ENGINEERING TRIPOS PART IB

Monday 31 May 2010 2 to 4

Paper 2

STRUCTURES

Answer not more than ***four*** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig. 5

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Single-sided graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

cjb04

SECTION A

1 The weightless pin-jointed structure shown in Fig. 1 has six members, I to VI, and is initially stress-free. Each member has cross-sectional area, A, Young's modulus, E, and all behaviour is linear elastic.

(a) Show that the structure has two redundancies. [3]

(b) A vertical force W is applied downwards as shown in Fig. 1. Specifying I and III as redundant bars, find:

(i) the particular equilibrium solutions for bar tensions; [3]
(ii) the possible states of self stress in the structure; [4]
(iii) the elastic solution for the bar forces; [8]
(iv) the horizontal displacement at the point of application of the load. [2]

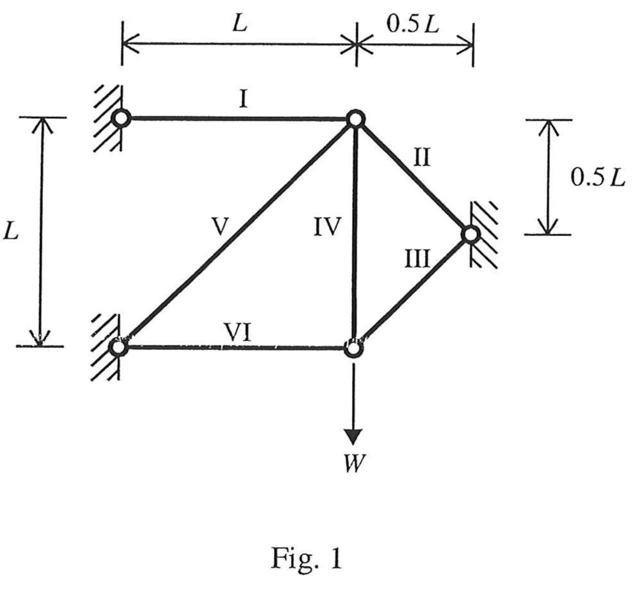


Fig. 1

2 The weightless frame shown in Fig. 2 consists of a continuous beam, BCE , that is rigidly connected to column AB at B and propped by a pin-ended column at C. Column AB is fixed to the ground at A. All members have flexural rigidity, EI , are axially rigid and behave elastically. The unloaded frame is unstressed.

(a) Determine the number of redundancies in the structure. [2]

(b) A uniformly distributed total load, W, is applied to BC and a point load, W, is applied at E, which is at a distance xL from point C.

(i) Determine x such that the vertical reaction at A is zero. [8]

For this value of x,

(ii) determine the moment at A and reaction at D, and [4]

(iii) determine the position and magnitude of the maximum sagging moment along BC. Sketch the bending moment diagram for the entire structure, indicating important values. [6]

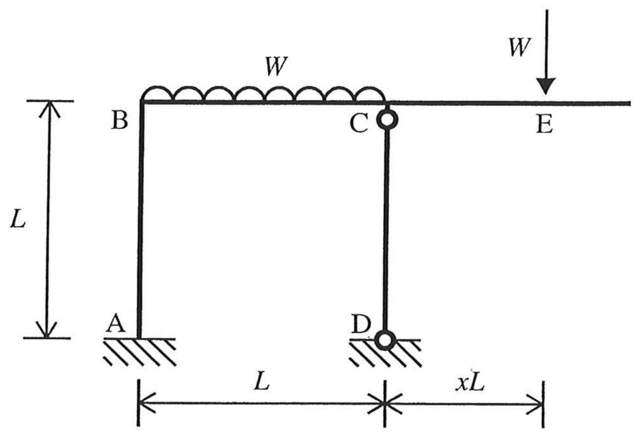


Fig. 2

3 Figure 3a shows a cantilever, fully restrained at one end, which carries a vertical point load of 20 kN at its free end. The beam has a right-angled triangular section throughout its length, with a constant wall thickness of 10 mm. The point load is applied eccentrically with respect to the cross-section as shown in Fig. 3(b). Two points A and B are of particular interest at the root of the cantilever. They are located at the mid-points of two sides, and at the mid-thickness, as shown in Fig. 3(b).

