



P_p, L_p as for (a)

at bottom of wall

$\sigma_v = 20 \times 9 = 180 \text{ kN/m}^2$

$u = 10 \times 9 = 90 \text{ kN/m}^2$

$\therefore \; \sigma_v' = \sigma_v - u = 90 \text{ kN/m}^2$

$\sigma_a' = 0.27 \times \sigma_v' = 0.27 \times 90 = 24.3 \text{ kN/m}$

$P_{A2} = \frac{1}{2} \times 24.3 \times 9 = 109 \text{ kN/m}$

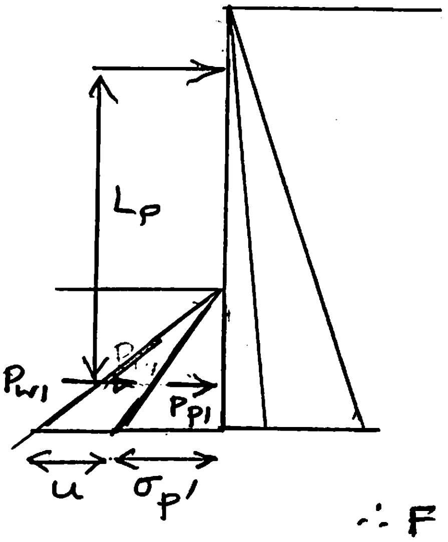
$L_A = 5\text{m}$

$P_W = \frac{1}{2} \times 10 \times 9^2 = 405 \text{ kN/m}$

$$\therefore \; F = \frac{281 \times 7}{(109 + 405) \times 5} = \underline{\underline{0.76}}$$ (wall will fail)

[5]

(b)(iii) Effect of further flooding



P_{A2}, P_W acting on active side as for (b)(ii)

On passive side.

$\sigma_v = 20 \times 3 = 60 \text{ kN/m}^2$ }
$u = 10 \times 3 = 30 \text{ kN/m}^2$ } $\Rightarrow \sigma_v' = \sigma_v - u = 30 \text{ kN/m}^2$

$\therefore \; \sigma_p' = k_p \sigma_v' = 3.68 \times 30 = 110 \text{ kN/m}^2$

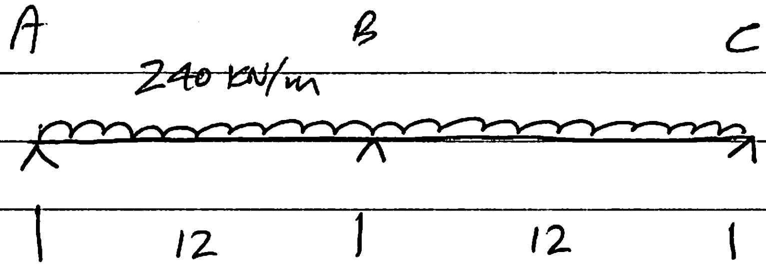
$\therefore \; P_{p1} = \frac{1}{2} \times 110 \times 3 = 166 \text{ kN/m}$

$P_{W1} = \frac{1}{2} \times 30 \times 3 = 45 \text{ kN/m}$

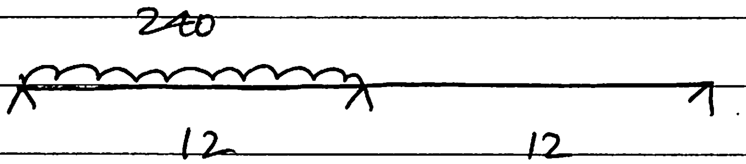
$L_p = 7\text{m}$ (as before)

$$\therefore F = \frac{(166 + 45) \times 7}{(109 + 405) \times 5} = \frac{1477}{2570} = \underline{\underline{0.57}}$$ (wall will fail)

[5]



Data book 4.5.3



$M_B = \frac{wL^2}{16}$ ∴ $R_C = -\frac{wL}{16}$ = −180 kN (i.e. downwards)

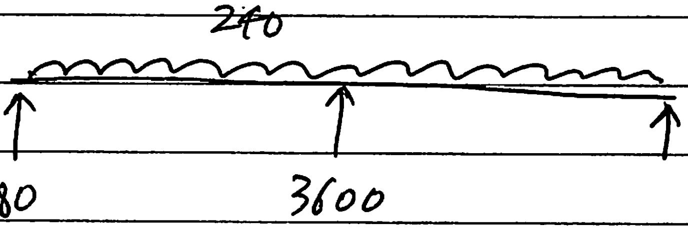
Overall equilibrium about B

$R_A.\cancel{12} + 180.\cancel{12} = 240.\cancel{12}.6$

~~[illegible]~~

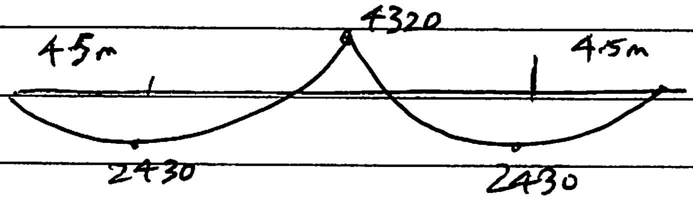
$R_A = 1260$

∴

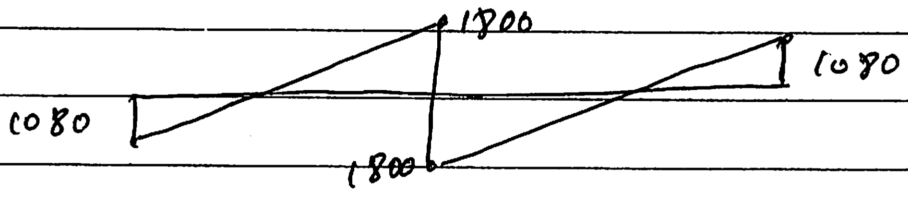


1260 − 180
= 1080

BM



SF



$0.15 \ f_{cu} \ bd^2 = 0.15 \,.\, 40 \,.\, 0.5 \,.\, 1^2 \,.\, 10^9$ (Nmm

$= 3 \,.\, 10^9$ Nmm

$= 3000$ KNm

∴ Sagging moment can be singly reinforced.

Guess $x = 0.5$ ∴ lever arm $= 0.75$ m

∴ $A_s . f_y . 0.75 = 2430$ KNm (datasheet)

$\Rightarrow A_s = 7043$ mm^2

$$x = 2.175 \ \frac{460}{40} \,.\, \frac{7043}{0.5 \,.\, 1 \,.\, 10^6} = 0.35$$

$$\text{lever arm} = 1\left(1 - \frac{0.35}{2}\right) = 0.823 \text{ m}$$

∴ $A_s = 6418$ mm^2

6 No 40mm bars $= 7536$ mm^2

8 No 32mm bars $= 6432$ mm^2.

Hogging moment (4320 KNm cannot be singly reinf)

$$\therefore 4320 - \underbrace{0.15 f_{cu} b d^2}_{= 3000} = 1320 \text{ KNm}$$

must be reinforced by compression steel.

$$\therefore 0.75 \,.\, f_y \,.\, A_s' \left(1 - \frac{d'}{1}\right) = 1320 \text{ KNm}$$

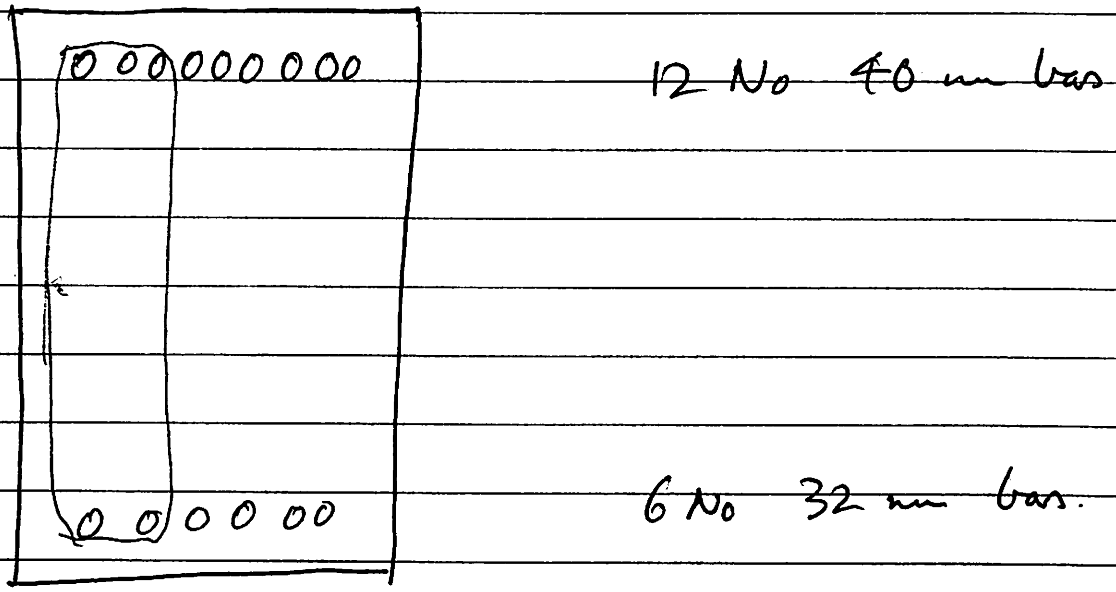
say $0.05 + 0.03 = 0.08$m

∴ $A_s' = 4160$ mm^2 = 6 No 32mm bars.

$0.87 f_y A_s = 0.75 . f_y . A_s' + 0.2 f_{cu} b d$

$0.87 . 460 . A_s = 0.75 . 460 . 4160 + 0.2 . 40 . 500 . 1000$

$\Rightarrow A_s = 13580 \text{ mm}^2$ ⇒ 11 No 40mm bars.
(probably use 12)



Shear max at support = 1800 kN.

$$v_c = 0.68 \left(\frac{100 A_s}{bd}\right)^{0.33} . \left(\frac{400}{d}\right)^{0.25}$$

$$= 0.68 \left(\frac{100 . 13580}{500 . 1000}\right)^{0.33} \left(\frac{400}{1000}\right)^{0.25}$$

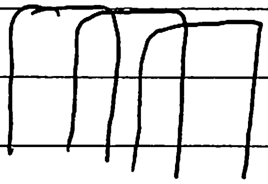
$$= 0.68 . 1.39 . 0.79 = \underline{\underline{0.75 \text{ N/mm}^2}}$$

Applied shear = 1800

$\therefore v = \frac{1800 \, 10^3}{bd} = 3.6$

$\therefore v_s = 3.6 - 0.75 = 2.85$ N/mm^2

With this number of bars probably use multiple links



Say 6 links . guess 20 mm bars.

