ENGINEERING TRIPOS PART IB

Wednesday 2 June 2010 2 to 4

Paper 5

ELECTRICAL ENGINEERING

Answer not more than **four** *questions.*

Answer at least **one** *question from each section.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

ajf05

SECTION A

Answer at least **one** *question from this section*

1 (a) State what is meant by the terms *Class A*, *Class B* and *Class AB* amplifiers. What Class of amplifier is the emitter-follower amplifier shown in Fig. 1? [4]

(b) Calculate the base current that flows into the bipolar junction transistor in Fig. 1 if the d.c. current gain h_{FE} is 50 and the base-emitter voltage V_{BE} is 0.7 V when the transistor is active and no input signal is applied. [4]

(c) The same transistor has small-signal parameters of $h_{ie} = 1$ kΩ, $h_{fe} = 50$ and $h_{oe} = 0.5$ mS. Derive expressions for the small-signal voltage gain and the output resistance of the circuit and calculate their numerical values. You may assume that the capacitors C_1 and C_2 have negligible reactance at the signal frequencies being used and that h_{re} is negligible. [8]

(d) What value of load resistor R_L should be connected across the output terminals of the circuit to maximize the output power? Estimate the resulting maximum ac power that can be delivered into this load for a sinusoidal signal. [4]

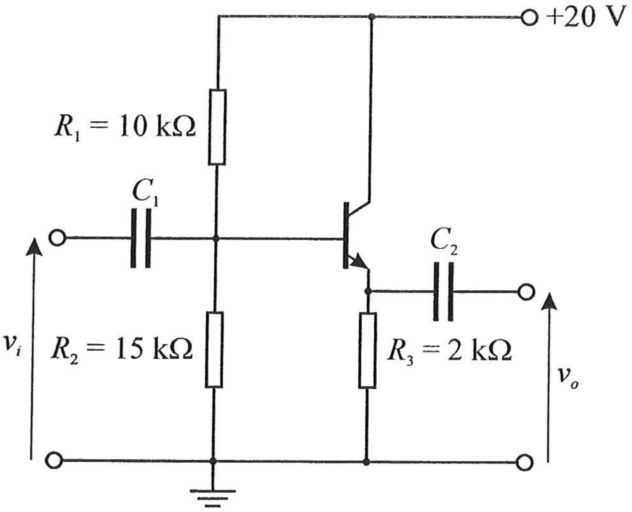

Fig. 1

2 (a) Fig. 2 shows a simple differential amplifier circuit for amplifying a high frequency (~10 kHz) signal from a light meter. Interference from the a.c. mains means that there is an unwanted additional signal at 50 Hz. Explain, with the aid of a diagram, the meaning of the terms *difference signal* and *common-mode signal*. Why is the differential amplifier in Fig. 2 suitable for this application? [5]

(b) The operational amplifier in the circuit shown in Fig. 2 is considered to be ideal. If $R_1 = R_2$ and $R_3 = R_4$, derive an expression for the differential gain of the circuit. [4]

(c) If the operational amplifier in the circuit shown in Fig. 2 actually has a finite gain A and finite input resistance R_i, but is otherwise ideal, show that, for $R_1 = R_2$ and $R_3 = R_4$, the differential gain of the circuit, G, is now given by

$$G = \frac{1}{\dfrac{R_1}{A}\left(\dfrac{1}{R_1} + \dfrac{1}{R_3} + \dfrac{2}{R_i}\right) + \dfrac{R_1}{R_3}} .$$ [7]

(d) The gain of the operational amplifier is $A = 10^7$ and the input resistance $R_i = 10\ \mathrm{M\Omega}$. Using the equation given in part (c), suggest suitable values for R_1 and R_3 if the difference signal $(v_2 - v_1)$ is to be amplified by a factor of 100 and the operational amplifier also has a small output resistance of 10 Ω. [4]

