ENGINEERING TRIPOS PART IB

Friday 4 June 2010 — 9 to 11.30
9 to 10.30 Foreign Language Option

Paper 8

SELECTED TOPICS

*Answer **one** question from Section A. In addition:*

*If you are not taking the Foreign Language option, answer **four** questions, taken from only **two** of Sections B – H. Not more than **two** questions from each section may be answered.*
*If you are taking the Foreign Language option, answer **two** questions from **one** of Sections B – H.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Data Sheet for Section B (6 pages)
Data Sheet for Section E (2 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A *Introductory Business Economics*

*Answer not more than **one** question from this section.*

1 (a) In the worldwide market for coffee, what effect would the following events have on the equilibrium price and quantity? Illustrate using a supply and demand diagram.

(i) An advertising campaign to promote the drinking of coffee. [3]

(ii) A period of bad weather, resulting in a lower coffee crop. [3]

(b) How does monopoly differ from perfect competition in terms of:

(i) the number of firms in the market; [1]

(ii) the profits of firms in the long run; [3]

(iii) consumer welfare. [4]

(c) Disequilibrium unemployment occurs when the labour market fails to clear, so that labour supply does not equal labour demand.

(i) Explain why the wage rate can be "sticky" in the short run, and how this can prevent the labour market from clearing. [3]

(ii) What policies can the government use to reduce unemployment. [3]

2 (a) Describe the model of perfect competition. [6]

(b) Consider a duopoly with two firms, X and Y, both of which produce a similar product, and charge a price of £2 per unit of their product. At this price both firms make an annual profit of £10 million. However, both firms are considering lowering the price for their product to £1.80, in order to increase their sales. The payoff matrix of the firms is shown below, where the first payoff in each cell is for firm Y, and the second payoff is for firm X.

		Firm X's price	
		£2.00	£1.80
Firm Y's price	£2.00	£10m, £10m	£5m, £12m
	£1.80	£12m, £5m	£8m, £8m

(i) Explain why each firm needs to consider the action of the other firm when setting its price. [2]

(ii) What is the Nash equilibrium in this game? Explain your answer. [6]

(c) What policies can government implement to increase long-run economic growth? [6]

SECTION B *Civil and Structural Engineering*

Answer not more than **two** *questions from this section.*
You may refer to the data sheet at the end of the examination paper as necessary.

3 You have been asked to advise on some of the key issues relating to a bus tunnel beneath the city centre of Cambridge. The ground conditions in the city centre are typically 5 m of fill, gravels, sands and silts overlying Gault Clay, with a water table close to the ground surface. The undrained shear strength of the Gault Clay is high, and can be taken to be 150 kN m^{-2}, constant with depth. The unit weight of all of the soils can be taken to be 20 kN m^{-3}.

You are to advise on the relative merits of two options: (a) a cut-and-cover tunnel, which would have to follow the route of existing roads, and (b) a bored tunnel. The cut-and-cover tunnel would be rectangular, 10 m wide and 6 m high, allowing buses to run in both directions. The bored tunnel alternative would be two separate 6 m diameter tunnels. The depth to the bottom of the tunnels would be variable, depending on which option was adopted, with a minimum of 10 m and a maximum of 20 m. The tunnel route would be adjacent to a number of historic buildings. Construction costs need not be considered.

(a) Discuss the advantages and disadvantages of cut-and-cover tunnel construction, how the ground might be supported during the excavation (give at least two options), and how the high water table would influence the chosen method of construction. [5]

(b) What methods of construction would be suitable for the bored tunnel option? Would the tunnels need to be constructed by open-face or closed face techniques? Justify your advice by considering the tunnel stability ratio. [4]

(c) Suggest two ways in which the bored tunnels could be lined, briefly describing the process. [2]

(d) Why does settlement potentially cause damage to buildings? Why might a tunnel directly beneath a masonry building be less damaging than one below but to one side of the building? [3]

(e) Suggest two methods of preventing damage to the buildings, giving brief descriptions of each method. [3]

(f) What role would instrumentation play in ensuring the tunnelling project is constructed safely and without problems? Give some examples. [3]

4 In order to construct a cut-and-cover tunnel, a 6 m deep excavation is required and a temporary sheet pile retaining wall is driven through dry sand to a depth of 9 m below ground level, as shown in Fig. 1. The toe of the wall just penetrates into the top of an impermeable clay layer. The wall is propped at a depth of 1 m below ground level. The dry sand has unit weight of 17 kN m^{-3} and its critical state angle of friction is $\varphi' = 35°$.

(a) Assuming plane strain conditions and the wall to be smooth, calculate the factor of safety against rotation about the prop if the active and passive pressures are fully mobilised. [7]

(b) Calculate how much the factor of safety reduces for each of the following unexpected events:

(i) Placing a 1.5 m high surcharge of sand with unit weight of 17 kN m^{-3} behind the wall. [3]

(ii) A flash flood, resulting in water quickly filling the sand behind the wall to ground level. Assume the unit weight of the saturated sand to be 20 kN m^{-3}. [5]

(iii) As for part (ii) but further flooding resulting in water also quickly filling the sand below the excavation up to the bottom of the excavation. [5]