$A_{sq} = 6 \times 314$
$= 1884$ mm^2

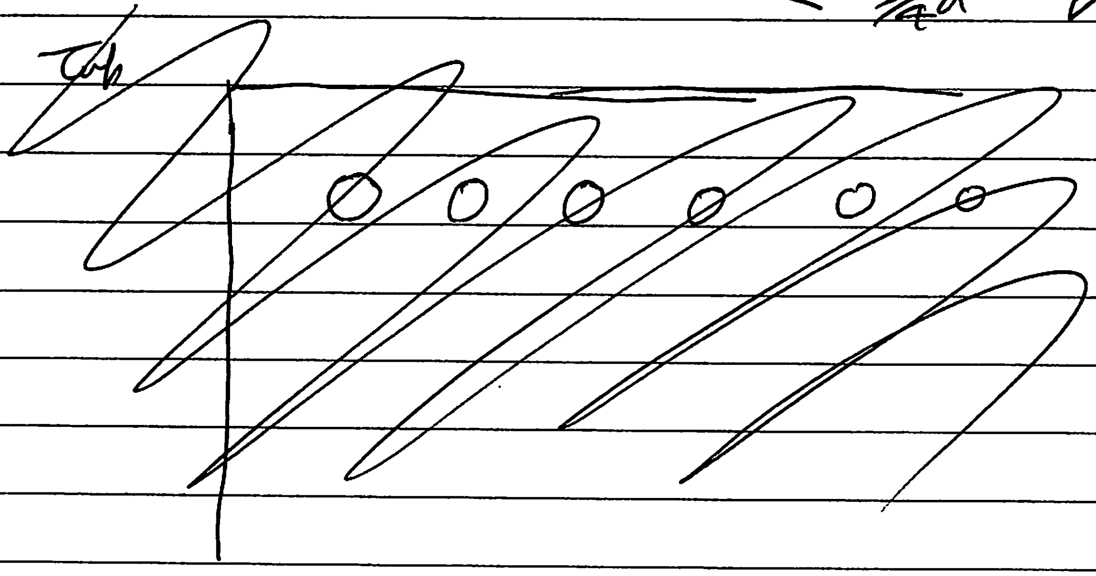
$$\therefore 3.6 = \frac{0.87 \, f_y \, A_{sq}}{b s}$$

$$= \frac{0.87 \, . \, 460 \, . \, 1884}{500 \, . \, s}$$

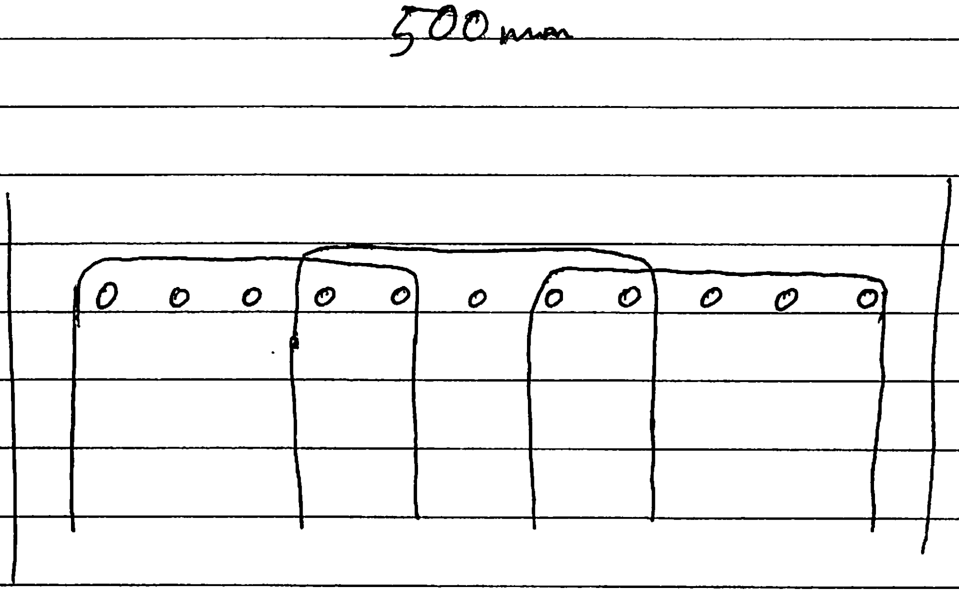
$\Rightarrow \underline{\underline{s = 418 \text{ mm}}}$ about right.

(say $s = 400$ mm
$< \frac{3}{4} d$ ✓✓

∴

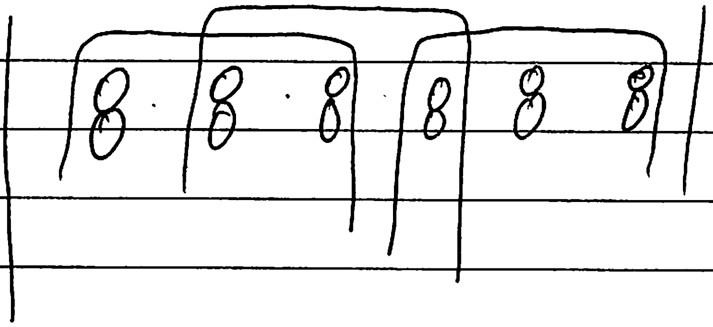


Check width



bars 11 x 40 = 440 mm No room for stirrups

12 No 40mm bars in pairs.



6 x 40 = 240
6 x 20 = 120
360
2x cover 100
260
5 gaps of 52mm

OK.

Paper 8 Selected Topics, Section C, 2010 Crib – MPFS

6 (a)

Overview:

- Need to move away from limited fossil fuels
- Need to control $C0_2$ emissions

Role of renewables

- Reduce usage
- Use energy sources with low $C0_2$ impact, including many renewables (e.g. tidal, solar ...)
- In practical terms renewables in the UK can only make a small contribution to the current usage, so need to reduce, especially in heating and transport.

Technical challenges

- durability, especially at sea
- efficient at low speeds and cope with storm loading, unpredictable power output
- aesthetics and noise considerations
- grid connection and generation
- cost of large structures
- power transmission from windy to required areas

Social factors

- acceptability
- amount of surface area available

This was answered well, drawing appropriately on material in the lectures. Marks were lost for not answering the various aspects raised in the question.

(b) (i) Cage rotor induction generator means that there is no possibility of varying the torque-speed characteristic of the generator eg as you could by adding in extra rotor resistance, or using slip energy recovery in a slip-ring generator. Thus, since the generator has its stator windings connected directly to the 3-phase 50 Hz grid, and given that it will operate on the steep part of its torque-speed characteristic then its speed is approximately synchronous speed irrespective of the torque provided by the turbine, hence a fixed speed system.

Advantages: Cage rotor generators have no rubbing contacts to wear out; cage rotors are more robust than wound rotors and are cheaper to manufacture; no slip frequency converter required; control much simpler.

Disadvantages: This system can't extract the maximum available power from the wind at all wind speeds since its power coefficient will vary depending on the tip-speed ratio; it can't generate reactive power and will therefore need a local source of reactive power.

(ii) Blade radius $R = 100/2 = 50$ m. Tip-speed ratio $\lambda = 8 = \omega R/v$ and $v = 6\text{ms}^{-1}$ giving $\omega = 0.96$ rads^{-1} = 9.2 rpm.

Assuming generator is operating very close to synchronous speed then its angular speed is $2\pi f/p = 2\pi.50/6 = 52.3$ rads^{-1} = 500 rpm.

(iii) Gearbox needs to transform the 9.2 rpm speed of the turbine into the 500 rpm speed of the generator so ratio is 9.2:500 = 1:54.3.

Ignoring any frictional losses in the main bearings and gearbox the output power of the turbine of 300 kW will be the same as the generator shaft power. From $P = 300000 = T\omega$ and with $\omega = 52.3$ rads^{-1} gives T=5.7 kNm.

Most students gave a good explanation of the virtually fixed speed nature of an induction generator connected directly to the 50 Hz grid, although very few mentioned that the cage construction precluded the use of speed control by slip energy recovery. On the calculation, the most common mistake was to mix up poles with pole-pairs when calculating the generator speed, which meant that answers thereafter were out by a factor of 2.

7. (a) Rate of flow of kinetic energy = $\frac{1}{2}\dot{m}V_0^2 = \frac{1}{2}(\rho A V_0)V_0^2$ so the denominator represents the kinetic energy available in the air stream.

(b) Use the standard control volume analysis to derive the Betz limit:
Bernouilli applied upstream and downstream of the rotor gives the pressure drop across the rotor as $\Delta p = \frac{1}{2}\rho\left(V_0^2 - V_1^2\right)$ where V_1 is the downstream velocity to give a thrust T on the rotor $A\Delta p$.
From momentum flux on a control volume the thrust applied to the rotor by the fluid is given by $T = \dot{m}(V_0 - V_1) = \rho A u(V_0 - V_1)$
Equating these two expressions for T gives $u = \frac{1}{2}(V_0 + V_1)$.
Power $P = Tu = \rho A u^2(V_0 - V_1)$

$$C_p = \frac{\rho A u^2(V_0 - V_1)}{\frac{1}{2}\rho A V_0^3} = 4\left(\frac{u}{V_0}\right)^2\left(1 - \frac{u}{V_0}\right) = 4(1-a)^2 a \text{ using } (V_0 - V_1) = 2(V_0 - u)$$

Differentiating with respect to a, $\Rightarrow -2a(1-a) + (1-a)^2 = 0$ to give a maximum for $a = 1/3$, with the corresponding value of $C_p = 16/27$.

(c) (i) Total extracted power:
$P = P_1 + P_2 = ½\rho A V_o^3 C_{P1} + ½\rho A V_1^3 C_{P2}$;

Where $C_{P1} = \frac{P}{½\rho A V_o^3}$ and $C_{P2} = \frac{P}{½\rho A V_1^3}$.

Power coefficient: $C_P = \frac{P_1 + P_2}{½\rho A V_o^3} = C_{P1} + (\frac{V_1}{V_o})^3 C_{P2}$.

Since $u_1 = ½(V_o + V_1) \Rightarrow \frac{V_1}{V_o} = 2\frac{u_1}{V_o} - 1 = 2(1-a_1) - 1 = 1 - 2a_1$.

We thus get $C_P = C_{P1}(a_1) + (1-2a_1)^3 C_{P2}(a_2) = 4a_1(1-a_1)^2 + (1-2a_1)^3 4a_2(1-a_2)^2$ (1)

(ii) The optimum value of C_{P2} is 16/27 with $a_2 = 1/3$, since $C_{P2} = C_{P2}(a_2)$. The optimum value of the total expression is obtained by differentiation: $dC_P / da_1 = 0$. We get
$4(1-a_1)^2 - 8a_1(1-a_1) - 6(1-2a_1)^2 C_{P2} = 0 \Rightarrow 6(1-2C_{P2})a_1^2 - 4(2-3C_{P2})a_1 + (2-3C_{P2}) = 0$.
Utilizing $C_{P2} = 16/27$ and solving for a_1, we get $a_1 = 1/5$. Inserting into eq. (1), we get
$C_{P1} = 4a_1(1-a_1)^2 = 4(\frac{1}{5})(\frac{4}{5})^2 = \frac{64}{125}$; $C_P = \frac{64}{125} + (\frac{3}{5})^3(\frac{16}{27}) = \frac{16}{25} = 64\%$

A popular question. Most candidates gave a satisfactory answer to part (a), though not all pointed out that the denominator is the kinetic energy flux in the stream tube. Part (b) was generally very well answered, full marks were awarded if the candidate correctly used Bernoulli's and momentum equations to determine the velocity at the rotor plane and to derive the expression for Cp before finding the optimum condition. Part (c) was less well answered. Although many candidates did find a correct expression for Cp for the two in-line wind turbines, very few realised that the presence of the second downstream turbine means that it is not optimal to run the first with an induction factor of 1/3.

8.

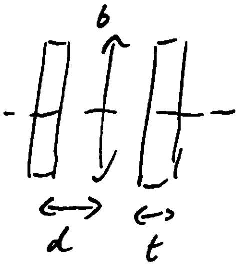
(a) Materials. Choose strong/stiff and lightweight materials such as CFRP, wood or bamboo.