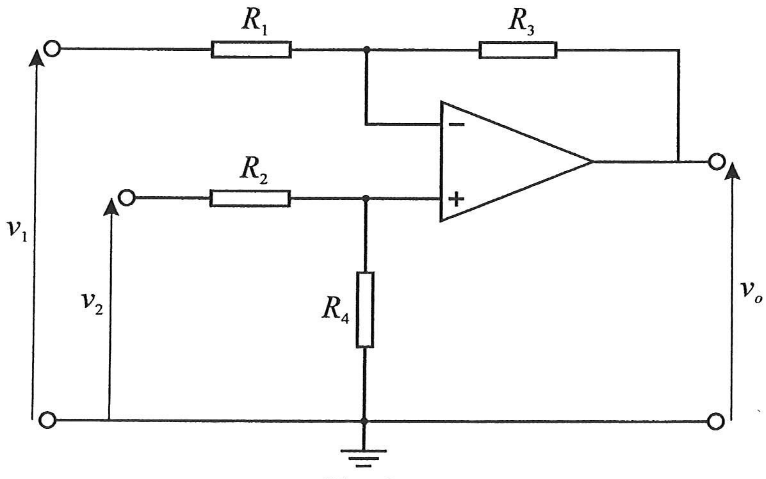

Fig. 2

SECTION B

Answer at least **one** *question from this section*

3 (a) Explain why power is commonly transmitted from a power station to a town using *high voltage three-phase a.c. electric power* rather than using *d.c. power*. Why is it necessary to consider a town as being a balanced load on the transmission line? [3]

(b) A set of three-phase a.c. transmission lines are connected to a town which acts as a balanced load consuming 100 MW of power at a power factor of 0.85 lagging. If the line voltage at the town is 33 kV, what is the line current per phase? [4]

(c) Each feeder line to the town has an impedance of $(0.02 + j0.05)\ \Omega$ per kilometer and is 100 km long. Calculate the power dissipated in the transmission lines and the line voltage at the sending end of the feeder. [8]

(d) By how much would power dissipation be reduced if the power factor of the town could be improved to 0.9 lagging? Describe one method by which this improvement could be achieved. [5]

(TURN OVER

4 (a) a.c. synchronous generators that are connected to a grid normally consist of a rotor and stator separated by an air gap. Sketch a graph showing how torque varies as a function of load angle. Identify the regions in which the system is *generating* and *motoring*. Also identify the points at which synchronicity is lost. [4]

(b) A star-connected, 4-pole synchronous generator which is rated at 750 MVA delivers 400 MW of power to a 50 Hz infinite bus with a line voltage of 22 kV and a power factor of 0.85 lagging. The synchronous reactance is 1.2 Ω per phase and the stator resistance is negligible. Calculate:

(i) the prime mover torque; [3]

(ii) the generator excitation voltage; [6]

(iii) the load angle. [2]

(c) Explain how the amount of reactive power delivered to the infinite bus can be controlled by the synchronous generator whilst maintaining a constant real power output. What is the maximum reactive power that can be delivered by the generator if it is limited by stator heating? [5]

5 An 8-pole, three-phase, star-connected induction motor is connected to a balanced 50 Hz supply with a line voltage of 415 V. The following tests are performed on the induction motor:

Line voltage, V_l	Line current, I_l	Input power, P_{in}	Speed
415 V	4.2 A	1100 W	750 rpm
90 V	65 A	4500 W	0 rpm

The stator winding resistance, R_1 , is 0.2 Ω. The stator leakage reactance, X_1 , is half that of the rotor leakage reactance referred to the stator, X_2' .

(a) Find the values of the following in the equivalent circuit of the induction motor:

(i) the iron loss resistance, R_i ; [3]

(ii) the magnetizing reactance, X_m ; [2]

(iii) the rotor winding resistance referred to the stator, R_2' ; [3]

(iv) the stator leakage reactance, X_1 . [2]

(b) Calculate the maximum torque and the slip and speed at which it occurs. [6]

(c) What resistance should be added to the rotor circuit via the slip rings to maximize the starting torque if the ratio of stator turns to rotor turns is 2? [4]

SECTION C

Answer at least **one** *question from this section*

6 (a) Fig. 3 shows the equivalent circuit for a small section δx of a lossless transmission line where C and L are the capacitance per unit length and inductance per unit length of the transmission line respectively. Using this circuit element, derive the Telegrapher's Equations:

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2}$$ [4]

(b) The capacitance per unit length and inductance per unit length of a lossless coaxial cable are given by

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(r_2/r_1)} \quad \text{and} \quad L = \frac{\mu_0\mu_r \ln(r_2/r_1)}{2\pi}$$

where r_1 is the outer radius of the inner conductor, r_2 is the inner radius of the outer conductor, and ε_r and μ_r are the relative permittivity and relative permeability of the filling dielectric respectively. Derive an expression for the velocity of a wave on this coaxial transmission line in the absence of reflections. Comment on the physical significance of your answer. [4]

(c) Calculate the ratio of the radii of the conductors, r_2 / r_1, if the coaxial cable is to have a characteristic impedance of 75 Ω, and PTFE is used as the dielectric filling which has a relative permittivity of 2.1 and a relative permeability of 1. [4]

(d) This coaxial cable is to be connected to a second coaxial cable with a characteristic impedance of 50 Ω which is itself terminated with a 50 Ω load. To eliminate reflections, a short section of a third coaxial cable is connected between the other two, which also uses PTFE as the filler dielectric. The system is to carry signals at a frequency of 10 MHz. Starting from the information in Section 3.10.1 of the Electrical and Information Data Book, calculate the characteristic impedance and length of this third (connecting) coaxial cable.

[5]

(e) What practical problems will be encountered if this system is being used to transmit a frequency modulated (FM) communications signal where the carrier frequency is 10 MHz?

[3]

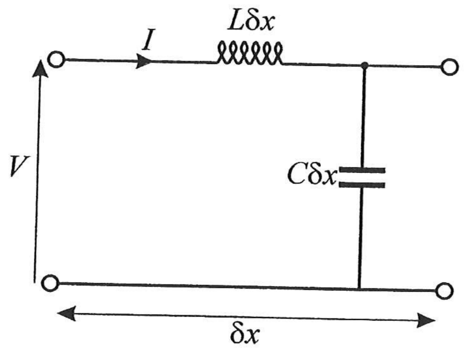

Fig. 3

7 (a) A plane electromagnetic wave propagates through free space along the z-axis in Cartesian coordinate space. The components of the electric field intensity along each Cartesian axis are given by:

$$E_x = E_{x0} \exp[j(\omega t - \beta z)]$$
$$E_y = E_z = 0$$

Given that in this simple case, the Maxwell equation $\nabla \times \mathbf{E} = -\dot{\mathbf{B}}$ reduces to

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t},$$

derive an expression for the y-component of the magnetic field intensity, H_y. [5]

(b) Wireless data (Wi-Fi) networks operating at a frequency of 2.4 GHz may be contained within a building by coating glass windows in a thin film of a transparent conductive material, such as indium tin oxide (ITO). ITO has a conductivity of $10^5\ \Omega^{-1}\ \mathrm{m}^{-1}$, a relative permittivity of 3.8 and a relative permeability of 1. Using the information in Section 3.10.4 of the Electrical and Information Data Book, calculate the propagation constant, γ, and intrinsic impedance, η, of the ITO seen by the Wi-Fi electromagnetic wave. [5]

(c) An electromagnetic wave passes from a medium with an intrinsic impedance η_1 into a medium with an intrinsic impedance η_2 where the direction of propagation is perpendicular to the plane of the surface. If the components **E** and **H** parallel to the boundary surface are continuous across the boundary, show that the transmission coefficient is given by

$$\rho_T = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_2 + \eta_1},$$

where E_t and E_i are the amplitudes of the transmitted and incident electromagnetic waves respectively. [5]

(d) If the incident Wi-Fi electromagnetic wave has a power density of 10^{-2} W m^{-2} in free space at the surface of the ITO, calculate the minimum thickness of ITO required to attenuate the power of the wave to less than 10^{-7} W m^{-2}. The power density decays with depth into the conductor, d, as $\exp(-2\alpha d)$ where α is the real component of the propagation constant. [5]

END OF PAPER