ENGINEERING TRIPOS PART IB

Monday 31 May 2010 9 to 11

Paper 1

MECHANICS: DYNAMICS OF RIGID BODIES

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

GC04

SECTION A

*Answer not more than **two** questions from this section*

1 A folding ruler consists of two identical sections A and B connected together by a light frictionless pin (marked Q), located at the centre of one edge. Each section consists of a rectangular wooden part of width D and mass M_w, and square brass ends of side length D and mass M_b. The total length of each section (wooden part plus two brass ends) is L. The arrangement is shown in Fig. 1. The thickness of the ruler is small with respect to the other dimensions, and so it can be considered a lamina.

(a) If $L = 10D$ what is the moment of inertia (I_G) of section A about an axis oriented normal to the plane of the lamina and passing through its centre of mass? [6]

(b) The ruler lies flat and fully folded ($\theta = 0$) on a smooth table. It is pulled open by applying forces, but no moments, to the tips marked P so that they move along line YY with constant speed of separation $2V$. The motion may be assumed planar.

(i) What is the angular velocity $(\dot{\theta})$ and angular acceleration $(\ddot{\theta})$ of section A as a function of θ? [6]

(ii) What is the shear force in the pin at Q as a function of θ, and in which direction does it act? [8]

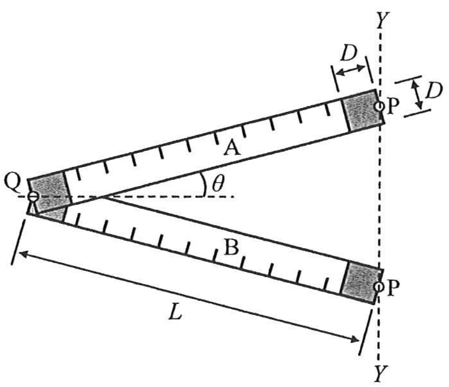

Fig. 1

2 A ball of mass M and radius R falls under gravity and strikes a slope inclined at angle θ to the horizontal (Fig. 2). Immediately before impact, the ball has velocity v_0 in the vertical direction.

(a) The slope is smooth, so that friction can be neglected, and collisions between the ball and the slope are perfectly elastic.

(i) What is the angular velocity of the ball immediately after impact with the slope? [3]

(ii) What is the magnitude and direction of the impulse imparted to the ball by the slope? [4]

(iii) A particular slope has angle $\theta = \frac{\pi}{4}$. Show that the ball travels initially in the horizontal direction after impact. [3]

(b) The ball is dropped in an identical manner onto a slope (inclined at θ to the horizontal) which is covered in carpet. The coefficient of friction between the ball and the slope is μ. The properties of the carpet are such that the component of velocity normal to the slope immediately after impact is zero.

What is the minimum value of μ to ensure no sliding occurs between the ball and the slope after impact? [10]

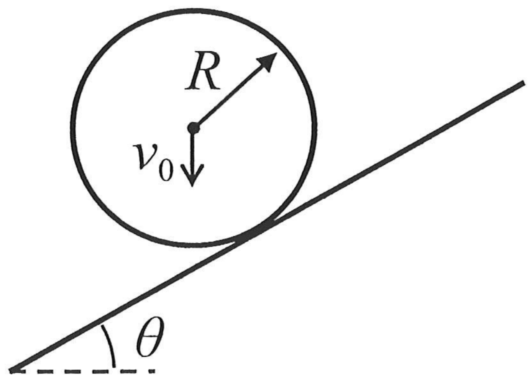

Fig. 2

3 A model of an epicyclic gearbox consists of a central 'sun' gear (S) which meshes with four 'planet' gears (P), see Fig. 3. All five gears have mass M and are circular in cross-section with radius R. The axles on which the planet gears freely rotate are connected together by a light, rigid ring (C). Friction may be neglected everywhere in the gearbox. The planets mesh with an outer annular gear ring (A) which is supported on frictionless ball joints (B).

(a) The sun gear rotates with constant angular velocity ω in the $\underset{\sim}{e}_3$ direction as shown in Fig. 3. Find the angular velocity of the planet gears. Hence show that the ring C rotates with angular velocity $\omega/4$. [5]

(b) A constant braking torque T is applied to the sun gear (S) to bring the system to rest. What is the energy dissipated during braking? [5]

(c) The sun gear again rotates with constant angular velocity ω in the $\underset{\sim}{e}_3$ direction. Now the whole gearbox is also rotated with constant angular velocity Ω in the $\underset{\sim}{e}_1$ direction.