Structural design. The shape of the outer shell will be strongly influenced by aerodynamics. However the effect of load distribution (e.g. amount of taper) will also affect the tradeoff between power, weight and cost. Use a shape efficient structure with the material at the edges, i.e. a beam structure, but allow for both flapwise and edge wise loading (worth adding a sketch here). Use a sandwich structure for the skins. Optimise the amount of the reinforcement going down the blade to account for the changing moment and effect on tip deflection. Use of composites also allows for easier manufacturing of complex shapes.

Using orthotropic materials (CFRP, wood or bamboo) allows for tailoring of the stiffness, so use unidirectional material (UD) along the spars to take the flapwise bending loads, UD at the leading and trailing edge for the edgewise bending and include off-axis components elsewhere for shear/torsional stiffness and strength.

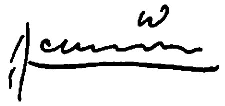
A slightly harder and correspondingly less popular question. Part (a) was not so well answered, with the obvious comments about shape and material not spelled out. Part (b) (i) had two parts, one where the beam second moment of area changed with position, one where it didn't. Very few did the integration as required where there was a change, simply using the databook formula for fixed I, though this gave the right answer in terms of dependence on L. (b) (ii) was again relatively standard structural analysis, reasonably well answered. The importance of the scaling with L was not well identified.

8(b)(i) Self-weight



$I_{sw} = \frac{1}{12} 2t.b^3 = \frac{tb^3}{6}$

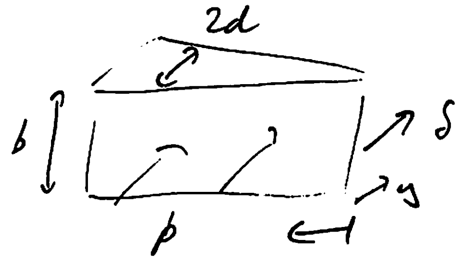
- I_{sw} is constant



$w = 2\rho g t b$ (ρ = density
E = Young's modulus)

Data book $\delta_{sw} = \frac{wL^4}{8EI} = \frac{3}{2}\frac{\rho g L^4}{Eb^2} \Rightarrow \delta_{sw} \sim L^2$ since $b \sim L$

Wind loading



$w = pb$

Now $I_{wL} = 2btd^2 = 2bt\, d_0^2 \left(\frac{x}{L}\right)^2$ is a function of x

$M = pb\frac{x^2}{2} \Rightarrow \frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{pL^2}{4Etd_0^2}$

key point

$\frac{dy}{dx} = \frac{pL^2(x-L)}{4Etd_0^2}$ (B.C. $x = L$, $\frac{dy}{dx} = 0$)

$y = \frac{pL^2}{4Etd_0^2}\left(\frac{x^2}{2} - Lx + \frac{L^2}{2}\right)$ (B.C $x = L, y = 0$)

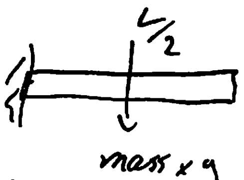
$\Rightarrow \delta_{wL} = \frac{pL^4}{8Etd_0^2}$

but t and d_0 scale with $L \Rightarrow \delta_{wL} \sim L$

[N.B most candidates failed to realise that I is a function of x, though the (incorrect) data book formula for constant EI gives the right scaling]

Relatively unpopular question, though reasonable average from those attempting it.

8(b)(ii) Self weight

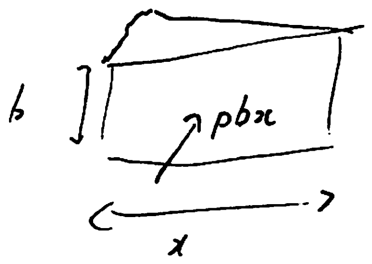


$$\sigma_{sw} = \frac{My}{I_{sw}} \Rightarrow \sigma_{sw_{max}} = \frac{\overbrace{2btL\rho}^{mass} g \cdot \frac{L}{2} \cdot \frac{b}{2}}{tb^3/6} = \frac{3\rho g L^2}{b}$$

$\nearrow$ assume max at root (I constant while M increases)

Wind loading

$$\sigma_{wL} = \frac{My}{I_{wL}} = \frac{pbx^2}{2} \cdot d \cdot \frac{L^2}{2btd_0^2 x^2} = \frac{pdL^2}{4td_0^2}$$



Max when $d = d_0$ at root

$$\Rightarrow \sigma_{max(wL)} = \frac{pL^2}{4td_0}$$

Combining stresses at root, where maximum for both load cases

$$\sigma_{wL} + \sigma_{sw} = \sigma_f = \frac{pL^2}{4td_0} + \frac{3\rho g L^2}{b}$$

$$\rightarrow t = \frac{pL^2}{4d_0} \frac{1}{\sigma_f - \frac{3\rho g L^2}{b}}$$

Most candidates got the idea but not the detail

$$\text{mass} = 2btL\rho = \frac{pL^3 b\rho}{2d_0} \frac{1}{\sigma_f - \frac{3\rho g L^2}{b}}$$

$\nwarrow$ self weight term

(c) As L increases so deflection due to self-weight and effect of self-weight on mass-to-avoid-failure increase.
⇒ switch to CFRP for long blades to reduce self-weight terms.

MPFS 22/6/10

9) a) $S = \frac{VL/D}{g\,SFC} \ln \frac{W_{start}}{W_{finish}}$, $V = M\sqrt{\gamma RT} = 0.85\sqrt{(1.4)(287.3)226}$

$= \underline{256.5 \text{ m/s}}$

$$S = \frac{(256.5)20}{(9.81)1.61} \ln\left(\frac{326.2 + 58.5 + 250.6}{326.2 + 58.8}\right) = 16300 \text{ km}$$

[2]

b) For $S/2 = 8150$ m

$$\frac{W_{start}}{W_{end}} = \exp\left\{\frac{S g\, SFC}{VL/D}\right\} = \frac{(8150\times10^3)(9.81)(1.61\times10^{-5})}{(256.5)(20)},$$

$$= 1.285$$

Amount of fuel used

$$W_f = W_{start} - W_{end} = \left(\frac{W_{start}}{W_{end}} - 1\right)W_{end}$$

$$= (1.285-1)385 = 109.7 \text{ tonne}$$

% fuel saving is $\left(\frac{250.6 - 219.4}{250.6}\right)100 = 12.4\%$

[2]

c) Disadvantages: longer journey time, more engine/ airframe strain and more environmental noise

Advantages: empty weight could be reduced because craft needs to support less weight, fuel savings and environmental benefit for CO_2 foot print.

[3]

d) $$S = \frac{VL/D}{g\,SFC} \ln \frac{W_{start}}{W_{end}} = \frac{VL/D}{g\,SFC} \ln\left\{\frac{W_e + W_p + W_f}{W_e + W_p}\right\}$$

So $$\frac{W_e + W_p + W_f}{W_e + W_p} = \exp\left\{\frac{S g\, SFC}{VL/D}\right\}$$

$$1 + \frac{W_f}{W_e + W_p} = \exp\left\{\frac{S g\, SFC}{VL/D}\right\}$$

$$W_f = (W_p + W_e)\left\{ \exp\left(\frac{S\, g\, SFC}{VL/D}\right) - 1 \right\}$$

Fuel burn per kg (payload) per km

$$F' = \frac{W_f}{S W_p} = \frac{1}{S}\left\{ 1 + \frac{W_e}{W_p} \right\} \exp\left(\frac{S\, g\, SFC}{VL/D}\right) - 1 \Bigg\} \quad [5]$$

e) To minimise F' make airframe and engine lighter (A380 → composite) avoid half empty planes (i.e maximise aircraft load factors). Minimise SFC through higher thermal efficiency and better engine-airframe match. Maximise VL/D. [3]

f) $$F' = \frac{W_f}{S W_p} = \frac{1}{S}\left(1 + \frac{W_e}{W_p}\right)\left(\exp\left(\frac{S g\, SFC}{VL/D}\right) - 1\right)$$

$$= \frac{1}{16300\times 10^3}\left(1 + \frac{326.2}{58.8}\right)\left\{ \exp\frac{(16300\times 10^3)(9.81)(1.61\times 10}{(256.5)(2.)} - 1 \right\}$$

$$= 0.262 \times 10^{-3} \ \mathrm{kg/(kg\, km)}$$

$$M_{CO_2} = 8 \times 44, \quad \dot{M}_{\text{fuel burnt}} = (8)(12) + 18$$

$$\frac{\dot{M}_{CO_2}}{S\, N_{pass}} = \left(\frac{M_f}{S M_p}\right)\left(\frac{M_{CO_2}}{M_f}\right) M_p = \frac{(0.262\times 10^{-3})(8\times 44)(100)}{(8\times 12) + 18}$$

$$= 81 \text{ g } CO_2 \text{ per passenger-km}$$

[4

Q4)

a) $\eta_p = \dfrac{\text{power to aircraft}}{\text{power to jet}}$

Power to aircraft = flight speed × net thrust $= VF_N$

$= V\{(\dot{m}_{air} + \dot{m}_f)V_j - \dot{m}_{air}V\}$

Power to jet = ke increase $= \frac{1}{2}\{(\dot{m}_{air} + \dot{m}_f)V_j^2 - \dot{m}_{air}V^2\}$

Above ignores energy going into raising jet temperature.

$$\eta_p = \frac{V\{(\dot{m}_{air} + \dot{m}_f)V_j - \dot{m}_{air}V\}}{1/2\{(\dot{m}_{air} + \dot{m}_f)V_j^2 - \dot{m}_{air}V^2\}}$$

Ignoring $\dot{m}_f$ since $\dot{m}_f \ll \dot{m}_a$ gives

$$\eta_p = \frac{\dot{m}_{air}}{\dot{m}_{air}} \frac{V(V_j - V)}{1/2\,(V_j^2 - V^2)} = \frac{2V}{V + V_j}$$

(i) Civil aircraft need high efficiency. When $V = V_j$ $\eta_p = 1$. If $V_j \sim V$ a large $\dot{m}$ is needed to get thrust. This means a high bypass ratio engine (BPR)

(ii) A high BPR ~~means~~ for a given core means a larger and heavier engine. The larger nacelle will increase drag. For very high BPR, wings would need to be higher from ground → more heavy undercarriage. Transportation of fan is an issue (spare engines moved in 747 freighter). (iii) Typical BPR 6-9.

(iv) Noise will generally decrease with BRR [7]

b) $\eta_p = \dfrac{2V}{V + V_j} = \dfrac{(2)260}{260 + 360} = 0.838$ [2]

c) $F = \dot{m}(V_j - V) \rightarrow \dot{m} = \dfrac{26 \times 10^3}{V_j - V} = \dfrac{26 \times 10^3}{360 - 260} = 260$ kg/s

$\dot{m}_{TOT,NEW} = \dot{m}_{TOT,OLD}(1.15)^2 = (260)(1.15)^2 = 343.85\ \frac{kg}{s}$

$V_{j,NEW} = \dfrac{F}{\dot{m}_{TOTAL,NEW}} + V = \dfrac{26000}{343.85} + 260 = 335.6$

$\eta_{p,NEW} = \dfrac{2V}{V + V_j} = \dfrac{(2)(250)}{250 + 335.6} = 0.853.$ [3]

d) The kinetic energy ~~thru~~ of the air through the engine divided by the thermal energy made available by burning the fuel gives η_{th}. It can be increased by increasing the turbine inlet temperature, (providing pressure ratio increases by a proportionate amount).

