ENGINEERING TRIPOS PART IB

Thursday 3 June 2010 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than **four** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig. 1.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

GV04

SECTION A

*Answer not more than **two** questions from this section*

1 Consider a car of mass M travelling in a straight line on a straight and level road. The only forces acting on it, in the horizontal plane, are assumed to be a propulsive force $T(t)$ and a resistive force $\lambda v(t)$, where $v(t)$ is the velocity of the car.

(a) If $T(t) = Ku(t)$, where $u(t)$ is a throttle input, find the transfer function from $\bar{u}(s)$ to $\bar{v}(s)$. [5]

(b) An adaptive cruise control system is now implemented, so that $u(t) = k_p(x_r(t) - x(t))$, where $x(t)$ is the position of the car and $x_r(t)$ is a reference position (in this case, a fixed distance behind the car in front). Draw a block diagram of the feedback system, and find the closed-loop transfer functions from $\bar{x}_r(s)$ to the position $\bar{x}(s)$ and to the error $\bar{e}(s) = \bar{x}_r(s) - \bar{x}(s)$. [5]

(c) If $x_r(t) = Ct$, then find an expression for the steady-state value of the error. [5]

(d) If $M = 1000$, $\lambda = 1200$ and $Kk_p = 2250$, then find the steady-state response to the reference $x_r(t) = 30t + \cos(1.5t)$. Comment on the result of an identically equipped following vehicle now taking *its* reference from the position of the vehicle we are considering here. [5]

2 A simple model of the human heart rate response to exercise on a stationary bicycle is given by

$$\bar{\delta}_{\mathrm{HR}}(s) = k_{\mathrm{HR}} \frac{\exp(-sT)}{s\tau + 1} \bar{p}(s)$$

where $\delta_{\mathrm{HR}}(t)$ is the difference between the current heart rate and the resting heart rate and $p(t)$ is the power being generated by the rider. k_{HR}, τ and T are parameters that would be expected to change from person to person.

(a) Sketch the response of heart rate, $\delta_{\mathrm{HR}}(t)$, to a step change in power, showing clearly the effect of the parameters T and τ. [5]

(b) Consider now a system designed to regulate the heart rate by means of adjusting the power absorbed by the machine:

$$\bar{p}(s) = -K\left(\bar{\delta}_{\mathrm{HR}}(s) - \bar{r}(s)\right)$$

where $r(t)$ is the desired change in heart rate. Draw a block diagram of the feedback system and sketch the Nyquist diagram of its return ratio. [6]

(c) For a particular non-athlete, it is found that $T = 12.1$sec, $\tau = 20$sec and $k_{\mathrm{HR}} = 0.2$. Show that if $K = 10$ then the feedback system has a phase margin of approximately $60°$. (You are not expected to draw an accurate Nyquist diagram.) [4]

(d) The same feedback system, with $K = 10$ again, is to be used by a trained athlete, for whom $T = 10.1$sec, $\tau = 15$sec and $k_{\mathrm{HR}} = 0.3$. What is the new phase margin? Comment on the likely result of this rider using the machine. [5]

3 The transfer function from torque $\bar{\tau}(s)$ to angular position $\bar{\theta}(s)$ of a particular satellite may be approximated as

$$G(s) = \frac{Js^2 + \lambda s + k}{s^2\left(J_0 J s^2 + (J + J_0)\lambda s + (J + J_0)k\right)}$$

where J_0 is the moment of inertia of the main body of the satellite. J is the moment of inertia of some attached solar panels and k is the stiffness, and λ the damping, of their attachment (all values are taken about the same longitudinal axis).

