ENGINEERING TRIPOS PART IB

Tuesday 1 June 2010 2 to 4

Paper 4

THERMOFLUID MECHANICS

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

Answer not more than ***two*** *questions from this section*

1 (a) Show that the thermal resistance of a spherical shell, of thermal conductivity λ, with inner and outer radii r_1 and r_2, respectively, is given by

$$R_{\text{th}} = \frac{1}{4\pi\lambda}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

You may assume that the surface area of a sphere is $4\pi r^2$. [4]

(b) A very thin-walled, high thermal conductivity, spherical vessel of diameter 0.25 m, is covered with a uniform layer of insulating material ($\lambda = 0.1$ Wm^{-1}K^{-1}), of thickness 0.025 m. The outer surface of the insulant exchanges heat with the environment via convection, with a heat transfer coefficient of $h = 10$ Wm^{-2}K^{-1}.

(i) Determine the value of the overall thermal resistance between the inner wall and the environment. [4]

(ii) The vessel is filled with wet saturated refrigerant R-134a at an absolute pressure of 1.06 bar, and it may be assumed that the inner surface of the vessel is at the fluid temperature. The environment is at 20 °C. Find the rate of heat transfer to the vessel contents. [4]

(c) (i) Explain why the addition of the insulant might lead to an increased heat transfer rate. [2]

(ii) Show that for a fixed inner radius, the condition that produces the maximum heat transfer is $r_2 = 2\lambda/h$. Can this condition exist here? Why? [2]

(d) The effect of the heat transfer is to gradually vapourise the vessel contents, and these vapours are vented via a throttle to atmosphere such that the pressure in the vessel remains at the initial value. How long does it take for all the contents to be vapourised? You may assume, as in (b), that the inner surface of the vessel is at the fluid temperature. [4]

2 Air and a hydrocarbon fuel, C_nH_m, are burnt under lean conditions.

(a) Show that the chemical balance is given by

$$C_nH_m + \lambda\left(n+\frac{m}{4}\right)(O_2 + kN_2) = nCO_2 + \frac{m}{2}H_2O + (\lambda - 1)\left(n+\frac{m}{4}\right)O_2 + \lambda\left(n+\frac{m}{4}\right)kN_2,$$

where λ is the ratio of the actual to the stoichiometric air fuel ratio, and $k = 3.762$ is the molar $N_2 : O_2$ ratio for air. [4]

(b) A measurement in the exhaust gases from a domestic central heating unit, burning natural gas (CH_4) shows that the oxygen concentration, measured on a dry basis, is 14% by volume.

(i) Show that the value of λ is then 2.79. [4]

(ii) The exhaust gases pass through a heat exchanger, and then to the atmosphere via a metal chimney. To avoid corrosion of the chimney, condensation of water from the exhaust gases is to be avoided. What is the minimum temperature (to the nearest degree) of the exhaust gases? Assume the ambient pressure is 1 bar. [6]

(iii) Many new central heating units avoid the corrosion problem by utilising plastic chimneys, which enables a lower exhaust gas temperature to be used. Assuming the exhaust temperature is now 25 °C, and the ambient pressure is 1 bar, find the mass of water condensed per kmol of fuel. Hence determine the increase in heat extracted, expressed as a % of the fuel's lower calorific value. (Assume the condensed water is also cooled to 25 °C, and that the dry exhaust gases have a molecular weight of $29.7\ \mathrm{kgkmol^{-1}}$ and a specific heat of $1.01\ \mathrm{kJkg^{-1}K^{-1}}$.) [6]

3 A steady flow steam plant works on the superheated Rankine cycle with reheat. The cycle operates with a boiler pressure of 100 bar (boiler entry is point (1)), with superheating to 550 °C (2), expansion to 5 bar (3), reheating back to 550 °C (4), and a final expansion to the condenser pressure (5), which is 0.05 bar. Inlet to the feed pump (6) is wet saturated. Assume all processes making up the cycle are reversible, the turbines are adiabatic, feed pump work may be neglected, and kinetic energy changes are small. (You are advised to use the steam chart where possible.)