The thermal efficiency is generally considerably lower than the propulsive. [3]

e) $\eta_{th} = \dfrac{\dot{m}_{air}(V_j^2 - V^2)}{2\dot{m}_{fuel}\,LCV} \longrightarrow \dot{m}_{air}V_j = \dfrac{2\eta_{th}\dot{m}_{fuel}\,LCV}{V_j}$

$$F_g = \frac{2\eta_{th}\dot{m}_{fuel}\,LCV}{V_j}$$ [2]

f) $\eta_o = \dfrac{\text{useful work}}{\text{Thermal energy from fuel}} = \eta_p\eta_{th}$

$$= \frac{\text{Thrust} \times \text{Speed}}{\dot{m}_f\,LCV} = \frac{\text{Thrust}}{\dot{m}_f}\,\frac{\text{Speed}}{LCV} = \frac{1}{SFC}\,\frac{V}{LCV}$$

$$\eta_o = \eta_{th}\eta_p = \frac{\dot{m}_a(V_j - V^2)}{2\dot{m}_f\,LCV}\,\frac{2V}{V+V_j} = \frac{\dot{m}_a(V_j+V)(V_j-V)V}{\dot{m}_f\,LCV\,(V+V_j)}$$

$$= \frac{\dot{m}_a V(V_j-V)}{\dot{m}_f\,LCV}$$ [2]

or

$$\eta_o = \frac{\text{Thrust Speed}}{\dot{m}_f\,LCV} = \frac{\dot{m}_{air}V(V_j-V)}{\dot{m}_f\,LCV}$$ [1]

3) 11 $$p_{01} = p_{amb}\left\{1 + \frac{\gamma-1}{2}M^2\right\}^{\frac{\gamma}{\gamma-1}} = 0.366 \text{ bar}$$

$$T_{01} = T_{amb}\left\{1 + \frac{\gamma-1}{2}M^2\right\} = 248.6K$$

a) $$\frac{\dot{m}\sqrt{c_p T_0}}{A p_0} = \frac{\gamma}{\sqrt{\gamma-1}} M \left(1 + \frac{(\gamma-1)M^2}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

For $M = 0.6$ $\quad \frac{\dot{m}\sqrt{c_p T_0}}{A p_0} = 1.0781 \quad A = \frac{\dot{m}\sqrt{c_p T_{01}}}{1.0781\, p_0}$

$$\frac{60\sqrt{1005(248.6)}}{(1.0781)\,0.366\times10^5} = 0.760 \text{ m}^2$$ [4]

b) $A = \pi(r_0^2 - r_i^2) \longrightarrow r_i = \sqrt{\underset{0.6}{r_0^2} - \frac{A}{\pi}} = 0.343 \text{ m}$ [1]

c) $r_{mean} = \frac{r_i + r_0}{2} = 0.47$, $U_{mean} = \Omega r_{mean} = \frac{(6000)2\pi}{60} r_{mean} = 295 \text{ m/s}$ [1]

d) $\frac{T_{02}^{isen}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 16^{1/3.5} = 2.2$, $T_{02,isen} = 2.2T_{01} = 546.92 K$

$$\eta_c = \frac{W_{ideal}}{W_{actual}} = \frac{T_{02}^{isen} - T_{01}}{T_{02} - T_{01}} \longrightarrow T_{02} = T_{01} + \frac{(T_{02}^{isen} - T_{01})}{\eta}$$

$$= 248.6 + \frac{(546.92 - 248.6)}{0.9}$$

$$T_{02} = 580K$$

$\frac{\Delta h_0}{U_{mean}^2} = 0.4 \quad \Delta h_{0,stage} = 0.36\, U_{mean}^2 = (0.36)(295)^2 = 31329 \text{ J/kg}$

$\Delta h_{0,compressor} = 1005(T_{02} - T_{01}) = 333057 \text{ J/kg}$

$$\text{Number of Stages} = \frac{\Delta h_{0,compressor}}{\Delta h_{0,stage}} = 11 \text{ stages}$$ [4]

e) No, the temperature rises will be. The pressure ratio is related to temperature ratio. The same temperature rise gives a smaller temperature ratio, as temperature increases, towards the rear stages [3]

f) $\text{Torque}\,\Omega = \dot{m}\,\Delta h_{o,compressor} = \text{Shaft power}$

$$\text{Torque} = \frac{\dot{m}\,\Delta h_{o,compressor}}{\Omega} = \frac{(60)^2(333057)}{(6000)\,2\pi} = 31804.6 \text{ Nm}$$

[2]

g) For the turbine $\frac{\Delta h_o}{(U_{mean})^2} = 1.5$ and 2 stages

$$W_{comp} = W_{turbine} \text{ or } \Delta h_{o,comp} = \Delta h_{o,turbine}$$

$$\Delta h_{o,turbine} = 3\,U^2_{mean,turbine} = \Delta h_{o,comp}$$

$$\Omega r_{mean,turbine} = U_{mean,turbine} = \sqrt{\frac{\Delta h_{o,comp}}{3}}$$

$$r_{mean,turbine} = \frac{1}{\Omega}\sqrt{\frac{\Delta h_{o,comp}}{4}} = \frac{1}{\frac{6000\,(2\pi)}{60}}\sqrt{\frac{333057}{3}}$$

$$= 0.53 \text{ m}$$

[3]

h) For the turbine the pressure gradient is favourable i.e. the blade flow more stable and so fewer stages are needed. For the compressor the pressure gradient is the opposite and this makes the flow unstable and hence the compression process needs to be done more gradually.

CRIB

12(a) $\sigma = N\,e\,\mu$

(b) I = 1.0e-6, W = 200 e-6, L= 20 e-6, t = 50e-9.

J = I/Wt = e-6/ 200e-6 * 50 e-9 = 1e+5 A/m2.

Vds = 5 V.

Eds = Vds/L = 5 / 20e-6 = 2.5e+5.

σ = J/E = 1e+5/ 2.5e+5 = 0.4

N = σ/e.μ = 0.4/ 1.6e-19 * 5e-5 = 5 e+22 m-3

(c)

If dielectric thickness = 0,
$Vg = Ne.t^2/2\varepsilon$, $Eg = N.e.t/\varepsilon$

E1 = 2.5e22 * 1.6e-19 * 5e-8/ (12 * 8.85e-12) = 1.88e+6 V/m

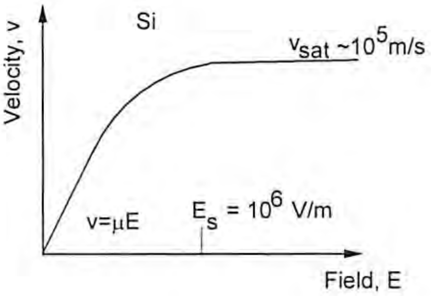
$V_{tot} = N*e*(t/\varepsilon)*(t/2 + d2) = E1*(t/2 + d_2) = 1.88\,e{+}6\,(5e{-}8/2 + 2e{-}7) = 0.424$ V

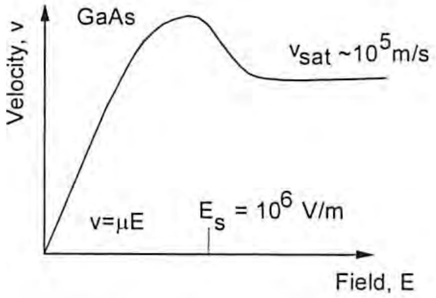
E

13 doping varies the carrier density in s/c
in metal, each atom 'donates' its conduction electrons, a fixed number of electrons, so the conductivity is ~ fixed.

b/
Si vs GaAs

Si native oxide vs bad native oxide
GaAs mobility much higher for 3-5.
Si cheaper, more established technology





phonon scattering causes limiting velocity

d/
$v = \mu E$, $t = L/v$, so $t = L/\mu E_d$

$F = ma = mv/\tau$, $F = eE$, $v = \mu E$, so $\mu = e\tau/m$.

Energy = $\frac{1}{2} mv^2$, find m from above, take v = critical velocity and hence find Energy.

$\tau = \mu m^*/e$ or $\tau = 9.1 \times 10^{-31} \times 0.15 / 1.6 \times 10^{-19} = 8.5 \times 10^{-13}$ s

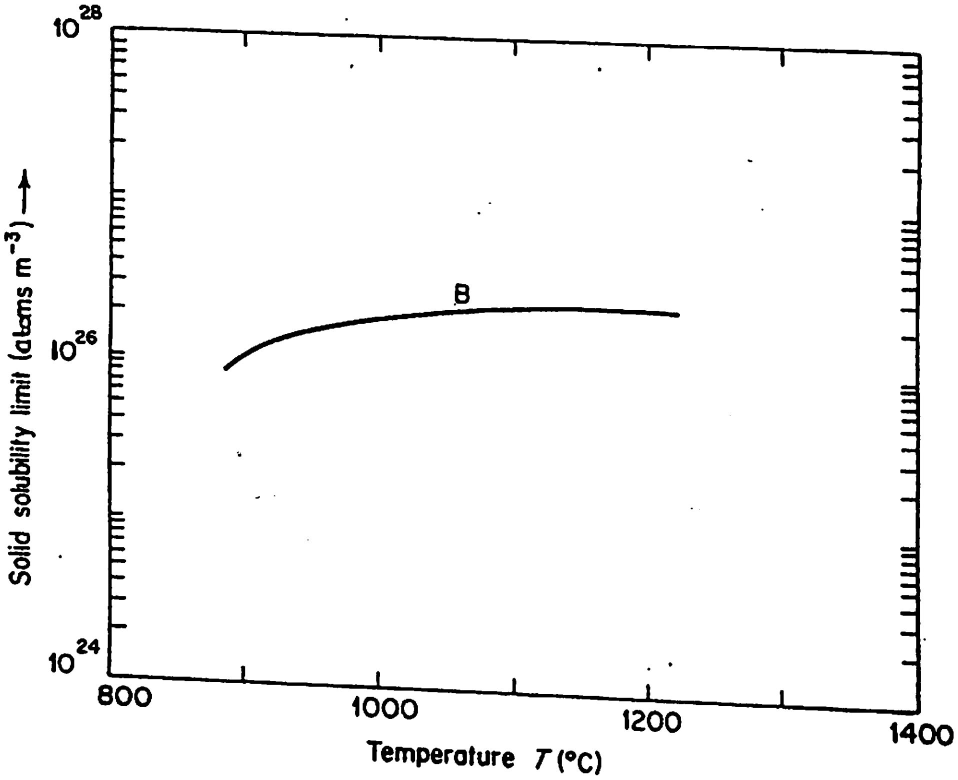
$E = \frac{1}{2} 9.1 \times 10^{-31} \times (1.5 \times 10^5)^2 = 1.02 \times 10^{-20}$ J = 0.064 eV.