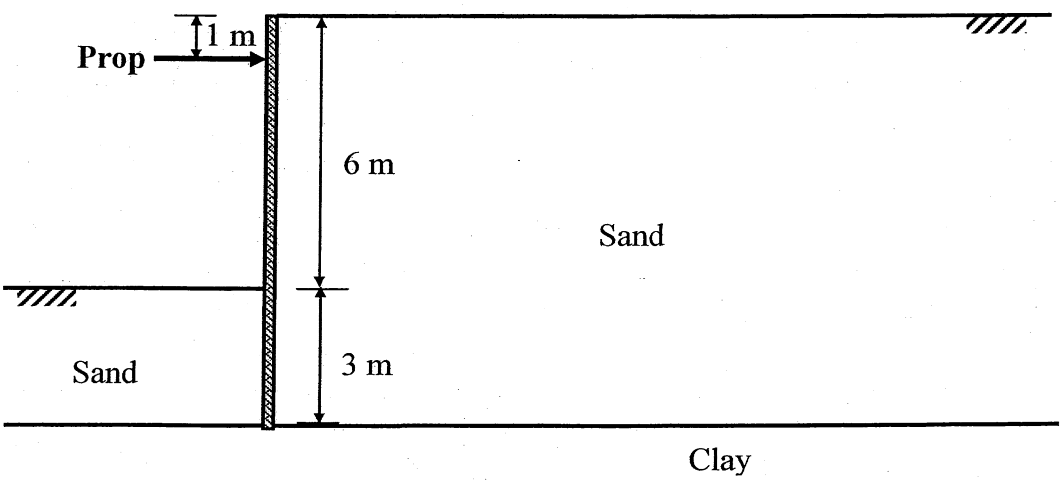


Fig. 1

5 A reinforced concrete beam, continuous over two 12 m spans as shown in Fig. 2, is to be designed to carry a uniformly distributed load of 240 kN m^{-1} (including its own self weight). It is to have an effective depth of 1.0 m and a breadth of 0.5 m.

(a) By using section 4.5.3 of the Structures data book, or otherwise, show that the end reactions are 1080 kN. [4]

(b) Sketch the bending moment diagram and calculate the maximum hogging and sagging bending moments. [5]

(c) Design suitable flexural reinforcement for the beam at the two critical locations. [4]

(d) Design suitable shear reinforcement for the beam. [4]

(e) Check whether the required reinforcement will fit into the beam. [3]

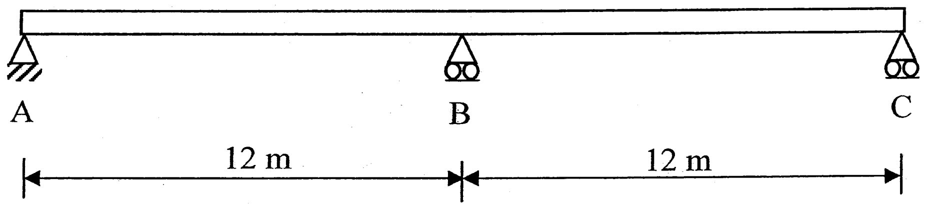


Fig. 2

SECTION C *Mechanics, Materials and Design*

Answer not more than **two** *questions from this section.*

6 (a) Discuss the potential role in the UK of renewable energy resources in the future, highlighting wind power generation. Discuss the particular challenges facing wind power generation. [10]

(b) A wind turbine of blade diameter 100 m is mechanically coupled to a 12-pole, 3-phase, star-connected cage rotor induction generator via a gearbox. The output from the generator is connected directly to the 3-phase, 50 Hz, 3.3 kV grid. At a wind speed of 6 m s^{-1} the turbine operates at a tip-speed ratio of 8 and produces mechanical power of 300 kW.

(i) Explain why this system is essentially a fixed speed system, and give two advantages and two disadvantages of this system, when compared to variable speed systems. [5]

(ii) Determine the angular speeds of the turbine and the induction generator, stating any assumptions. [3]

(iii) Find the gearbox ratio required, and the torque acting on the shaft of the generator, stating any assumptions. [2]

7 The power coefficient C_P for a wind turbine is defined as

$$C_P = \frac{P}{\frac{1}{2}\rho A V_0^3}$$

where P is the power extracted from a stream tube of air of density ρ, velocity V_0 and area A.

(a) Explain the physical meaning of the power coefficient and hence explain why the denominator in the expression for C_P takes the form given. [2]

(b) If the wind turbine has an axial induction factor $a = 1 - (u/V_0)$, where u is the axial velocity in the rotor plane, derive the following expression for the power coefficient:

$$C_P = 4a(1-a)^2$$

and show that this has a maximum value of $16/27$ corresponding to $a = 1/3$. [8]

(c) Two turbines are located along a common axis, as shown in Fig. 3. Because of the presence of the first turbine the air flow is decelerated at turbine 2, such that $V_0 > u_1 > V_1 > u_2$, where V_0 and V_1 are the flow velocities upstream of turbines 1 and 2, respectively, and u_1 and u_2 are the corresponding axial velocities in the rotor planes. It may be assumed that $u_1 = ½(V_0 + V_1)$.

(i) Derive an expression for the overall power coefficient $C_P = P/(\frac{1}{2}\rho A V_0^3)$, where P is the total power extracted, in terms of axial induction factors defined by $a_1 = 1 - u_1/V_0$ and $a_2 = 1 - u_2/V_1$. [6]

(ii) What is the optimum overall power coefficient and corresponding operating conditions for the two turbines? [4]

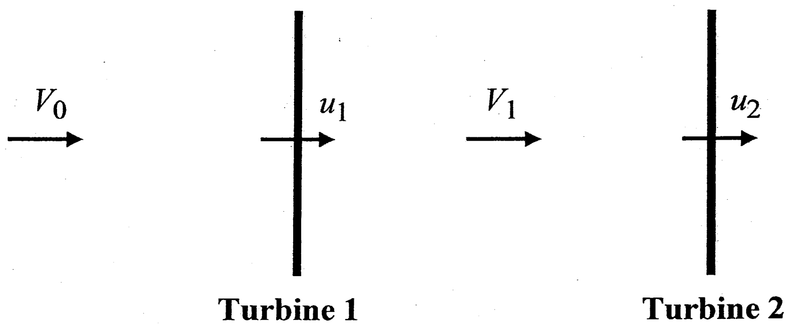


Fig. 3

8 (a) Discuss briefly how large wind turbine blades can be designed to ensure adequate stiffness while minimising weight. [3]