(a) Figure 1 shows some experimentally determined points of the frequency response corresponding to this transfer function. Assuming there exists a means of applying a torque to the satellite, explain briefly how these points might have been obtained. Add asymptotes to the provided magnitude diagram, and use these to estimate the combined moment of inertia $J_0 + J$. [6]

(b) A feedback position control system for the satellite is now to be designed, of the form:

$$\bar{\tau}(s) = K(s)\left(\bar{\theta}_d(s) - \bar{\theta}(s)\right)$$

where $\theta_d(t)$ is the demand signal. If a proportional controller, $K(s) = k_p$, is used then estimate the maximum phase margin that can be obtained, and the corresponding value of k_p. [6]

(c) Consider instead a controller of the form:

$$K(s) = k_p \frac{(10s+1)}{(2.5s+1)}$$

Sketch the form of the Bode diagram of this controller, and add the resulting return ratio $G(s)K(s)$ to the provided Bode diagram for a suitable value of k_p. Hence find the value of k_p that gives, approximately, the maximum phase margin subject to the requirement that $|G(j\omega)K(j\omega)| \geq 1$ for $\omega \leq 0.2$. What is this phase margin, and what is the significance of this requirement? [8]

An extra copy of Fig. 1 is provided as an attachment. This should be handed in with your answer.

