ENGINEERING TRIPOS   PART IB

Tuesday 1 June 2010   9 to 11

Paper 3

MATERIALS

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments to this paper.

STATIONERY REQUIREMENTS
Single-sided paper
Single-sided graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 (a) The time-dependent deformation of polymers under stress can be represented by mechanical models of elastic springs and viscous dashpots.

(i) Using the governing equations for a linear elastic spring of modulus E and a linear dashpot of viscosity η, derive the following governing equation between the applied macroscopic stress σ and overall strain ε for the 3-element model of Fig. 1:

$$\frac{d\varepsilon}{dt}+\frac{E_2}{\eta_3}\varepsilon=\frac{E_1+E_2}{\eta_3 E_1}\sigma+\frac{1}{E_1}\frac{d\sigma}{dt}$$

where t is time. [5]

(ii) Determine an expression for the creep compliance $C(t)$ of the 3-element model where $C(t)=\varepsilon(t)/\sigma_o$ and σ_o is a constant stress applied at time t = 0. Hence, define the limits of creep compliance corresponding to very short and very long times. [4]

(iii) Why is this model unable to account for the time-dependence of the yield stress of a polymer? [3]

(b) An extruded pipe of outside diameter 168 mm is to be made of polyvinyl chloride (PVC). It should withstand an internal pressure of 1.2 MPa whilst supplying hot and cold water. Given that the tensile yield strength of PVC is 50 MPa as determined by a simple tensile test at room temperature, estimate the minimum wall thickness after applying a safety factor of 2 on the load. [4]

In order for the PVC pipe to last 50 years, British Standard (BS) 3505 states that the minimum acceptable wall thickness must not be less than 9.2 mm. Account for the difference between the calculated value and that specified by BS 3505. [4]

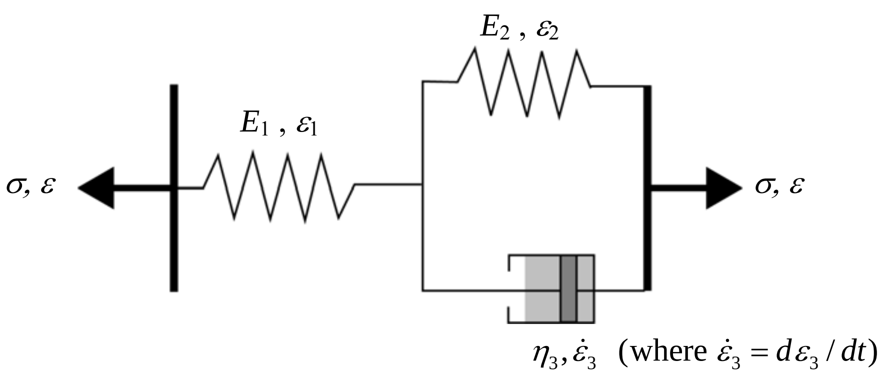

Fig. 1

2 (a) A casting is made from plain carbon steel containing 0.4 wt% of carbon.

(i) Sketch carefully the microstructure of the "as-cast" 0.4 wt% carbon steel and label the phases present. Estimate the proportions of the phases present. [3]

(ii) Propose and justify a particular heat-treatment in order to convert the "as-cast" microstructure to one that is completely martensitic. Explain why the Fe-C equilibrium diagram (Materials Data Book page 30) is of no use in accounting for this thermal treatment. [4]

(iii) Explain the effects upon microstructure of tempering the martensitic carbon steel at 550 ^{o}C for 1 hour. [3]

(b) Account for any changes in *hardness* and *toughness* of the 0.4 wt% carbon steel that may have taken place from its initial cast state to its final tempered condition. [5]

(c) At what stage in the complete casting and heat-treatment process might cracking occur in a component of complex shape and variable thickness? Give a possible cause of such cracking and propose a method for its prevention. [5]

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3 Figure 2 shows a schematic of a pair of rolls for the cold rolling of wide steel strip of width w (normal to the plane of Fig. 2). A roll force P is applied to each roll of radius R rotating at an angular velocity ω. The strip of initial thickness h_1 enters the rolls at velocity v_1 and exits with a final thickness h_2 and velocity v_2. The distance between the point at which the metal first enters the rolls and finally exits the rolls is L.