(b) Figure 4 is an idealisation of a wind turbine blade of length L, comprising two load-carrying flanges of constant thickness t and constant breadth b separated by a distance $2d$, taken to the mid-thickness of the flanges. The separation distance $2d$ varies linearly from zero at the tip to a maximum of $2d_0$ at the built-in root of the blade. It can be assumed in all the subsequent analysis that t is small compared with d. The flanges are made of material with Young's modulus E, density ρ and failure stress σ_f. There is a uniform transverse wind loading pressure p acting on the front face of the blade and the gravitational acceleration is g.

(i) In order to assess the effect of blade length, first assume that all the beam dimensions (b, d_0 and t) scale with the blade length L. Derive expressions for the tip deflections of the blade due to wind loading alone and due to self-weight loading alone. Hence show that the deflection associated with wind loading scales with L while the deflection due to self-weight loading scales with L^2. [9]

(ii) For a specified length L the flange thickness is now varied so as to avoid failure. Derive an expression for the mass of the blade required to avoid failure due to combined wind and self-weight loading (considering the blade in the horizontal orientation as shown in Fig. 4). [6]

(iii) Comment briefly on the importance of the results found in (b)(i) and (ii) for design of large wind turbine blades. [2]

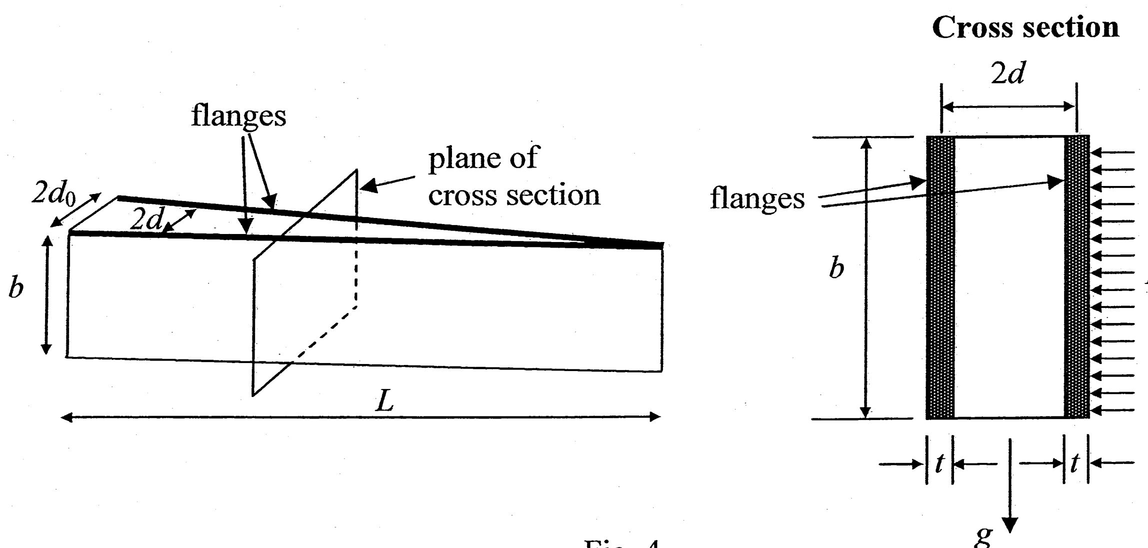


Fig. 4

SECTION D *Aerothermal Engineering*

Answer not more than **two** *questions from this section.*

9 A single shaft turbojet engine operates on an aircraft moving at a Mach number of 0.86 and at an altitude of 11000 m ($T = 216.6$ K, $p = 0.226$ bar).

(a) If the mass flow rate into the engine is 60 kg s^{-1} and the Mach number at the compressor inlet face is 0.6, find the face's area. [4]

(b) If the outer radius of the compressor's 1st blade row is constant at 0.6 m calculate the corresponding hub radius. [1]

(c) If the engine rotational speed is 6000 rpm calculate the mean blade speed for the 1st blade row. [1]

(d) The isentropic efficiency and pressure ratio of the whole compressor are 0.9 and 16, respectively. For a design stage loading factor, $\Delta h_0/U^2$, of 0.36 find the minimum number of compressor stages. [4]

(e) Will the pressure ratio across all these stages be equal? Justify your answer. [3]

(f) Find the torque needed to drive the compressor. [2]

(g) For the turbine $\Delta h_0/U^2 = 1.5$ and there are two stages. Find the mean radius of the turbine. [3]

(h) Comment on the disparity between the number of turbine and compressor stages. [2]

Take $\gamma = 1.4$, $R = 287.3$ J kg^{-1} K^{-1} and $g = 9.81$ m s^{-2}.

10 The table below gives some mathematical expressions for different efficiency parameters.

Efficiency characterisation	Mathematical expression(s)
Froude propulsive efficiency	$\eta_p = \dfrac{2V}{V + V_j}$
Thermal efficiency	$\eta_{th} = \dfrac{\dot{m}_{air}\left(V_j^2 - V^2\right)}{2\dot{m}_{fuel} LCV}$
Overall efficiency	$\eta_{ov} = \dfrac{V}{SFC\, LCV} = \dfrac{\dot{m}_{air} V\left(V_j - V\right)}{\dot{m}_{fuel} LCV}$

In the above, V is the flight velocity and V_j the propulsive jet velocity. Also, $\dot{m}_{air}$ is the air mass flow rate and $\dot{m}_{fuel}$ the fuel mass flow rate. The parameter, LCV, is the fuel's lower calorific value and SFC the specific fuel consumption.