(a) Sketch the cycle on a $T-s$ diagram. [3]

(b) Determine, per unit mass flow rate:

(i) the work output from each of the turbine stages; [2]

(ii) the heat addition in the boiler, the superheater and the reheater; [2]

(iii) the thermal efficiency of the cycle. [2]

(c) If the flow between the superheater and the first turbine is throttled to 50 bar and all other pressures remain unchanged, calculate the percentage reduction in output work and the new thermal efficiency. [5]

(d) Confirm that the feed pump work is negligible. [3]

(e) Re–sketch the original (unthrottled) cycle on a $T-s$ diagram, and indicate on it the areas which represent the boiler heat input and the condenser heat rejection. What combination of these areas represents the thermal efficiency of the cycle? [3]

SECTION B

*Answer not more than **two** questions from this section*

4 A plate is slowly lowered at a constant speed V through oil as shown in Fig. 1. The plate is of length $2L$ and is a distance H from the base of the large tank containing the oil. It is assumed that $L \gg H$ and that the tank and plate are of infinite extent in the z−direction, measured into the page. The dynamic viscosity of the oil is μ.

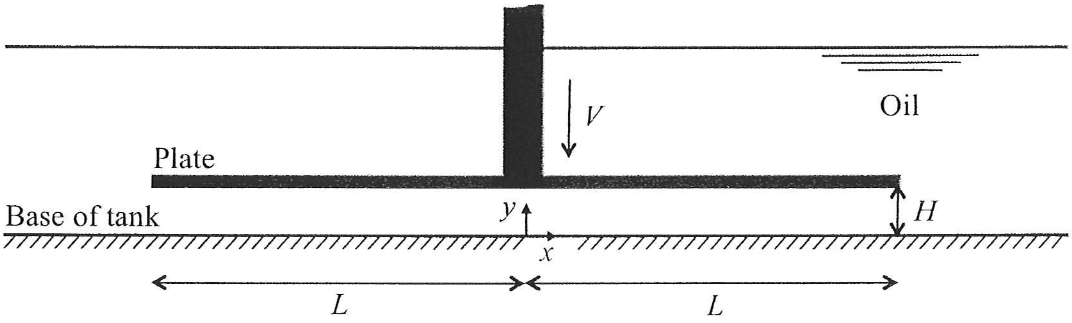

Fig. 1

(a) By applying the principle of conservation of mass show that Q, the volume flow rate of fluid between the plate and the base of the tank per unit distance into the page, is given by $Q(x) = V|x|$. [4]

(b) By considering a force balance on an elemental section of the fluid or otherwise, show that the velocity of fluid between the plate and the base of the tank, u, satisfies

$$\mu \frac{d^2u}{dy^2} = \frac{dp}{dx},$$

where p is the pressure under the plate. Neglect inertial forces. [4]

(c) Solve for u for $x \geqslant 0$ by applying appropriate boundary conditions. Hence show that

$$\frac{dp}{dx} = A\frac{\mu V x}{H^3},$$

and find the value of the constant A. [6]

(d) Find an expression for F, the total force per unit distance in the z−direction required to push the plate down at speed V. [6]

5 Consider a two–dimensional jet of air impinging on a cylinder as shown in Fig. 2. The cylinder and jet are of infinite depth into the page. The cylinder has diameter D and the jet has initial width w_0 and initial speed u_0. The jet eventually separates from the cylinder, with the total contact angle α as shown in the figure. Inviscid, incompressible flow can be assumed and $w_0 \ll D$. The jet does not mix with the ambient air and does not break up. The short transition regions after the flow comes into contact with the cylinder and before it separates can be neglected for this problem. Within the contact region the flow streamlines can be assumed to be circular.

(a) Prove, from first principles, that the pressure gradient across a streamline with radius of curvature R is

$$\frac{\mathrm{d}p}{\mathrm{d}r} = \frac{\rho u^2}{R},$$

where the symbols have their usual meaning and should be defined. [4]

(b) The pressure in the jet before it contacts and after it leaves the cylinder is equal to the ambient atmospheric pressure, p_A. Within the region where the jet contacts the cylinder, the jet velocity is $u(r)$ and the pressure is $p(r)$, where r is the radial distance from the centre of the cylinder. By considering any streamline, write down an equation relating p_A and u_0 to $p(r)$ and $u(r)$. Hence show that

$$ru(r) = M,$$

where M is a constant to be found. [7]

(c) Find an expression for w. Note that for $\varepsilon \ll 1$, $\ln(1+\varepsilon) \approx \varepsilon - \frac{1}{2}\varepsilon^2$. [5]