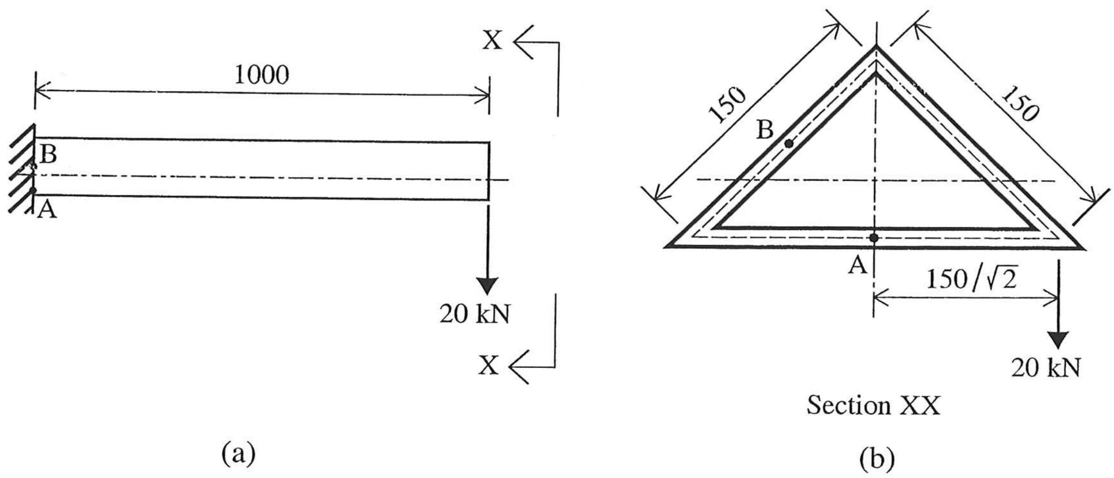
(a) (i) Determine the normal longitudinal stresses at both A and B. [6]

(ii) Calculate the shear stresses at both A and B. [6]

(b) The beam is fabricated from S275 steel (Structures Data book, Section 10.1) and is designed to carry a load at full yielding of 20λ kN, where λ is a safety factor. Considering only points A and B, determine λ using:

(i) the Tresca Yield Criterion; [4]

(ii) the von Mises' Yield Criterion. [4]



(Not to scale – all dimensions in mm)
Fig. 3

SECTION B

4 Figure 4 shows a continuous beam. The beam has a fully-plastic moment capacity of 1000 kNm and a self-weight of 5 kN/m. It carries loads as shown in the figure.

(a) Using a collapse mechanism consisting of plastic hinges at B and at a point located a distance x from A, calculate the best estimate of the load factor λ that can be applied to the live loads that will just cause the beam to fail. [8]

(b) For the mechanism defined in (a), calculate the support reactions at A, B and C. [7]

(c) Have you calculated an upper bound, a lower bound, or the true value of the collapse load? Justify your answer. If you believe you have not found the true collapse load, suggest an alternative mechanism that should be checked, but DO NOT perform any further calculations. [5]