Address the following and state all assumptions made.

(a) Account for the *neutral plane* in the rolling process. [5]

(b) Sketch the distribution of roll pressure along the roll-strip interface. Account for the effect of varying the coefficient of friction μ between the roll surface and steel strip upon the distribution of roll pressure. [6]

(c) Derive an expression for the *roll force* P for perfectly lubricated rolls. [5]

(d) Describe briefly how the *roll force* is influenced by work hardening of the steel strip and the strip temperature. [4]

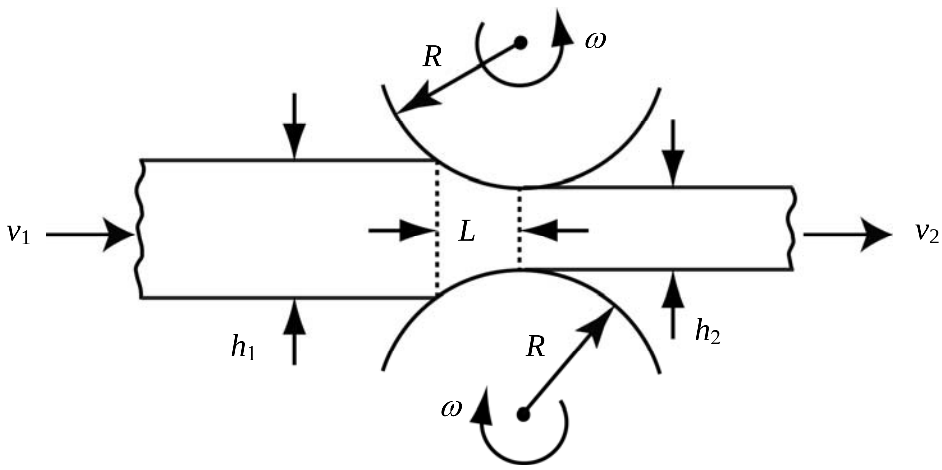

Fig. 2

SECTION B

4 (a) Figure 3 shows the phase diagram for the Al-Zn system.

(i) The phase diagram contains regions labelled A, B, C, D and E. Write down the phases involved in *each* of these regions;

(ii) The phase diagram contains two reactions. For *each* reaction, state the type of reaction, temperature and composition;

(iii) List the hardening mechanisms available to Al-Zn alloys. [7]

(b) An Al-Zn alloy is prepared by mixing 20 g of Al and 30 g of Zn. The alloy is melted and then allowed to cool slowly so that equilibrium is maintained. At each of the following temperatures, write down the phases present, their composition and the proportions of each phase.

(i) 500 °C;

(ii) 300 °C;

(iii) 250 °C. [6]

(c) Sketch schematic plots of the Gibbs free energy as a function of composition at 600 °C, labelling the composition range over which the different phases are stable. [7]