$\lambda = v\tau = 1.27 \times 10^{-7}$ m ε

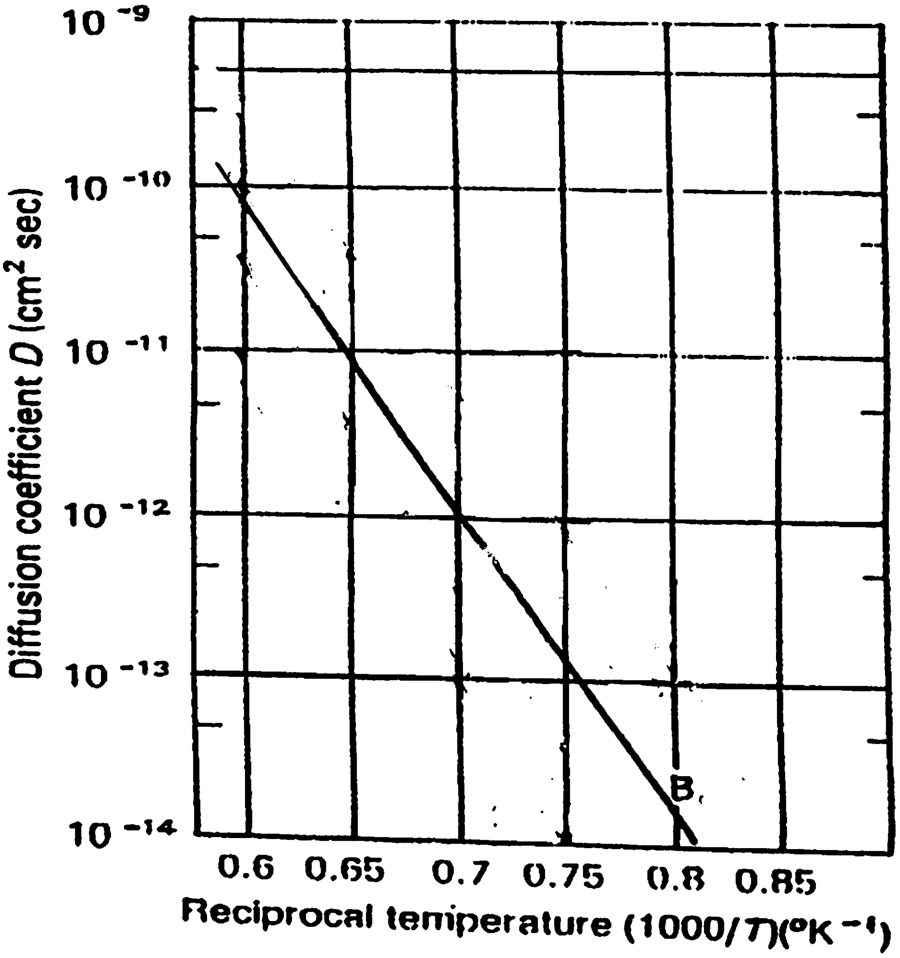
14 (a) In order to manufacture n-channel MOST devices n^+ doped source and drain regions must be produced. Compare and contrast two methods of producing these doped regions, indicating which would be preferable for application in devices to operate at GHz frequencies. (8)

(b) Give two examples of dopants that could be used to produce the n^+ regions and one, other than boron, that can be used to dope the substrate p-type. (3)

(d) Using the data in figures X (a) and (b) determine whether or not a successful n-pocket isolation diffusion is obtained when Boron is diffused through a 2 micron thick n-type silicon layer containing 6 x 10^{15} donor impurity atoms /cm^3. Assume that the diffusion takes place at 1100C from a saturated boron vapour for 45 minutes. State all assumptions made. (9)



The solid solubility versus temperature data for a range of atoms on silicon ε


10 $^{-9}$
10 $^{-10}$
10 $^{-11}$
10 $^{-12}$
10 $^{-13}$
10 $^{-14}$
Diffusion coefficient D (cm^2 sec)
B
0.6
0.65
0.7
0.75
0.8
0.85
Reciprocal temperature (1000/T)(°K^{-1})

E

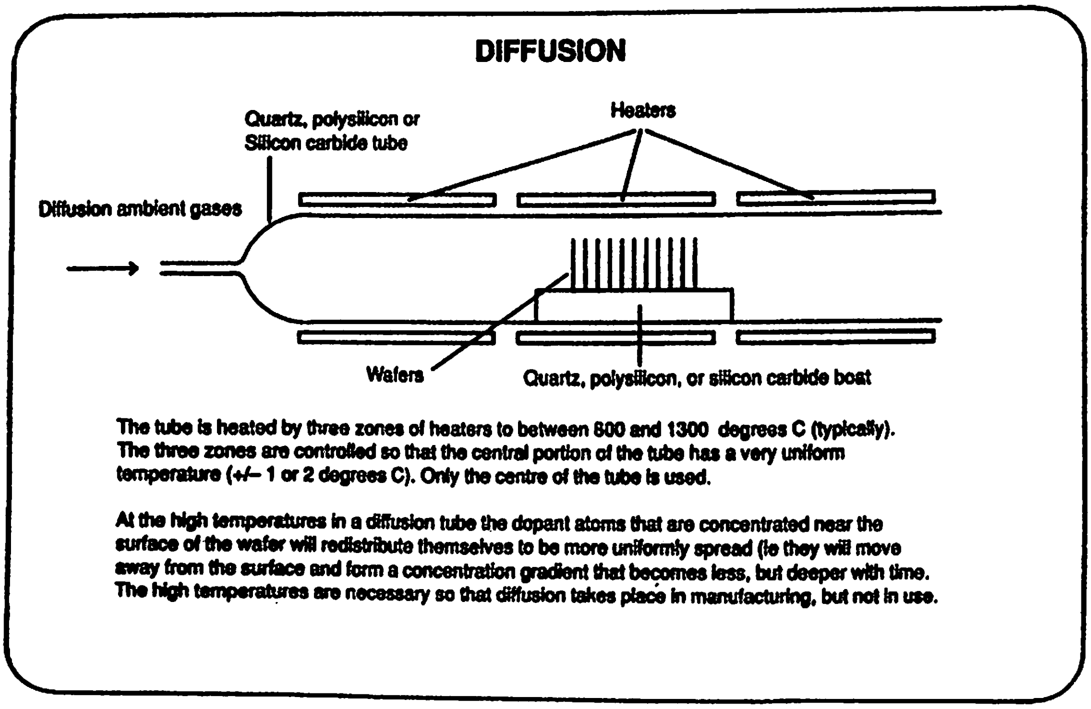
14

Solution:

Diffusion and Ion Implantation

Diffusion -

Two types of diffusive processes are relevant to device formation – (1) diffusion from an infinite source where a constant surface concentration is maintained and (2) diffusion from a constant total dopant. Both are carried out in a furnace as detailed below.



The tube is heated by three zones of heaters to between 800 and 1300 degrees C (typically). The three zones are controlled so that the central portion of the tube has a very uniform temperature (+/– 1 or 2 degrees C). Only the centre of the tube is used.

At the high temperatures in a diffusion tube the dopant atoms that are concentrated near the surface of the wafer will redistribute themselves to be more uniformly spread (ie they will move away from the surface and form a concentration gradient that becomes less, but deeper with time. The high temperatures are necessary so that diffusion takes place in manufacturing, but not in use.

In the first case the dopants are transported from a source vapour onto the silicon surface and diffuse into the wafer.

In the second case the small amount of dopant deposited either onto the silicon surface or incorporated into a doped oxide on top of the silicon diffuses into the bulk silicon crystal.

The solution that satisfies the above boundary conditions is

$$C(x,t) = C_S \, erfc\left(\frac{x}{2\sqrt{Dt}}\right)$$

where C_S is the constant surface concentration (in atoms/cm^3), D is the constant diffusion coefficient cm^2/s.

Supposing now that a thin layer of dopant is present on the surface of our silicon wafer and that there is a fixed total amount of dopant S per unit area and that the dopant diffuses into the silicon.

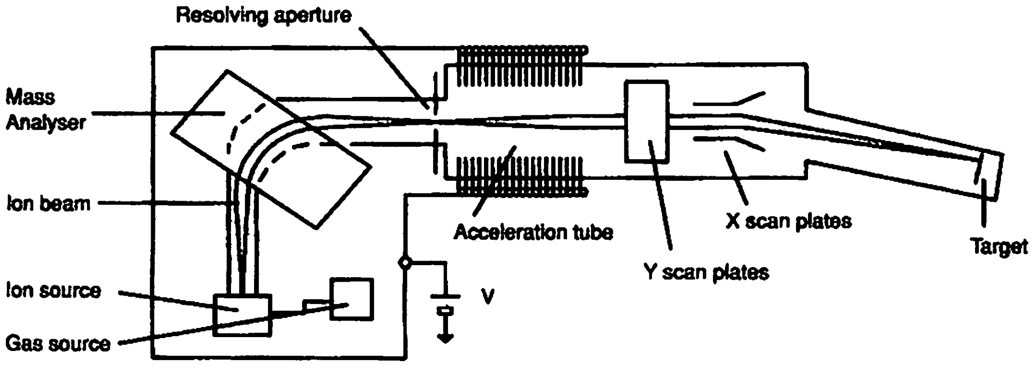
The solution that satisfied the boundary conditions for situation (2) gives

$$C(x,t) = S/(\pi Dt)^{1/2} \exp\left[\frac{-x^2}{4Dt}\right]$$

Ion Implantation:

This is the introduction of ionised projectile atoms into samples. The energies of the atoms are sufficient to cause penetration beyond the surface region. It has the advantage over diffusion in that it has the capability of precisely controlling the number of implanted dopant atoms and it takes place at much lower temperatures. A Schematic of such a system is shown in the next diagram.

ION IMPLANTATION



Ion implantation fires dopant ions into the surface of the target (a wafer) to achieve a uniform, accurately controlled dopant concentration.

The ion beam is generated by ionising the source (a gas or vapour at low pressure), selecting the correct mass/charge ratio of ion, and accelerating them through 10 – 200 keV (typically). The beam is focussed and scanned across the target to achieve good uniformity. The total dose is monitored by Faraday cups placed just outside the target area, and implantation continues until the correct dose is reached.

The use of 10-500keV energies for boron, phosphorus or arsenic ions is sufficient to implant ions from about 100-10,000Å below the silicon surface. The depth and type of implantation can be selected to meet a particular application.

As it is a directional process it also reduces the amount of sideways spread through apertures, that is associated with the standard diffusion process.

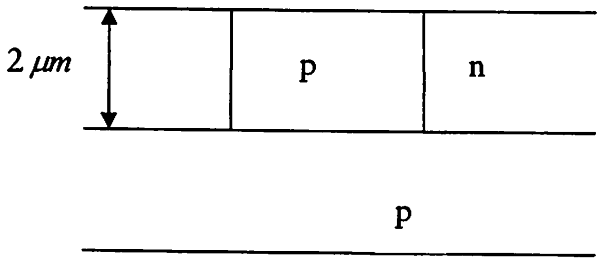
However the ions cause a certain amount of damage to the crystal surface as they impact upon it. In some instances the energies being sufficient to turn the surface into an amorphous layer.

In addition the atoms, as implanted, will be mostly inactive because on average they will tend to occupy interstitial rather than substitutional positions in the lattice.
Because of both of these problems a post implantation heat treatment is needed. If a shallow diffusion is required the heat treatment is short and at very low temperatures, but if a deeper diffusion is required, then a drive in stage is needed.

To summarise ion implantation has the advantage that it produces very uniform profiles and is very controllable. However, it is time consuming and is not practical for large doses. The damage to the lattice also increases as the dose increases and this also limits the highest practical dose (exact amount depends on the atom used as atom size is important factor).

(b) Phosphorus and Arsenic for n-type
Boron and Antimony for p-type

(c) Assume all infinite source of dopants therefore use erfc profile. For successful isolation diffusion the density of the B impurities which penetrate the 2 micron thick n-layer must be > than the density of the n-type dopant throughout.



So work out the concentration of the B at 2 μm from the surface and if > than 6×10^{15} then successful isolation is achieved.