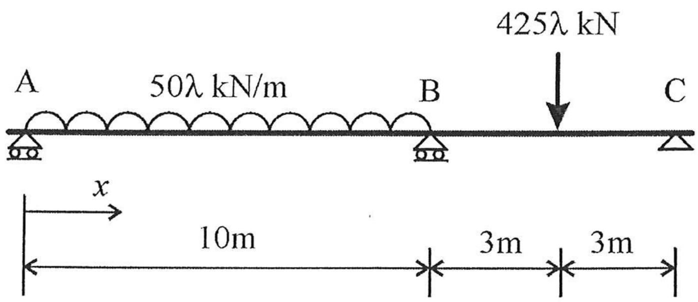


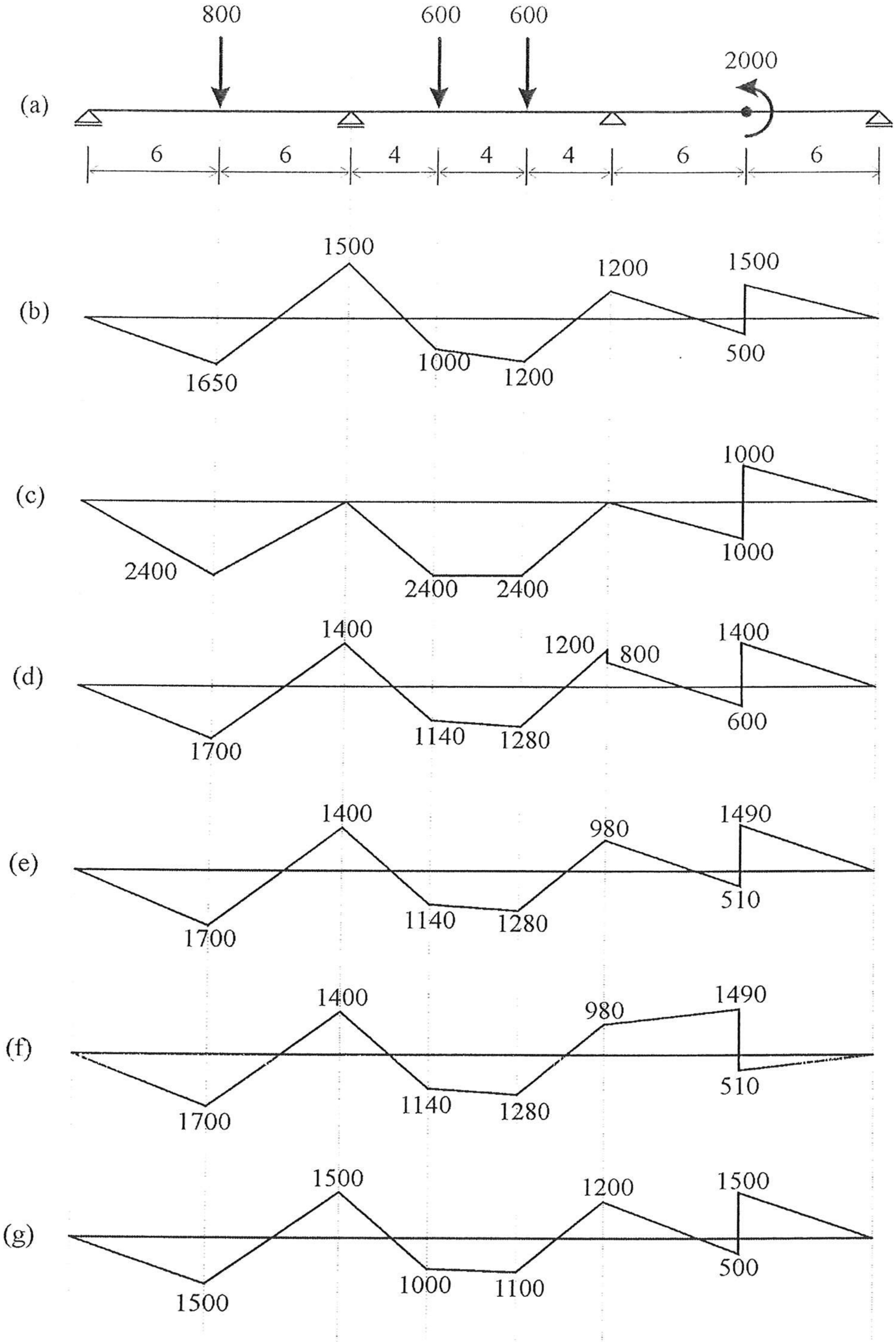
Fig. 4

5 (a) Quote the Lower Bound Theorem. [4]

(b) Figure 5(a) shows a continuous beam with three spans. It is subjected to a number of loads, as shown. Its self-weight is negligible. The beam has a fully plastic moment capacity of 3000 kNm in sagging or hogging bending. Figures 5(b) to 5(g) show six potential bending moment distributions. Which ones satisfy the requirements of the Lower Bound Theorem for the loading shown? Justify your answers. (An additional copy of Fig. 5 is attached at the end of this paper and can be annotated and handed in with your solution.) [8]

(c) For any that are satisfactory, estimate the load factor at collapse. [4]

(d) Suggest another distribution of bending moments that would give the highest possible estimate of the collapse load. [4]



(all dimensions in m, loads in kN, moments in kNm)

Fig. 5

6 The collapse load of a very large reinforced concrete slab, subject to a point load, can be estimated by a *local* collapse mechanism consisting of n identical segments, as shown in Fig. 6 (which is drawn for n=7). The segments of the mechanism all meet at the point of application of the load.

a) If the fully-plastic moment capacity of the slab is m , for both sagging and hogging bending, determine the collapse load in terms of n and R . [12]

b) Determine the optimum values for n and R . [8]