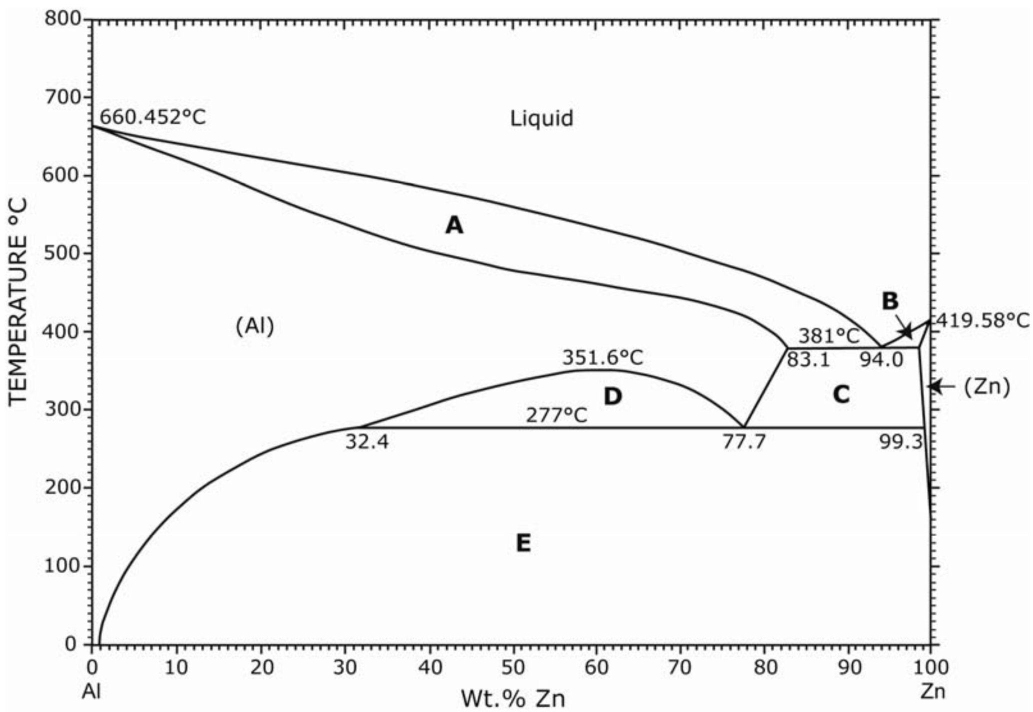

Fig. 3

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5 Derive the governing equations for transient one-dimensional mass and thermal diffusion in the Materials Data Book (page 8), assuming conservation of mass and energy respectively. Comment on their form. [8]

A 1 mm thick metallic wafer, made of a single crystal, is placed in a carbon-containing atmosphere at a temperature of 1000 °C such that the carbon concentration on all surfaces of the wafer is maintained at 0.1 wt%.

For the diffusion of carbon in the sample, the activation energy Q is 87 kJ mol^{-1} and the corresponding pre-exponential factor D_o is 1×10^{-6} m^2 s^{-1}.

(a) Calculate the diffusion coefficient D of carbon in this sample at 1000 °C. [2]

(b) The concentration of carbon C at a distance x below the surface after time t, is described by

$$C(x,t) = C_s - (C_s - C_o)\,\mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

where C_s is the concentration of carbon at the surface, C_o is the initial concentration in the material, and D is the diffusion coefficient.

If the diffusion coefficient is independent of carbon concentration, calculate the concentration at a depth of 0.1 mm below the surface of the wafer after 1 minute. [4]

(c) Plot the carbon concentration profile through the thickness of the wafer at this time stating any assumptions you make. Explain how you might expect the diffusion rate to differ if the wafer were polycrystalline. [6]

6 (a) Explain why yield criteria for metals are usually independent of hydrostatic pressure. [4]

(b) Sketch the failure surface of dry soil in τ - σ space, where τ and σ are respectively the shear and normal stresses (compressive stresses are positive quantities), showing the regions at which failure occurs. Using sketches to illustrate your answer, explain why this failure criterion might be appropriate. [6]

(c) A wet soil is found to obey the following failure criterion

$$\tau = \tau_* + \sigma_n \tan\phi$$

where τ and σ_n are respectively the shear stress and the normal compressive stress on the plane of failure, τ_* = 20 kPa and ϕ = 35°.

(i) Sketch the soil failure surface in τ - σ_n space, showing the regions at which failure occurs. What is the physical meaning of τ_*? [4]

(ii) The soil is found to slip when subjected to a state of stress with a shear component τ equal to 50 kPa along the slip plane. Determine the normal stress σ_n on the slip plane, and the maximum and minimum principal stresses. [6]

END OF PAPER