Temperature 1100°C	therefore	Cs from Fig.(a) is ~2 $\times 10^{20}$ cm^{-3}
	and	D from fig (b) is ~ 3 $\times 10^{13}$ $cm^{2}5^{-1}$.

$$C(x,t) = C_S \, erfc\left(\frac{X}{2\sqrt{Dr}}\right)$$

$$= 2\times10^{20} erfc\left(\frac{2\times10^{-6}}{2\times\sqrt{3\times10^{-13}}\times2700}\right)$$

$$= 2\times10^{20} \times erfc\ 1.11$$

$$\approx 2\times10^{20} \times \cdot 2$$

$$\approx 4\times10^{19} / m^3$$

therefore successful isolation

P8 Information Engineering Option

15

a) RGB represents the 3 colour components, red, green & blue independently and are usually either real numbers in the range 0 to 1, or integers in the range 0 to 255.

YUV is a simple linear transformation such that

$$\begin{bmatrix} Y \\ U \\ V \end{bmatrix} = C . \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

where C is a ~~[illegible]~~ 3×3 matrix such that Y represents the apparent brightness or luminance of the pixel (0·3R + 0·6G + 0·1B from measurements of the human visual system) & U & V represent the difference of B & R respectively from the luminance Y. U & V are known as chrominance components.

HSV is a non-linear transformation representing the hue, saturation and value of the pixel. The value V is the maximum of R, G and B; and the saturation is the ratio of the minimum of R, G & B to V. The hue is a number from 0 to 1 which indicates which of R, G or B is the maximum & where the colour lies between these 'pure' colours. It is continuous & cyclic from Red through

Yellow, Green, Cyan, Blue, Magenta and back to Red.

b) Since the apparent colour (hue & saturation) depends only on the ratios of R, G & B components, rather than on their absolute values, we may modify the brightness of a pixel in RGB space by simply scaling all 3 compents by the same factor g. Hence the new values R', G', B' are given by

$$R' = g R \qquad G' = g G \qquad B' = g B$$

Hence in YUV space:

$$\begin{bmatrix} Y' \\ U' \\ V' \end{bmatrix} = C \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} = g C \begin{bmatrix} R \\ G \\ B \end{bmatrix} = g \cdot \begin{bmatrix} Y \\ U \\ V \end{bmatrix}$$

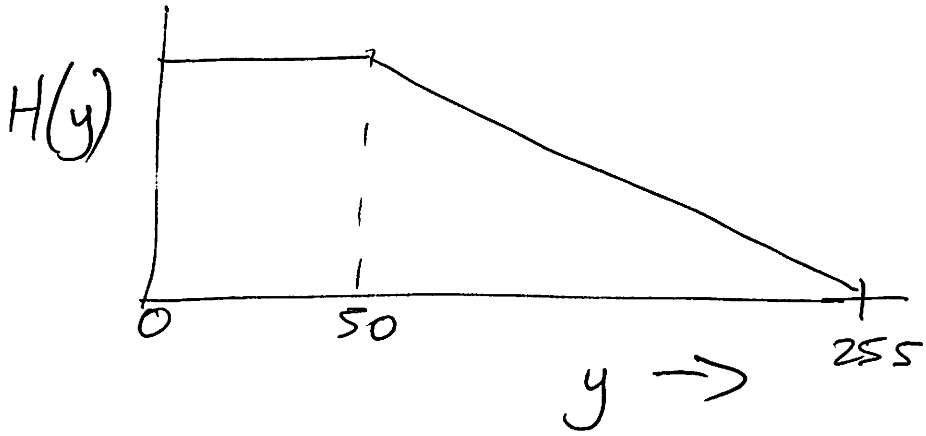
so the same applies for YUV components.

In HSV space, H & S depend only on the ratios of R, G & B, so they will remain constant and ~~a~~ only V needs to be scaled by g.

Hence

$$H' = H \qquad S' = S \qquad \text{and } V' = g V.$$

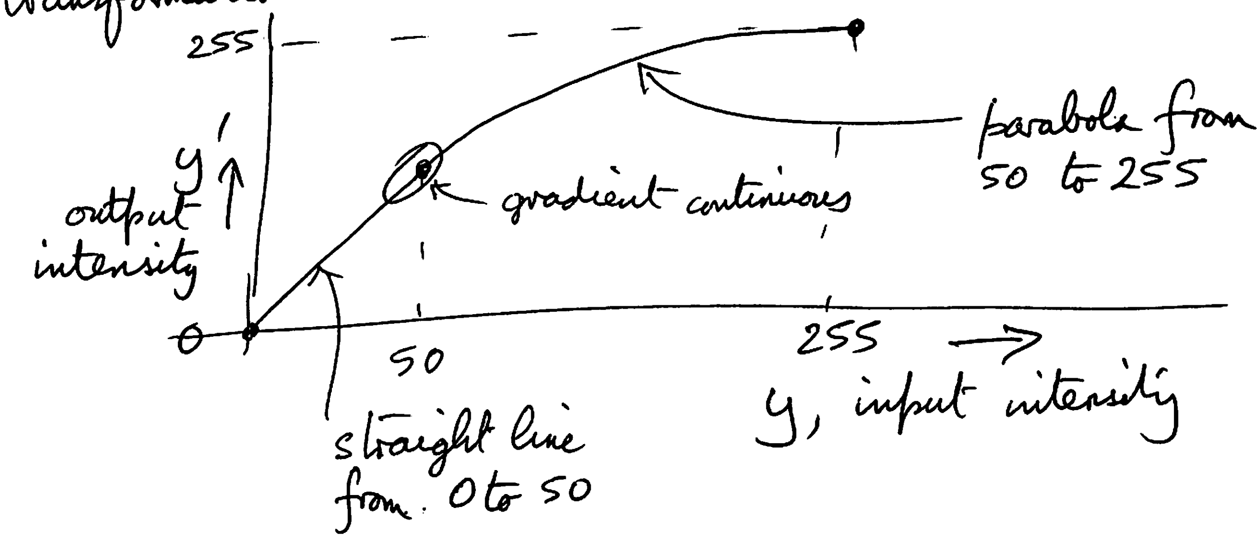
c)



~~To get a more well balan~~

This shows that the image will be quite dark as there are more pixels with low intensity than with high intensity.

To get a more well-balanced image, we need to brighten most pixels so as to produce a relatively uniform histogram $H'(y')$. Hence the equi-spaced bins of H' need to correspond to equal-area regions of H. This will require the gradient of the transformation from y to y' to be proportional to $H(y)$. Therefore the transformation curve will look like



d) The above curve, which transforms y to y', will improve contrast in all parts of the image where the gradient of the curve is greater than 1 (which is good) but it will reduce contrast in regions of high intensity where the gradient is small. By blurring the intensity image which moves us up & down this curve we ensure that the desired gain value g, $= y'/y$ that scales each pixel, cannot change rapidly in a small (local) region. Since $g = \frac{y'}{y}$ is always greater than 1, this means that local contrast changes are preserved even in high intensity regions, and the image looks less 'washed out' in these regions. However it can lead to some 'halo' effects near high-contrast edges in the image.

F

P8: ~~Q2~~.16

(a) Interest points found by band-pass filtering using a Laplacian of a Gaussian over multiple scales and looking for max/min locally in image and over scales.

— set up image pyramid by smoothing with Gaussian kernels

$$\sigma(o, s) = \sigma_0 \, 2^{(o + s/S)} \quad s \in [0, \dots S-1]$$

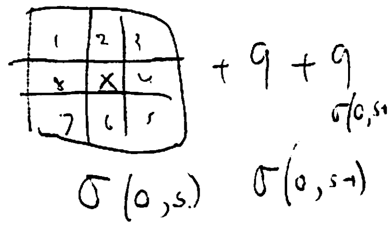
Each octave has S images smoothed by $\sigma(o,s)$

— Compute approx to LOG using DOG using neighbour images in pyramid

$$\nabla^2 G_\sigma \simeq G_{k\sigma} - G_\sigma$$

— localize max/min of $\nabla^2 G_\sigma$ and mark as blob centre.

by looking at 26 neighbour values –



— scale for $\sigma(o,s)$.

(b) Orientation estimated from image gradients — histogram of gradient angles, weighted by magnitude. Window size? Bins?

(c) SIFT descriptor (see notes)

(d) Sample image + product a SIFT descriptor for each pt. Histogram into finite number of words by K.NN.. Histogram is a vector describing images, F

Part IB

17 ANSWERS Paper 8 , 2010

(a) $\log p(S|\mu) = \log \prod_n p(x_n|\mu)$

$$= \log \prod_n \prod_m p(x_{nm}|\mu)$$

by independance.

Now $p(x_{nm}|\mu) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(x_{nm}-\mu_m)^2\right\}$

so

$$\boxed{\log p(S|\mu) = -\frac{1}{2}\sum_n \sum_m (x_{nm}-\mu_m)^2 - \frac{NM}{2}\log(2\pi)}$$

(b) Maximising $\log p(S|\mu)$ as a function of μ_m gives:

$$\frac{\partial}{\partial \mu_m} \log p(S|\mu) = 0$$

which is solves as follows

$$\frac{\partial}{\partial \mu_m} \log p(S|\mu) = -\frac{1}{2}(-2)\sum_{n=1}^{N}(x_{nm}-\mu_m) = 0$$

$$\sum_n x_{nm} - N\mu_m = 0 \quad \Rightarrow \quad \boxed{\mu_m = \frac{1}{N}\sum_n x_{nm}}$$

which makes sense as the arithmetic mean.

In vector form: $\boxed{\underline{\mu} = \frac{1}{N}\sum_n \underline{x}_n}$

(c) If we assume all μ_m equal (to say, μ) then

$\frac{\partial}{\partial \mu} \log P(S|\mu) = 0$ gives ...

(continued)

(c) $\sum_n \sum_m (x_{nm} - \mu) = 0$

Therefore $\sum_n \sum_m x_{nm} = NM\mu$

$$\boxed{\mu = \frac{1}{NM} \sum_n \sum_m x_{nm}}$$

(d) You would have to place a prior on μ, $P(\mu)$ and then use Bayes rule to infer the posterior $P(\mu|S)$ given data.

$$P(\mu|S) = \frac{P(S|\mu)\,P(\mu)}{P(S)}.$$

For example, if your prior $P(\mu)$ is Gaussian, then the posterior is also Gaussian with means and variances that are easy to compute.

Advantages:
- easy to incorporate prior knowledge
- gives a distribution over values of μ indicating uncertainty
- does not suffer from overfitting which ML does.

Disadvantages
- incorporates subjective information
- can be computationally expensive depending on the prior.

9

13

y (a) Write short notes on:

(i) temporal difference learning;

Answer: Temporal difference learning is an algorithm that implements a type of reinforcement learning. An agent can take actions and receives rewards. Temporal difference learning is a way to calculate the value (discounted sum of all future rewards) of a given policy. It does this by calculating the temporal difference error which is the difference between the estimated value of the current state and the sum of the current reward and estimated (discounted) value the next state. This error can be used to update the value functions online.

(ii) direct inverse learning;

Answer: Direct inverse learning is an an algorithm that tries to learn an inverse model by adapting a set of parameters in a model such as weights in a neural network. Given the system in which actions are mapped into states, the inverse model tries to learn the mapping from states to actions that will achieve them. It does this by observing the system often under random inputs (actions) and tries to learn the mapping of the achieved state to action that caused it by using a supervised learning rule such as back propagation. The problems with such learning is that it is not goal directed, required rewiring for use and fails for many nonlinear systems.