(a) Stating all assumptions, derive the equation for η_p. Comment on: (i) why commercial aircraft use bypass engines, (ii) factors that limit the bypass ratio (BPR), and (iii) how noise generally varies with BPR. [7]

(b) For a turbofan engine designed to give 26 kN thrust, with $V = 260$ m s^{-1} and $\dot{m}_{air}$ = 260 kg s^{-1}, calculate η_p. Assume that the bypass jet velocity and the core jet velocity are equal to V_j. [2]

(c) A new, lower noise engine is proposed. For this, the fan diameter is increased by 15% relative to the engine described in part (b). If the flight speed, V, and the net thrust are unchanged from part (b), find η_p for the new design. Assume both the air mass flow rate per unit flow area and the hub-to-tip ratio of the fan are unchanged. Also, assume that the bypass jet velocity and core jet velocity are equal. [4]

(d) What does η_{th} in the table physically characterise and how can it be increased in engine design? Which is typically larger out of η_{th} and η_p? [3]

(e) Show that the relationship between gross thrust, F_G, and V_j for a static jet engine can be expressed as

$$\frac{F_G}{\dot{m}_{fuel}} = \frac{2\,\eta_{th}\, LCV}{V_j}$$ [2]

(f) Derive the equations for η_{ov} given in the table above. [2]

11 (a) The Breguet equation for aircraft range s is given below

$$s = -\frac{V_\infty (L/D)}{g\ SFC} \ln\left[\frac{W_2}{W_1}\right]$$

where V_∞ is the cruise speed, W_1 and W_2 the aircraft mass at the start and end of cruise, SFC the specific fuel consumption, g the acceleration due to gravity, and L and D are the lift and drag forces. An aircraft is to cruise at 10000 m (where the ambient temperature is 223 K) at $M_\infty = 0.85$ with $L/D = 20$ and $SFC = 1.61 \times 10^{-5}\,\text{kg s}^{-1}\text{N}^{-1}$. Find s. Note, the aircraft's empty weight $W_e = 326.2$ tonne, the payload, $W_p = 58.8$ tonne and the fuel weight $W_f = 250.6$ tonne is negligible at landing. [3]

(b) If s in the above were achieved with two flights of $s/2$ with a stopover, estimate the fuel saving. [2]

(c) State the practical advantages and disadvantages of having the stopover. [3]

(d) Show that F', the mass of fuel needed to move unit mass of payload by unit distance, can be expressed as:

$$F' = \frac{1}{s}\left(1 + \frac{W_e}{W_p}\right) \times \left(\exp\left\{\frac{s\ g\ SFC}{V_\infty L/D}\right\} - 1\right)$$ [5]

(e) Discuss the necessary practical logistic, engine and airframe design aspects that need to be addressed to minimise F'. [3]

(f) Assuming an idealized combustion reaction of the form

$$C_8H_{18} + 12.5\ O_2 \rightarrow 8CO_2 + 9\ H_2O$$

calculate the amount of CO_2 produced per passenger-km for the part (a) flight. Assume the typical passenger with baggage weight is 100 kg. The molecular weights are $CO_2 = 44$, C = 12 and H = 1. [4]

Take $\gamma = 1.4$, $R = 287.3\ \text{J kg}^{-1}\ \text{K}^{-1}$ and $g = 9.81\ \text{m s}^{-2}$.

SECTION E *Electrical Engineering*

*Answer not more than **two** questions from this section.*
You may refer to the data sheet at the end of the exam paper as necessary.

12 (a) Derive the relationship between mobility and electrical conductivity. [4]

(b) A typical amorphous silicon (a-Si) thin film transistor consists of a thin layer of a-Si with a gate insulator of silicon nitride (Fig. 5). The transistor dimensions are $W = 200$ μm, $L = 20$ μm and a-Si thickness is $t = 50$ nm. The mobility of a-Si is $5\times10^{-5}\,m^2V^{-1}s^{-1}$. Assuming that the gate induces a uniform carrier density, N, in the a-Si, what is the value of N if the saturated current is 1 μA for a source-drain voltage of 5 V. [10]

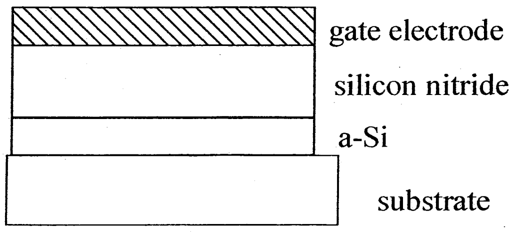


Fig. 5

(c) From part (b), calculate the gate operating voltage. The gate voltage V of a FET with a minimally thick gate dielectric is given by

$$V = \frac{Net^2}{2\varepsilon}$$

where e is the electronic charge, t is the a-Si thickness and ε is the permittivity. In the present case, the gate dielectric is 200 nm thick. Assume that the a-Si and silicon nitride both have a relative dielectric constant of 12. [6]

13 (a) Explain how conductivity is controlled in a semiconductor, as compared to a metal. [3]

(b) Compare the advantages and disadvantages of Si and GaAs for use as channel materials for metal - insulator field effect transistors. [4]

(c) Sketch the velocity vs electric field characteristic for an electron in Si and GaAs, and explain the various features. What causes the limiting velocity? [3]

(d) Derive the expression relating the carrier transit time t to the carrier mobility μ, channel length L, and source-drain field E_d. Under what conditions does the expression hold? [4]