(d) By applying the steady flow momentum equation, work out the force per unit depth into the page exerted by the jet on the cylinder. State clearly which direction the force acts in. (Note that the force acts only in the vertical.) [4]

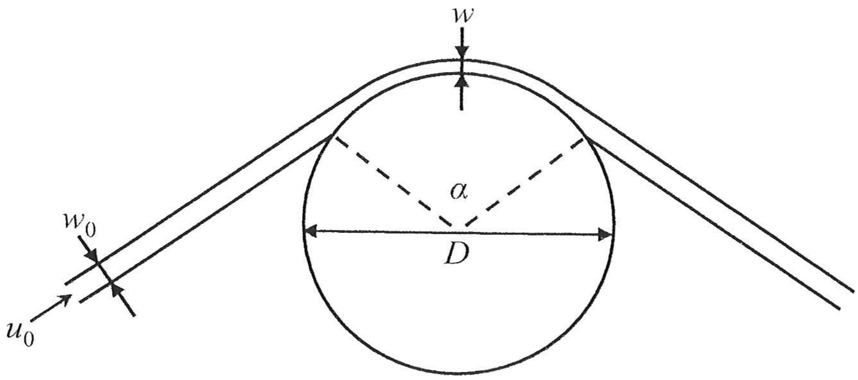

Fig. 2

6 (a) Show that the rate of work transfer for a reversible adiabatic pump or turbine conveying an incompressible fluid at volumetric flow rate Q is given by

$$-\dot{W}_x = Q(p_{0,2} - p_{0,1}),$$

where $p_{0,1}$ and $p_{0,2}$ are the total pressures at points 1 and 2, respectively, and

$$p_0 = p + \frac{1}{2}\rho u^2 + \rho g z,$$

where the symbols have their usual meaning. [4]

(b) Figure 3 shows a diagram of a pump storage scheme. During electrical power generation, the water flow from the upper to the lower reservoir is 120 $\mathrm{m^3 s^{-1}}$, whilst during pumping, the flow rate is 80 $\mathrm{m^3 s^{-1}}$ in the other direction. (The pump and turbine are the same physical machine.) The difference in height between the upper and lower reservoir water surfaces may be assumed constant, and equal to 320 m. The water is conveyed between the pump/turbine machine and the upper reservoir via 4 parallel pipes, each of internal diameter 2.8 m. Assume that the coefficient of friction in the pipes is 0.005, and the distance travelled by the water between the upper reservoir and the pump/turbine is 1600 m. The pipes between the pump/turbine and the lower reservoir are short and of very large diameter.

You may assume that the height difference between points 1, 2 and 3, and between 4 and 5 are negligible.

(i) What are the gauge pressures at points 2, 3 and 4 for the pumping and for the generation cases? [6]

(ii) If the machine efficiency (i.e. the ratio formed from the electrical power and the reversible adiabatic pump or turbine power) is 90% for both pumping and generation, determine the electrical power required for pumping, and the electrical power generated. Hence find the overall installation efficiency, in terms of the energy supplied vs. energy consumed. [6]

(iii) At the operating point for generation given, find the proportional change in output power for a proportional change in flow rate, i.e.

$$\frac{\mathrm{d}\dot{W}_x}{\dot{W}_x} \bigg/ \frac{\mathrm{d}Q}{Q}.$$

[4]

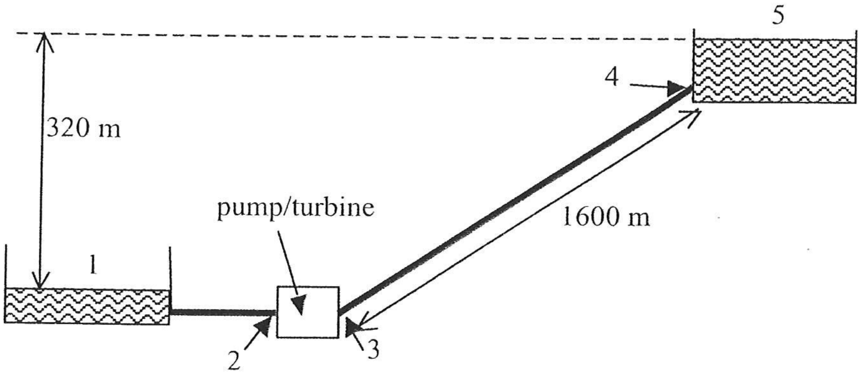

Fig. 3

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