(i) If the moment of inertia of the gearbox about $\underset{\sim}{e}_1$ is I_{GB}, what is the total moment of momentum of the gearbox? Express your answer in vectorial form. [4]

(ii) Hence, or otherwise, determine the additional reaction forces at the ball joints due to the motion of the gearbox. With the aid of a sketch show the direction in which these reaction forces act. [6]

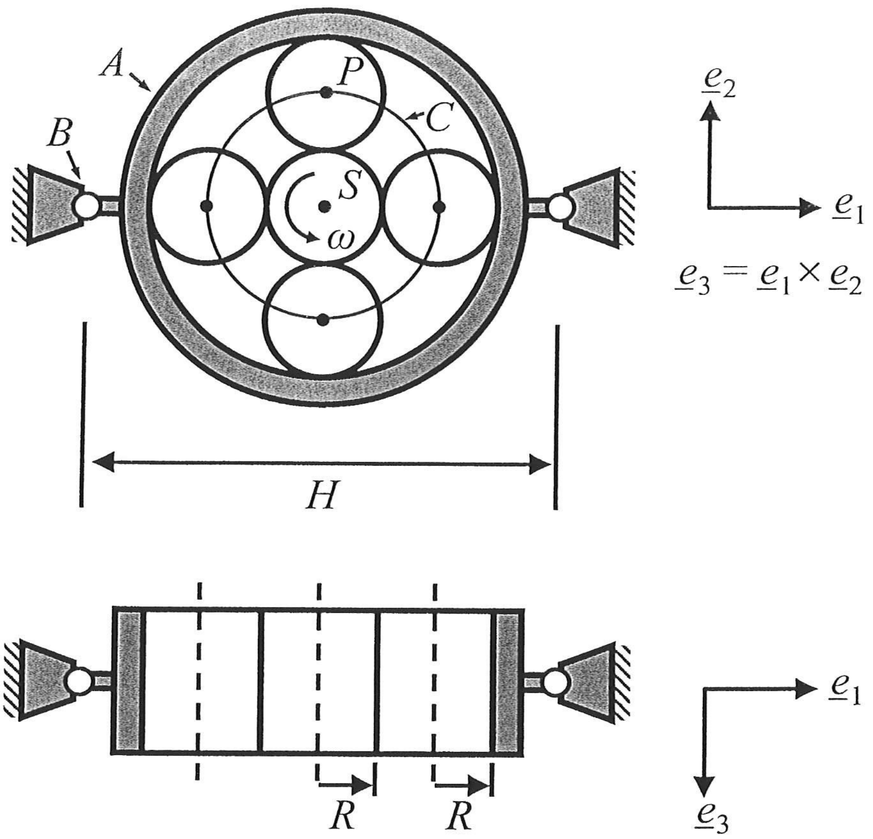
A
B
P
C
S
ω
H
R
R
$\underline{e}_1$
$\underline{e}_2$
$\underline{e}_3 = \underline{e}_1 \times \underline{e}_2$
$\underline{e}_1$
$\underline{e}_3$

Fig. 3

SECTION B

*Answer not more than **two** questions from this section*

4 Ice skaters are modelled as a central point mass M and two point masses m each rigidly connected to the central mass at a distance d as shown in Fig. 4, moving without friction on the plane.

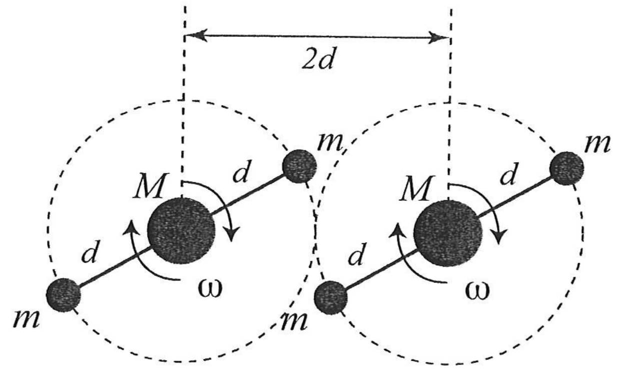

Fig. 4

a) What is the moment of inertia of such a skater about its centre of mass? [2]

b) Two such skaters are spinning in the same direction at a distance $2d$ apart, initially with angular velocity ω and with their centres of mass at rest. At time $t = 0$ their hands, represented by the masses m, meet in a clap, which is taken to be a perfectly elastic collision. Calculate the total moment of momentum before and after the collision. [8]

c) Determine the velocities of the skaters after the collision and sketch the subsequent motion. [10]

5 A mechanical tachometer to measure angular velocity is constructed using two identical masses m at the end of two rigid light rods ABC, as shown in Fig. 5. The rods are of length $2l$, and are joined to the top of the shaft at points A. A frictionless slider of mass M is connected to the centre of the rods at B by two struts BD each of length l. The pin joints at A, B, and D are also frictionless. The shaft is rotating with a constant angular velocity ω. The distances AA and DD are negligibly small.