(iii) loss functions in neuroscience. [25%]

Answer: A loss function specifies how errors are converted into an internal scalar number representing how bad that error is for the agent. For example it would specify when pointing at a target how bad a 2cm error is compared with a 1cm error. Studies on human show that the loss function for the human reaching is less than quadratic showing that humans are more robust to outliers than statistical methods such as linear regression.

(b) Answer the following questions in the context of learning.

(i) Describe briefly the differences between supervised and reinforcement learning.

Answer: The key differences are that supervised learning receives the desired output and gets a direct measure of error between the inputs and

9

outputs (vector error) and is therefore suitable for classification and regression problems. In contrast reinforcement learning only receives a scaler signal, that is reward and the aim is to increase the reward accumulated over time. It is suitable for the control of complex systems.

(ii) Describe what a forward model is and how it can be learned and used. [25%]

Answer: A forward model maps commands into the state or sensory feedback they will cause. Forward models make a prediction of upcoming information, supervised learning can be used to adapt the parameters by comparing the prediction to the actual information. In the electric fish that is done to anti-hebbian learning They can be used in a number of ways. First they can be used to predict upcoming states and therefore ameliorate time delays of system (example of manipulation and grip force). Second, they can be used to predict sensory feedback and attenuate this due to one's own motions. Third, they can be used to estimate the state system such as the rotation of the eye.

(c) Answer the following questions in the context of stiffness control of the human arm.

(i) Why does the human arm have innate stiffness?

Answer: Muscles have spring-like properties and therefore for a fixed activation level they resist motion away from an equilibrium position therefore producing stiffness.

(ii) How can the stiffness ellipse of the arm be measured?

Answer: By using a robot to produce a step displacements of the hand of around 1 cm and measuring the restoring force in different directions of perturbation and then fitting a stiffness ellipse to the data. Critically the perturbation needs to be completed before voluntary control can start so within 100 ms.

(iii) Under what conditions, and why, is it sensible to use stiffness or to use inverse model control to resist force perturbations? [25%]

Answer: Inverse model control is generally used when the perturbation is predictable in magnitude and direction whereas stiffness controllers is used for unpredictable perturbations but is more energy consuming.

(TURN OVER for continuation of Question 1

(d) Answer the following questions in the context of noise.

(i) How can noise on the motor command determine how movements are generated?

Answer: Signal-dependent noise (SDN) corrupts the commands and leads to variability of the movement. Choosing different ways to move produced different motor command and different noise leading to different variability of the movement. By tailoring the statistics of the outcome to a particular task an optimal way of movement can be chosen because the motor system is highly redundant. Such a model has been used to predict goal-directed movements such as movement of the eye or arm.

(ii) How can noise on sensory inputs determine how stimuli are perceived?

[25%]

Answer: Sensory inputs are noisy making estimation hard. Bayesian methods combine prior knowledge about the state of the world with evidence from sensory input to produce optimal estimate. That is by achieved by multiplying the prior by the likelihood and normalising to produce a posterior. Therefore as sensor inputs become more noisy our brains rely more on the prior (example of motion illusion).

19

2 (a) Sketch the mechanical responses for the following, and describe the underlying microstructures and physical processes that give rise to the observed responses.

(i) Show on the same plot the stress-strain response for a collagenous soft tissue loaded in uniaxial tension both parallel and perpendicular to the primary collagen fibril orientation.

Answer: Collagenous tissue stress-strain responses are approximately quadratic. The response longitudinally (i.e. parallel to the collagen fibrils) will be roughly an order of magnitude stiffer than that transversely (i.e. perpendicular to the collagen fibrils) There are two key features of soft tissue mechanical response that are illustrated here, (a) nonlinearity and (b) anisotropy. The nonlinearity arises due to the underlying fibrillar nature of the collagen fibrils, which are contained in a viscous (sugar-based) ground substance in most soft tissue microstructures. On mechanical loading, the fibrils both uncrimp and re-align in the direction of applied load. This phenomenon is comparable to the "recruitment" of individual spring elements in a sequential manner, and gives rise to the increasing stiffness with increasing tensile strain. The anisotropy occurs in that the path to realignment of the fibrils with the loading direction is significantly smaller for the case where fibrils are already primarily oriented in this direction.

(ii) Show the evolution of a soft tissue response with time following the application of a step stress and a step strain.

Answer: Soft biological tissues are largely water, on the order of 70 v/v. As such their mechanical response is one of poroelasticity: the flow of fluid through a porous solid network. The solid network is in this case the proteins and sugars that make up the tissue itself. On application of a step stress, the strain in the tissue immediately increases to an initial value, and then increases progressively with time in a phenomenon known as creep. Conversely, when an instantaneous strain is applied, the stress in the tissue immediately increases to an initial value and then decreases progressively with time in a phenomenon known as stress-relaxation. The curves are roughly exponential exp(-t/tau) with time where the time constant tau is related to physical characteristics of the flow process including a physical length scale, the (shear) modulus of elasticity of the porous network and the hydraulic permeability related to the fluid travelling through the elastic

(TURN OVER for continuation of Question 2

skeleton.

[40%]

(b) Describe the following in the context of tissue engineering.

(i) The make-up of a natural tissue, with examples.

Answer: Natural tissues consist of cells residing in an extracellular matrix. The proportion of cells to ECM depends on the type of tissue, with 3/4 tissue types largely cellular (epithelial, muscle, nervous) and 1/4 largely ECM (connective tissue). The ECM itself consists of proteins and sugars (and minerals in the case of mineralized tissues such as bone) and water and is characterized by properties overall related to the properties of these non-living materials.

(ii) The overall approach of tissue engineering.

Answer: Tissue engineering is a general approach based on adding living cells to a scaffold material, with a third additional component such as stimulatory factors (growth factors, mechanical loading, etc.) to encourage the cells to thrive. The scaffold may be biodegradable. The hope is to start with something that looks more like a tissue than would an implant made of engineering materials, where the scaffold takes the ECM role. Over time, it would be hoped that the tissue engineered implant would be incorporated into the body, ideally with the clearance of the engineered scaffold and perfect integration into the surrounding tissue for complete "healing"

(iii) The role that bioreactors play in the tissue engineering process, with examples.

Answer: Bioreactors have several roles in tissue engineering. The use of bioreactors for culturing the cell-scaffold construct has been shown to produce a far superior tissue, which is hypothesized to be related to both the physical loading of cells due to fluid motion and also the improved diffusion of nutrients through the scaffold. Example include spinner flasks, where the tissue spins, rotating wall vessels, where the entire container spins, perfusion chambers, either hollow fibre or direct flow, or direct mechanical loading. The bioreactor has to maintain sterile conditions for keeping the cells alive and the nutrient broth available. Bioreactors also have a likely role in the scale-up and commercialization of tissue engineering, in which cells from different people

(cont.

would need to be used to make individualized implants with cells not only kept under sterile conditions but also labelled according to whose cells these are. This is likely related to the development of multi-chamber bioreactors with automated material handling capabilities.

(iv) The mechanical and non-mechanical requirements for an osteochondral tissue engineered implant.

Answer: An osteochondral implant should replicate the baseline mechanical properties of the two attached tissue layers that it is replacing, namely bone and cartilage. Bone is close to linearly elastic, slightly anisotropic (L = 1.5 T), E = 10 GPa, and poroelastic with kappa = 10-20 to 10-17 m4 (Ns)-1. Cartilage is Nonlinearly elastic, anisotropic (L > 10 T), E < 1 MPa, and poroelastic with kappa = 10-14 m4 (Ns)-1. Further mechanical considerations include the attachment of the two layers together, along with the attachment of the implant into the joint itself for integration with surrounding tissue. The tribological/frictional characteristics of the cartilage surface are also extremely important. Non-mechanical key factors include biocompatibility, including non-mutagenicity, sterilizability, and the chemical make-up of the scaffold by-products as it degrades.

[60%]

(TURN OVER

20 3 Write brief notes on each of the following:

(a) The system of navigation used by bacteria such as *E. coli* in search of nutrients. Be sure to mention the main components of the system. [20%]

Answer:

(a) For chemotaxis of, e.g., E.coli, see lecture notes.

Main components :-

- Reversible rotary motors, embedded in cell wall.
- Flagellar filaments, driven by motors.
- Receptor proteins embedded in cell wall: each sends a signal to the cell's interior when a single nutrient molecule is bound to it.
- Machinery (not detailed in lectures) for integrating such signals, working out the time rate-of-change of incidence of nutrient molecules, and then sending signals to reverse the sense of rotation of motors.

(b) The way in which an organism's cells "specialise", in order to construct different parts such as limbs, organs, etc., even though every cell contains the full DNA prescription for the entire organism. [20%]

Answer:

(cont.

(b) As cells in an embryo divide, they become more numerous. Each cell contains the complete DNA of the organism, and thus all of the genes required to build specialised parts, such as limbs, eyes, nerves, kidneys, etc.

In the development of embryos a stage is reached when cells begin to specialise into different components, depending on their location in the organism's "body plan". Cells contain machinery for "switching on" ("transcribing") only those (relatively) few genes necessary for the specialised function.

(c) The significance of the principle of self-assembly in the process of building biological structures. Give a few specific examples. [20%]

Answer:

(TURN OVER for continuation of Question 3

(c) In building (say) a house, the way in which the various physical components are put together is prescribed by an overall "assembly drawing".
By contrast, in biological systems the entire information for constructing an organism is contained in the DNA within every cell. The cells manufacture specific protein molecules which can self-assemble into the structures

required. The molecules adhere to one another by reason of their geometrical shapes, and the electric charges distributed over their surfaces: Brownian movement of these small objects in their aqueous environment ensures that they are constantly bumping into one another and adhering when in the right relative location.

Examples:

- Bacterial flagellar filaments – self-assembled from the special bi-stable protein "flagellin" so as to form long, corkscrew-like filaments.
- Spherical coats for viruses.
- Bacterial flagellar motors – built from ~ 20 different sorts of protein, which are produced in the right sequence to self-assemble into stator & rotor rings, etc.
- Protein molecules (see (d)).

(d) The overall conformation of protein molecules as determined by the weaker rather than the stronger chemical forces. [20%]

Answer:

(cont.

(d) Protein molecules consist of chains of specific amino acids (as programmed by the DNA). The links of the chain consist of very strong covalent bonds. But although they are strong, these bonds have a degree of rotational freedom; and there are 2 degrees of geometric freedom per amino acid.

The way in which the chain folds up (into a unique conformation, usually) is determined (in a way not yet fully understood) by "weak" forces such as hydrogen bonds and hydrophobic effects.

(The Young's modulus of proteins (E) is determined by the characteristics of the hydrogen bond --- which is why, e.g. wood has E values of the same order of magnitude as ice.)

(e) The different lines of evidence which have been used by scientists to establish the "tree of life". Do the resulting trees differ in form? [20%]

Answer:

(TURN OVER for continuation of Question 3

(e) Darwin used fossil evidence and the similarity of bone-layout of 5-fingered "hands" of many vertebrates to argue for a "tree" springing from a single primordial ancestor.

-3-

Details of the part of the tree involving vertebrates can be established by studying the skeletons of different species – including tracking the details of skulls and individual bones.

In the last ~50 years technologies for sequencing DNA and proteins have made possible the establishment of highly detailed portions of the tree of life.