(e) From (c), take the electron mobility of Si as $0.15\,m^2V^{-1}s^{-1}$ and the limiting velocity as $1.5\times10^5\,m\ s^{-1}$. Using Newton's laws or otherwise, relate the mobility to the electron mass and the mean free time between collisions. Hence, from the limiting velocity, calculate the energy lost per collision in eV. Assume m*= 1. [4]

(f) Calculate the value of the mean free path between collisions. [2]

14 (a) In order to manufacture n-channel MOST devices n^+ doped source and drain regions must be produced. Compare and contrast two methods of producing these doped regions, indicating which would be preferable for application in devices to operate at GHz frequencies. [8]

(b) Give two examples of dopants that could be used to produce the n^+ regions and one, other than boron, that can be used to dope the substrate p-type. [3]

(c) Using the data in Fig. 6 (a) and (b), determine whether or not a successful n-pocket isolation diffusion is obtained when boron is diffused through a 2 micron thick n-type silicon layer containing 6×10^{15} donor impurity atoms cm^{-3}. Assume that the diffusion takes place at 1100° C from a saturated boron vapour for 45 minutes. State all assumptions made. [9]

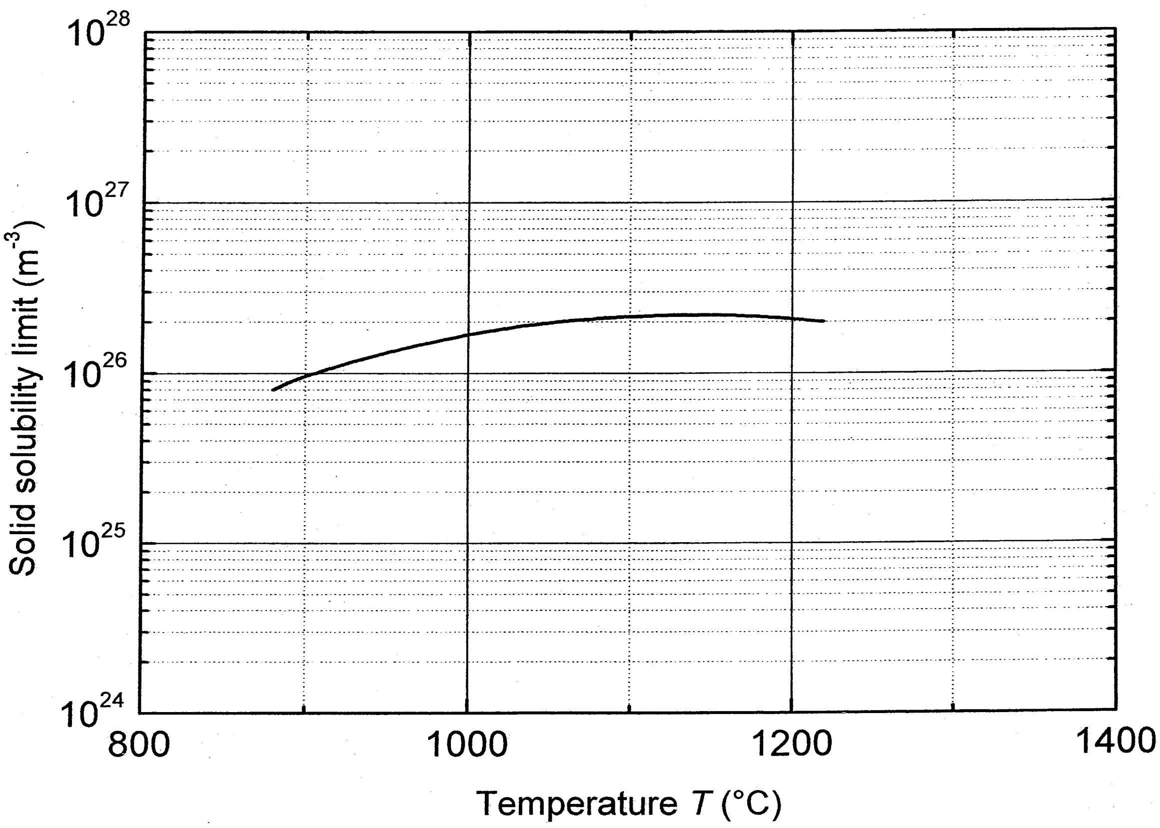


Fig. 6 (a) The solid solubility versus temperature for boron in silicon

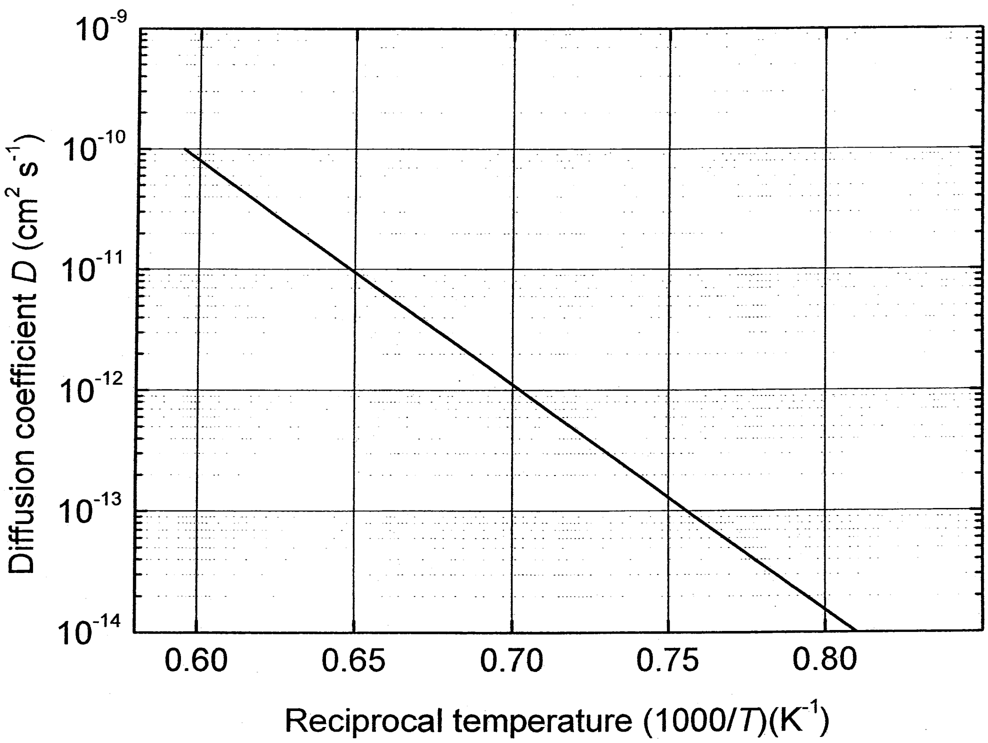


Fig. 6 (b) Diffusion coefficient for boron in Si vs reciprocal temperature

SECTION F *Information Engineering*

Answer not more than **two** *questions in this section.*

15 (a) Describe how the colour representations YUV and HSV each relate to the more usual RGB (red, green, blue) representation for each pixel in an image. [5]

(b) It is desired to brighten a pixel by a factor g (where $g > 1$), and to keep its apparent colour (hue and saturation) unaltered.

Give expressions for how the RGB, YUV and HSV components should be modified to achieve this for each of these three colour representations. [5]

(c) The histogram of intensities, $H(y)$, of the pixels in a particular image is found to show that the density of intensities is approximately uniform in the region $y = 0$ to 50, and that the density falls off linearly from this level, when y exceeds 50, towards zero density as y approaches 255.

Explain how the intensities of the pixels of this image should be modified in order to produce a better "balanced" image, and sketch the approximate curve of output intensity versus input intensity which would achieve this. [6]

(d) It can be advantageous to modify the above intensity correction scheme such that the intensity image, which forms its input, is a blurred version of the intensity of the original image. Explain what problem this can overcome and how it does it. [4]

16 A mobile phone application sends an image of a poster to a server to compare it with images of known posters in its database.

(a) Outline an algorithm for detecting and localising *interest points* in the image by *band-pass* filtering. Describe how the scale of the feature can be computed so that image feature size can be normalized. [6]