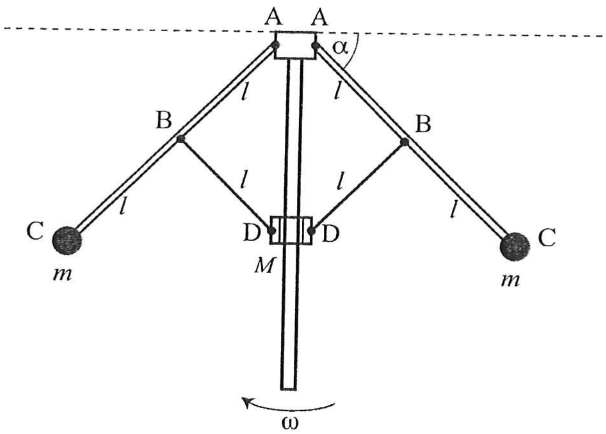

Fig. 5

a) Show that the angle α of the ABC rods with respect to the horizontal plane, is given by

$$\sin\alpha = \frac{(M+2m)g}{4m\omega^2 l}$$ [8]

b) What is the minimum value of ω which results in the masses m lifting up and away from the shaft? [2]

c) Suddenly the two struts BD break and the slider falls away. What is the initial angular acceleration $\ddot{\alpha}$ of the ABC rods? [4]

d) Given that the initial value of α is α_0, determine the equation satisfied by the angle α_{min} corresponding to the maximum height the masses will reach during the subsequent motion. [6]

6 Figure 6 shows a rocker that is constructed by drilling a cylindrical hole of diameter r in a solid cylinder of diameter $2r$. The rocker is placed on the edge of a curved well, as shown in Fig. 6a. The mass of the rocker after drilling is M. The well has a cross section that is described by two quarter-circles and a semicircle, each with radius $R = 1.75r$. When released, the rocker rolls down the surface without slip and without losing contact.

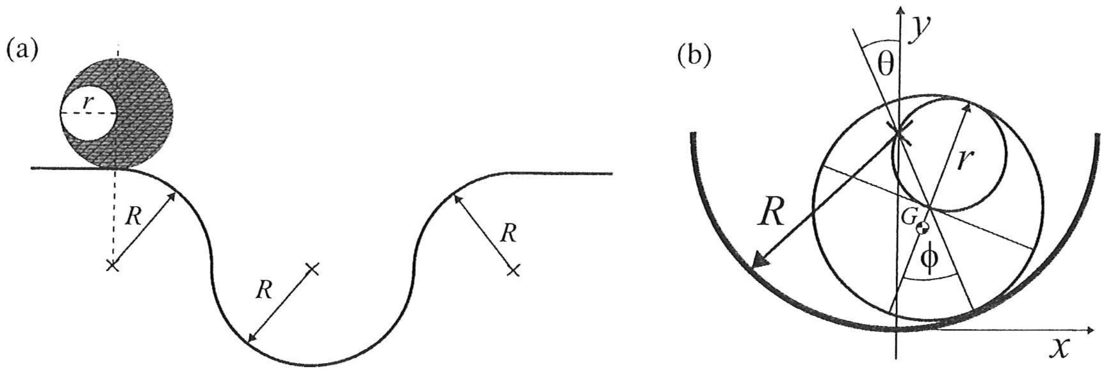

Fig. 6

a) Without doing any calculations, sketch the approximate position and orientation of the rocker when it momentarily stops at the other side of the well. [2]

b) Determine the location of the centre of gravity of the rocker in relation to its geometric centre, and calculate its moment of inertia, I_G, about its centre of gravity. [4]

c) Now the rocker is placed vertically in its stable position at the bottom of the well and is set in motion, as shown on Fig. 6b. The angle θ is the deflection of the perpendicular line at the contact point from the vertical, and the angle ϕ is the angle between the same perpendicular line and the symmetry axis of the rocker. Show that the position vector of the centre of gravity of the rocker is given by

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (R-r)\sin\theta - \frac{r}{6}\sin(\phi-\theta) \\ R-(R-r)\cos\theta - \frac{r}{6}\cos(\phi-\theta) \end{pmatrix}$$ [6]

d) Hence write down the mechanical energy of the rocker for *small angle deflections.* By comparing to that of the simple harmonic oscillator, determine the frequency of small amplitude oscillations of the rocker. [8]

END OF PAPER