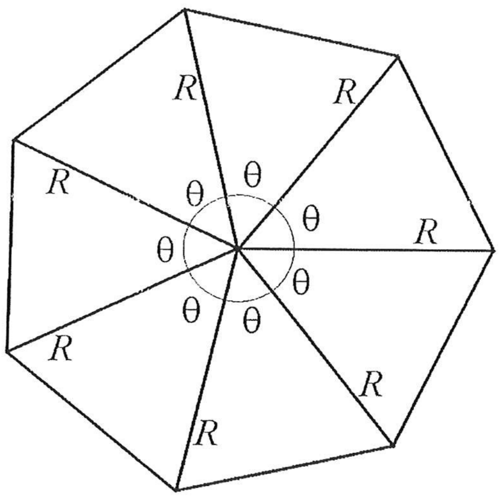
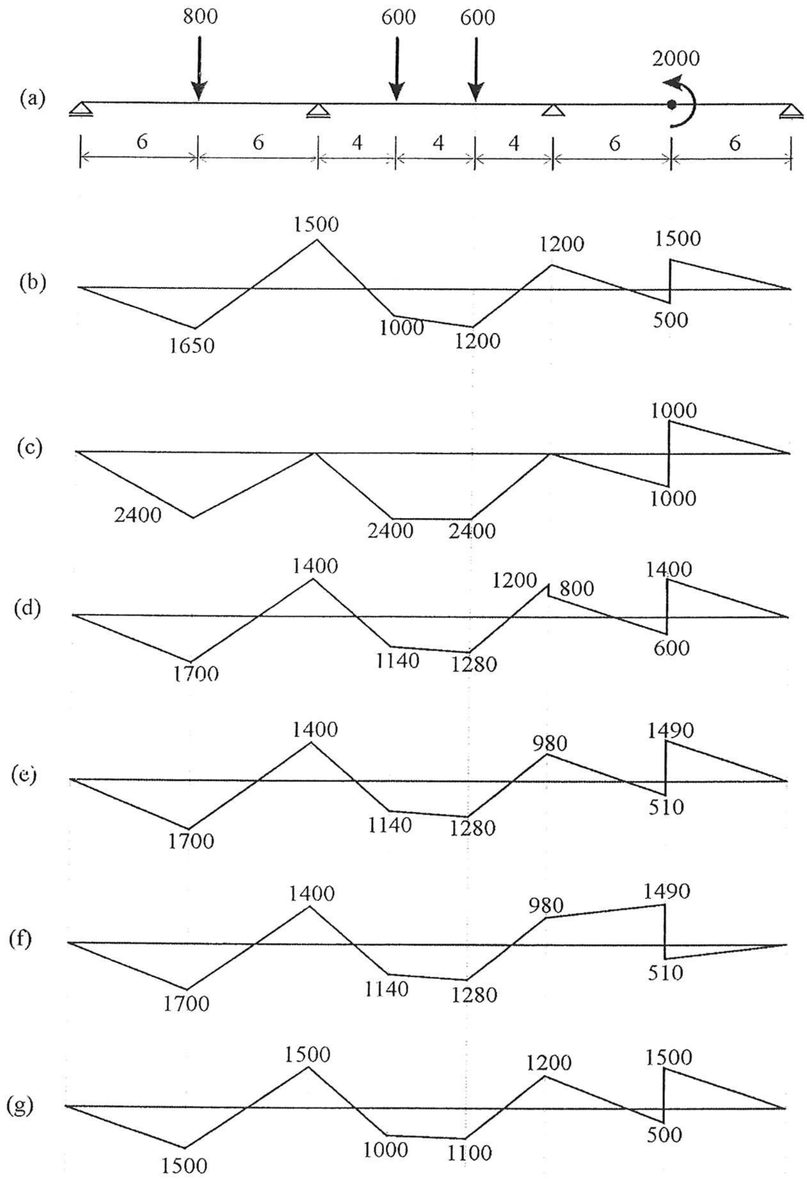


Fig. 6

END OF PAPER

ENGINEERING TRIPOS PART IB
STRUCTURES

Additional copy of Fig. 5 that can be annotated and should be handed in with your solutions. Additional copies are available from the Invigilator.



(all dimensions in m, loads in kN, moments in kNm)