(b) How can an orientation for image feature be assigned from image gradients in the neighbourhood of the interest point? [4]

(c) Describe a suitable descriptor which can be used to match the image feature to features of known objects in the database. [4]

(d) Outline an extension of the matching scheme to describe the similarity of images of the same object category. [6]

17 Consider a data set of N images $S = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N\}$ where each image is represented as a vector of M real-valued features, e.g. $\boldsymbol{x}_n = (x_{n1}, \ldots, x_{nM})$.

(a) Assume a probabilistic model where each feature is independent and has a Gaussian distribution with distinct mean μ_m and variance 1. Write down the expression for the log likelihood as a function of the vector of parameters $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_M)$ and the data points $\{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N\}$. [5]

(b) Derive the maximum likelihood estimator for the vector $\boldsymbol{\mu}$ given the data set S. [5]

(c) Rederive this maximum likelihood estimator under the assumption that all elements of $\boldsymbol{\mu}$ are equal. [3]

(d) Describe how you would *learn* $\boldsymbol{\mu}$ using Bayesian inference, and some advantages and disadvantages of using Bayesian inference as compared to maximum likelihood. [7]

SECTION G *Engineering for Life Sciences*

*Answer not more than **two** questions in this section.*

18 (a) Write short notes on:

(i) temporal difference learning;

(ii) direct inverse learning;

(iii) loss functions in neuroscience. [5]

(b) Address the following in the context of learning.

(i) Describe briefly the differences between supervised and reinforcement learning.

(ii) Describe what a forward model is and how it can be learned and used. [5]

(c) Answer the following questions in the context of stiffness control of the human arm.

(i) Why does the human arm have innate stiffness?

(ii) How can the stiffness ellipse of the arm be measured?

(iii) Under what conditions, and why, is it sensible to use stiffness or to use inverse model control to resist force perturbations? [5]

(d) Answer the following questions in the context of noise.

(i) How can noise on the motor command determine how movements are generated?

(ii) How can noise on sensory inputs determine how stimuli are perceived? [5]

19 (a) Sketch the mechanical responses for the following, and describe the underlying microstructures and physical processes that give rise to the observed responses.

(i) Show on the same plot the stress-strain response for a collagenous soft tissue loaded in uniaxial tension both parallel and perpendicular to the primary collagen fibril orientation.

(ii) Show the evolution of a soft tissue response with time following the application of a step stress and a step strain. [8]

(b) Describe the following:

(i) The make-up of a natural tissue, with examples.

(ii) The overall approach of tissue engineering.

(iii) The role that bioreactors play in the tissue engineering process, with examples.