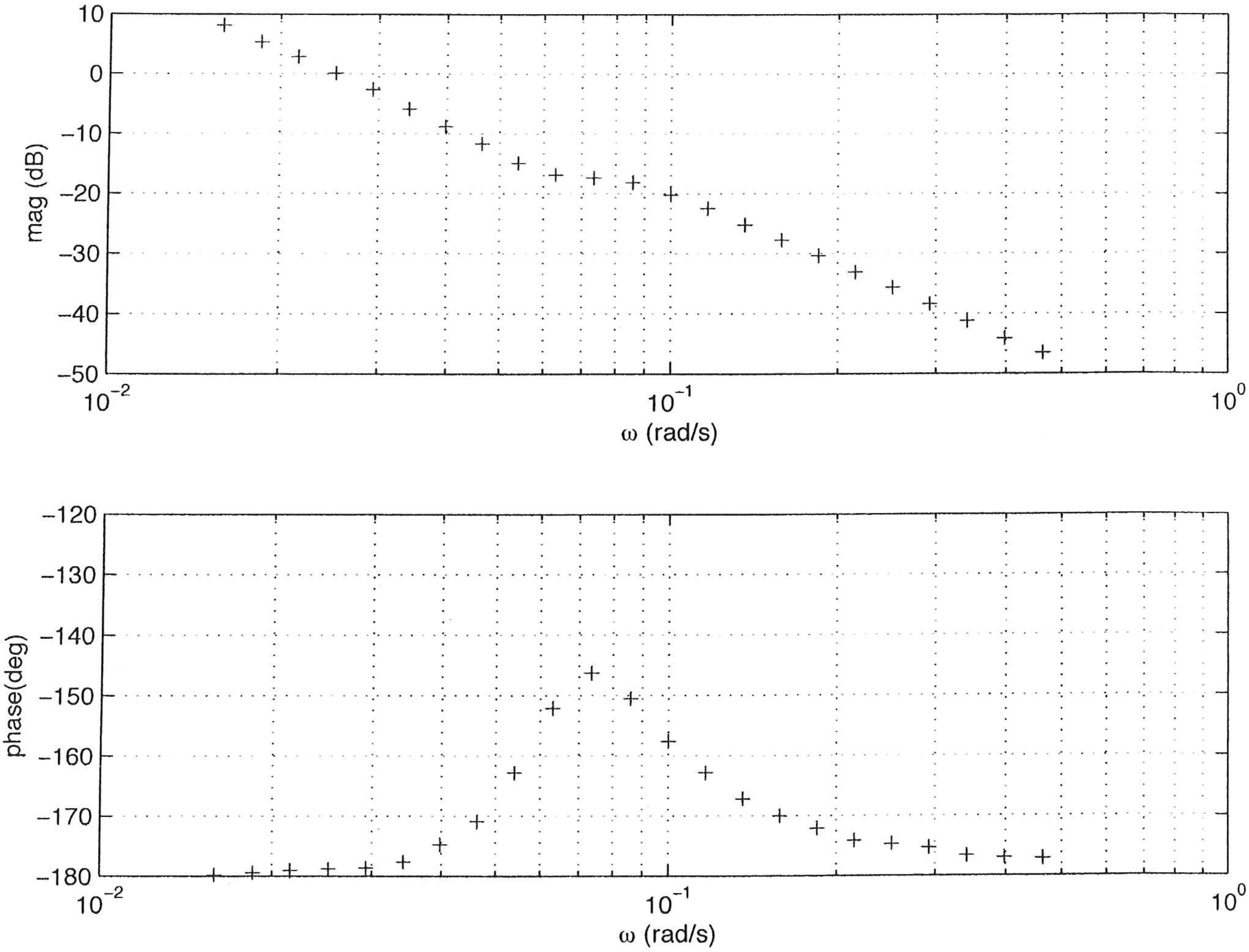

Fig. 1

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SECTION B

Answer not more than **two** *questions from this section.*

4 (a) Define the energy and the average power for a signal $f(t)$. [2]

(b) Determine from first principles the inverse Fourier Transform of the function $F(\omega)$:

$$F(\omega) = \begin{cases} h, & |\omega| < B \\ 0, & \text{otherwise} \end{cases}$$

[4]

(c) For a general function $f(t)$, show the following relationship between the function and its Fourier transform $F(\omega)$:

$$\int_{-\infty}^{+\infty} f(t)^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) F(-\omega) d\omega$$

[8]

(d) A signal is defined as

$$f(t) = \frac{\sin(\alpha t)}{\pi t}$$

Determine the energy of this signal. Consider what happens to this value as $\alpha \rightarrow \infty$ and hence determine a fundamental property of the δ-function. [6]

5 (a) Describe the correct procedure for sampling a continuous-time signal $f(t)$, whose maximum frequency component is at BHz, to give a discrete time signal $f(nT)$, and explain how to recover the original continuous time signal exactly from the digital samples. You should assume ideal δ-function sampling and perfect reconstruction filters throughout, and your description should include an explanation of the signal bandwidth, the Nyquist sampling frequency and aliasing. How would this procedure be modified in practice for the discretisation and reconstruction of a real-world signal? [7]

(b) A signal $f(t)$ is sampled with sampling period T. If ideal sampling is used, explain why the sampled signal may be written as:

$$f_s(t) = f(t)\delta_p(t)$$

where $\delta_p(t)$ is a special function which you should carefully define. [3]

(c) Show that the Fourier transform of the sampled signal may be written as:

$$F_s(\omega) = \sum_{n=-\infty}^{+\infty} f(nT)\exp(-jn\omega T)$$

Prove that this spectrum is a periodic function of ω and give its period. [6]

(d) Hence or otherwise show that the samples $f(nT)$ may be obtained from $F_s(\omega)$ via the formula:

$$f(nT) = \frac{T}{2\pi}\int_{-\pi/T}^{+\pi/T} F_s(\omega)\exp(+jn\omega T)d\omega$$

[4]

(TURN OVER

6 (a) Describe the principal techniques of modulation, stating the advantages and disadvantages of each, and giving examples of when each would be used in practice. [6]

(b) A speech signal $f(t)$, whose value is limited to lie within the range ± 1V, is to be transmitted via modulation on a radio channel centred at 800kHz. The modulation index is 0.4. The expression for the modulated signal is of the form:

$$s(t) = \big(10 + af(t)\big)\cos(\omega_c t)$$

What type of modulation scheme is being used? State the values of the constants a and ω_c. Show that the spectrum of the modulated signal for positive frequencies is given by:

$$S(\omega) = 10\pi\delta(\omega - \omega_c) + \frac{a}{2}F(\omega - \omega_c), \qquad \omega > 0$$

where $F(\omega)$ denotes the Fourier transform of $f(t)$. What bandwidth should be allocated to this channel if $f(t)$ is a medium quality speech signal and the gap between each adjacent channels in the spectrum must be 3kHz? [8]

(c) A signal modulator circuit for a carrier frequency ω_c is to be made by taking a certain signal $x(t)$ defined as follows:

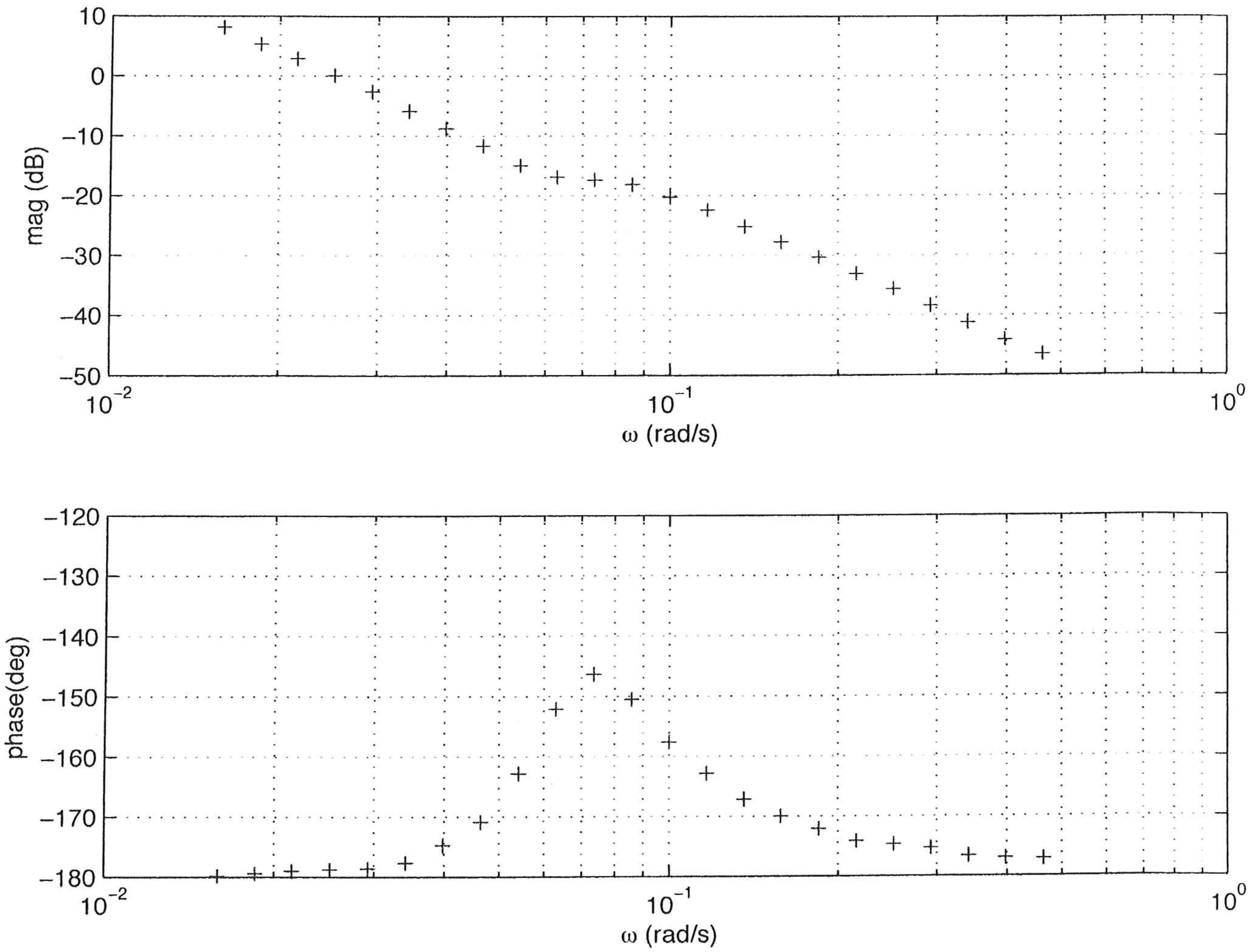
$$x(t) = 1 + f(t) + \cos(\omega_c t)$$

and squaring it. Show that, under appropriate assumptions, an amplitude modulated (AM) signal may be obtained from $x(t)^2$ by bandpass filtering; suggest also a suitable centre frequency and bandwidth for the required filter. [6]

END OF PAPER

ENGINEERING TRIPOS PART IB

Thursday 3 June 2010, Paper 6, Question 3.



Extra copy of Fig. 1 for Question 3.

(to be handed in with your answer)