In general the layout of trees set up by different methods agree – though the present level of detail obtainable by sequencing is orders of magnitude larger than for the earlier trees, of course

H

21 Crib:

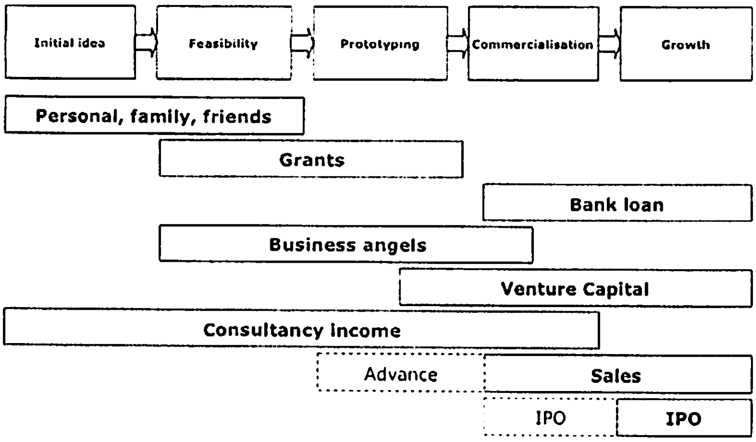
(a) Revenue = money from selling something. Source = actual / future customers

Grant = a 'gift'. Source = national govt (UK Technology Strategy Board) or regional govt (EEDA) or other org such as NESTA (National Endowment for Science, Technology and the Arts) or charity (Gates Foundation).

Debt = borrowing money. Source = a bank or specialist finance organisation.

Equity = selling part of the ownership (shares) of your business in return for cash. Source = venture capitalist, business angel or public markets.

(b)



(c) (i) investment banks, high net worth individuals, endowments, pension funds

(ii) firms with some proven market traction that show potential for rapid and substantial growth if given additional capital and access to expertise

(iii) from management fee (for raising the fund) and carried interest from profits made by selling shares in investee companies – via trade sale or IPO.

(d) •The market –Who has the problem that you attempting to fix?

•Product / service –What solutions are going to be used to address the problem?

•Management team –Who is going to do it? What is their track record?

•Business operations –How are you going to do it?

•Financial projections –How and when will money be made?

•Marketing strategy –How will get people to buy your product / service

•Resources required –What do you need to start your business?

•Exit opportunities –How will your investors get their returns?

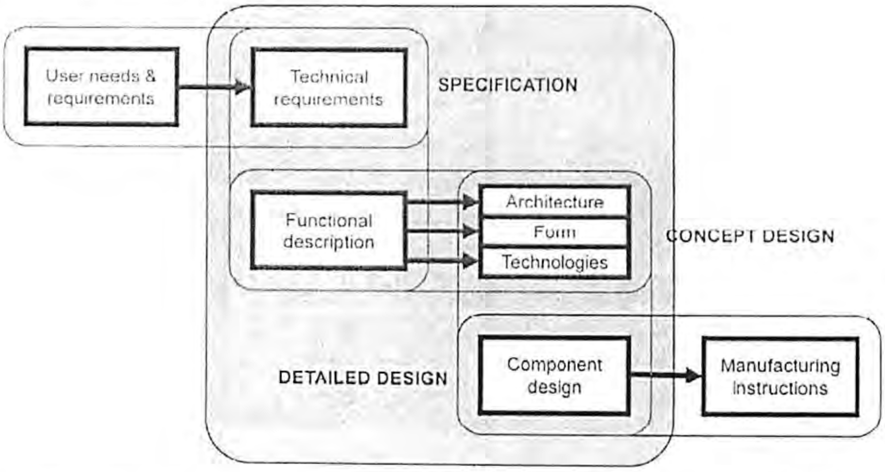
Key area is usually reckoned to be the management team. The often heard quote from VCs is that they'd prefer to invest in an A team with a B idea, rather than a B team with an A idea. The market focus rather than technology focus of the business idea is also important. For many businesses seeking to exploit some novel IP, the protection of the idea (IP) is also critical.

22 Crib:

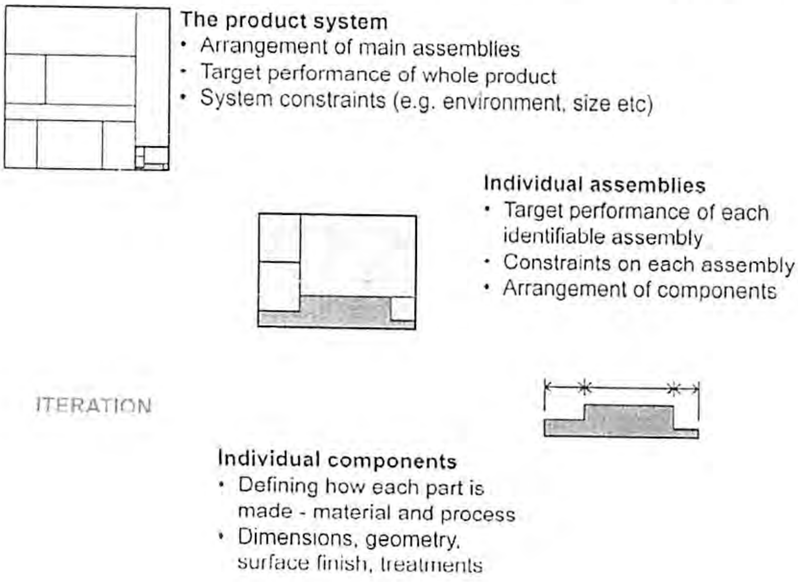
(a) (i) Sell a product: Pros = Increase revenue with higher volumes; Con = Need to develop, make, distribute, support, etc. Sell a service: Pro = No manufacturing costs; Con = Can be hard to scale up the business (in many cases can only grow through recruiting and training lots of people).

(ii) Licence the idea to someone else: Pros = Get the cash quite fast, little responsibility for building the business/achieving success, little further funding /capital investment needed. Cons = Get relatively small % of value, need to find and manage licensees. In comparison, making and selling a product is a high cost, high risk approach. Requires investment up front, and all the challenges of designing, testing, making a new product before any revenue is generated.

(b) Overall process:



Specification: Many design projects focus on the technical aspects of the 'core product', with a detailed technical specification, forgetting that first and foremost the product needs to deliver benefits to the consumer, and in addition, there are many other issues that might also need to be carefully specified. Thus, it is rarely sufficient to focus just on the technical aspects of a design.



The product system
- Arrangement of main assemblies
- Target performance of whole product
- System constraints (e.g. environment, size etc)

Individual assemblies
- Target performance of each identifiable assembly
- Constraints on each assembly
- Arrangement of components

Individual components
- Defining how each part is made - material and process
- Dimensions, geometry, surface finish, treatments

Concept design: should be considered at the system, module and component level; links requirements to working principles. These can then be taken forward to prototyping.

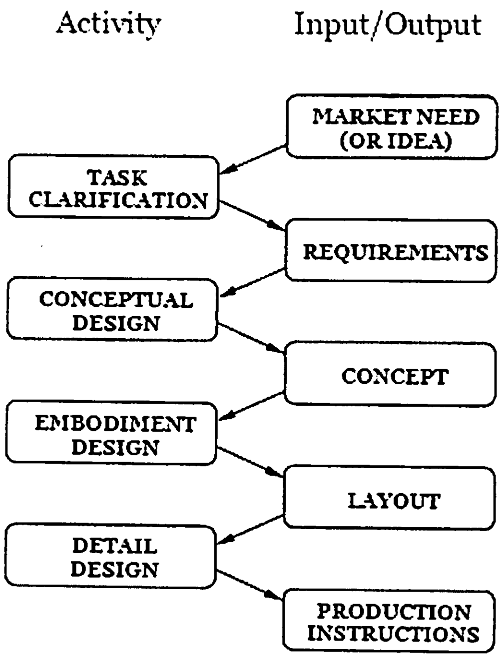
TYPE	*FIDELITY*				*COST*
	Function / performance	***Appearance***	***Producibility***	***Usability***	
Simple Sketch	*Low*	*Medium*	*Low*	*Low*	*Low*
Block model	*Medium*	*Medium*	*Low*	*Med-high*	*Low*
Visual (physical) model	*Low*	*High*	*Low*	*Medium*	*Medium*
3D CAD model	*Low*	*High*	*Medium*	*Low*	*Medium*
Functional (technical) model	*High*	*Low*	*Medium*	*Medium*	*High*
Pre-production prototype	*High*	*High*	*High*	*High*	*High*
Analytical / virtual models	*High*	*Low*	*Low*	*Low*	*Varies*

All stages need to be linked through a design process:

Optimise principle
- Understand customer needs
- Clarify requirements

Optimise layout
- Generate alternatives
- Evaluate alternatives
- Establish 'architecture'

Optimise details
- Design for production
- Evaluation
- Documentation



23 Crib:

(a) The basic problem of supply and demand is that the process of producing the product takes time – both due to production and due to the need to acquire the materials and components required to make it. If the customer is not prepared to wait for this process to occur, a stock of finished goods must be held so that whenever a customer arrives, there is a complete product ready to be bought. However, this stock of finished goods is expensive – all the costs of production have been paid for, and extra costs are incurred due to the need to store the product. The ideal is clearly that a steady stream of customers should arrive at exactly the same rate as the factory produces the products – so all the factory's resources are continuously in use, but no stock is kept.

•Customers would like to be able to buy any volume of a product without delay at any time

•Owners of the business would like to see all their resources fully and efficiently used all the time without having to pay any of the costs associated with holding stock

(b)

(i) production of cars, electronics, pharmaceuticals (high volume, low variety)

(ii) racing cars, top-end Swiss watches, wedding cakes (low volume, high variety)

For high volume, standardised products, automation is an attractive option - the high initial cost of automated equipment will be paid off through faster production rates with fewer (expensive) manual workers. However, automation is far from a perfect solution:

- It is generally difficult to set-up (programme) a machine for the first run of a new product/part
- All automated machines have a relatively limited flexibility - with limits based on the number of degrees of freedom, size etc.
- Automated systems are not particularly robust - the software is only as good as the specification of anticipated problems
- Automated assembly is generally difficult due to the difficulty of handling and aligning parts
- Some material types are not easily handled by machines - sewing remains a manual operation.

High variety products with lower volumes tend to have shorter life cycles and more customisation even within 'standard' products. It is therefore very difficult to automate production for such products, and automated production technology remains a 'tool' for use by humans rather than a substitute.

In contrast, people:

•are highly flexible

•have low capital and set up costs, higher hourly cost but can the number of them active in a system can be changed relatively easily
•can be involved in assessing and assuring the quality of production and can improve the system through wider awareness
•but they do get bored when doing the same thing too many times.

(c) Four general strategies have developed for trying to smooth the pattern of demand:

1. •The simplest strategy is to design a system with capacity to supply the average demand, and use the quiet periods to build stock to meet the periods of peak demand.
2. •An attractive possibility is to find a second product with the inverse demand cycle so that the two combined have more level demand
3. •Price can be used to reduce demand at the peak, and increase it at off-peak times - for instance by charging more for toys at christmas
4. •Reservations can be used to allow advance booking of peak demand, so that later requests must be met at other times

Bicycle lights are likely to also show seasonal demand, i.e., demand is likely to go up in winter and down in summer. Strategies for dealing with this could be to find markets in opposite hemisphere, promoting the use of lights not only for use at night (bad weather safety), encourage out of season buying through price promotions, etc.

Supply can be balanced by increasing flexibility. Clearly, the overall capacity of the system must at least be sufficient to meet the average demand.

Increasing flexibility requires: Appropriate tools and equipment, and operators who are sufficiently skilled and motivated to switch between tasks Product design options where some production can be done in advance of peak demand - ideally long task time production of standard components - to reduce the amount of work required at times of peak demand.