(iv) The mechanical and non-mechanical requirements for an osteochondral tissue engineered implant. [12]

20 Write brief notes on each of the following.

(a) The system of navigation used by bacteria such as *E. coli* in search of nutrients. Be sure to mention the main components of the system. [4]

(b) The way in which an organism's cells specialise, in order to construct different parts such as limbs, organs, even though every cell contains the full DNA prescription for the entire organism. [4]

(c) The significance of the principle of self-assembly in the process of building biological structures. Give a few specific examples. [4]

(d) The overall conformation of protein molecules as determined by the weaker rather than the stronger chemical forces. [4]

(e) The different lines of evidence which have been used by scientists to establish a 'tree of life'. Do the resulting trees differ in form? [4]

SECTION H *Manufacturing, Management & Design*

*Answer not more than **two** questions in this section.*

21 (a) List the four types of funding available to a new business and give one example of a possible source for each of the four types of funding. [4]

(b) Sketch a diagram that illustrates which source of funding is likely to be appropriate at each of the different stages of the commercialisation of a new idea. [5]

(c) Describe the characteristics of Venture Capital (VC) funds in terms of:

(i) who puts money into VC funds;

(ii) what type of firms VC funds invest in; and

(iii) how VC funds make their profits. [5]

(d) You have started a new business and you are seeking to raise investment from a VC fund. Describe the kind of information that a VC fund would want to see in your business plan. Identify which areas you think the VC fund would regard as most important for deciding whether to invest in your business. [6]

22 (a) List and discuss the advantages and disadvantages of business models which seek to make money from:

(i) making and selling a product, compared with selling a service;

(ii) licensing intellectual property, compared with making and selling a product. [8]

(b) You have decided to build a business around developing, making and selling a novel portable container for hot drinks. Describe, using diagrams where appropriate, the stages you should go through to convert an initial idea into a fully specified product ready for manufacturing. [12]

23 (a) Describe the conflicting objectives of customers and owners of a business that need to be considered when designing any production system. [4]

(b) Give three examples in each case for production systems that typically have:

(i) high levels of automation;

(ii) high levels of manual labour.

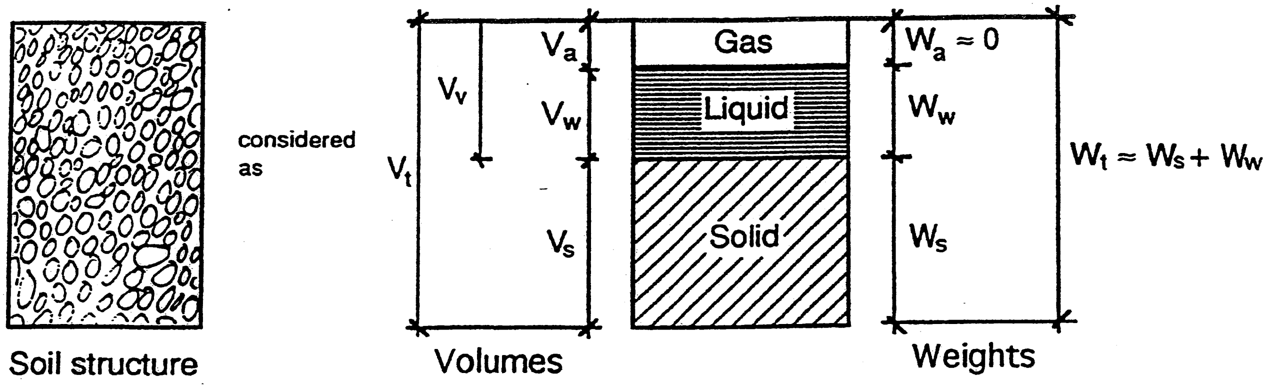
Describe the advantages and disadvantages of automated production systems in comparison with manual production systems. [8]

(c) Discuss possible strategies that could be used to help balance supply and demand in a production system for making bicycle lights. [8]

END OF PAPER

Data sheet – Soil Mechanics (6 pages)

General definitions



Specific gravity of solid G_s

Voids ratio $e = V_v / V_s$

Specific volume $v = V_t / V_s = 1 + e$

Water content $w = (W_w / W_s)$

Degree of saturation $S_r = V_w / V_v = (w G_s / e)$

Unit weight of water $\gamma_w = 9.81\ \text{kN/m}^3$ (although we assume $10\ \text{kN/m}^3$)

Unit weight of soil $\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e} \right) \gamma_w$

Buoyant (effective or submerged) unit weight

$$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w \quad \text{(soil saturated)}$$

Unit weight of dry soil $\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e} \right) \gamma_w$

Relative density $I_d = \frac{(e_{max} - e)}{(e_{max} - e_{min})}$

where e_{max} is the maximum voids ratio achievable in the quick tilt test (for sands), and
e_{min} is the minimum voids ratio achievable by vibratory compaction (for sands).

Classification of particle sizes

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D equivalent diameter of soil particle

D_{10}, D_{60} etc particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

Stress components

Principle of effective stress (saturated soil):

total stress	σ	=	effective stress σ' +	pore water pressure u
	τ	=	τ' +	0

and

σ_v = vertical stress

σ_h = horizontal stresss

τ = shear stress

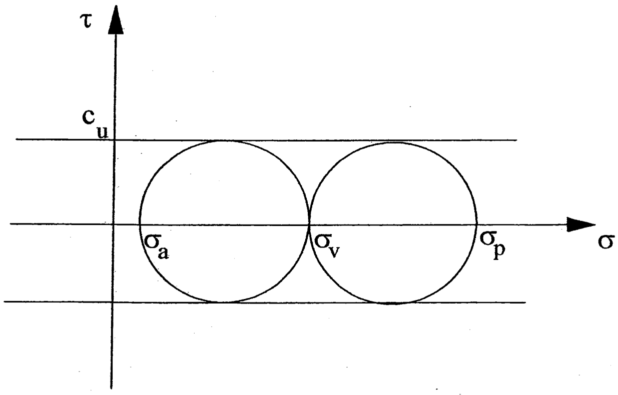
Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the *undrained shear strength* c_u which is mobilized when the shear stress $\tau = c_u$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, σ_a and σ_p respectively, are given by

$$\sigma_a = \sigma_v - 2\,c_u$$

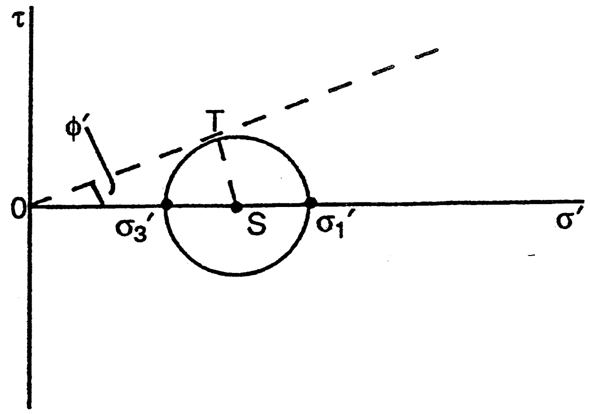
$$\sigma_p = \sigma_v + 2\,c_u$$

where σ_v is the total vertical stress.



Strength of sands

Mobilised angle of shearing ϕ'
where $\tau = \sigma' \tan \phi'$



$$\sin\phi' = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

$$\therefore \phi' = \sin^{-1}\left[\frac{\left(\sigma_1'/\sigma_3'\right) - 1}{\left(\sigma_1'/\sigma_3'\right) + 1}\right]$$

Earth pressure coefficient K: $\sigma_h' = K\sigma_v'$

Active pressure: $\sigma_v' > \sigma_h'$ $\therefore$ $\sigma_1' = \sigma_v'$

$\sigma_3' = \sigma_h'$

$K_a = (1 - \sin\phi'/1 + \sin\phi')$

Passive pressure: $\sigma_h' > \sigma_v'$ $\therefore$ $\sigma_1' = \sigma_h'$

[We assume principal stresses $\sigma_3' = \sigma_v'$

are horizontal and vertical] $K_p = (1 + \sin\phi')/(1 - \sin\phi') = \dfrac{1}{K_a}$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left(\sigma_1'/\sigma_3'\right)_{max}$

at critical state ϕ'_{crit} after large strains.

Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi' = \phi'_{max}$ and

$$\phi'_{max} = \phi'_{crit} + \phi'_{dilatancy}$$

where ϕ'_{crit} is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero $(\phi'_{dilatancy} -> 0)$ as $\phi'_{max} -> \phi'_{crit}$.

ϕ'_{crit} is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values (±2°):

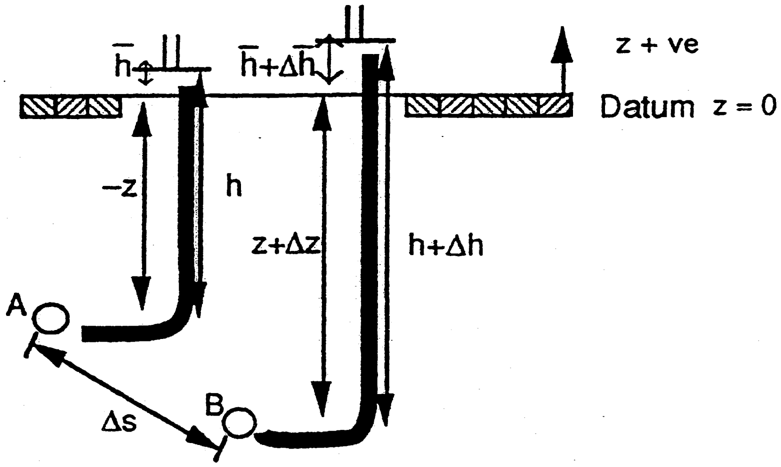
	ϕ'_{crit}	ϕ'_{max}	
feldspar	40°		
quartz	33°	53°	($I_d = 1$, and mean effective stress OS < 150 kPa)
mica	25°		

Seepage

Excess pore water pressure

Head $h = u/\gamma_w$

Potential $\bar{h} = h + z$



Total pore water pressure head at A: $u = \gamma_w h = \gamma_w (\bar{h} - (-z))$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta\bar{h} + \Delta z)$

[Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w (\bar{h} + \Delta\bar{h})$]

Hydraulic gradient A–B $$i = -\frac{\Delta\bar{h}}{\Delta s} = -\frac{\Delta\bar{u}}{\gamma_w \Delta s}$$

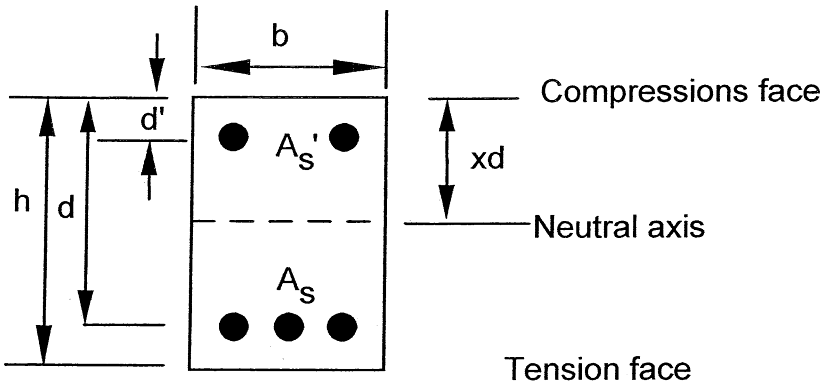
Darcy's law v = ki
v = average or superficial seepage velocity
k = coefficient of permeability

Typical permeabilities

$D_{10} > 10\text{mm}$	:	non-laminar flow
$10\text{ mm} > D_{10} > 1\mu\text{m}$	:	$k \cong 0.01(D_{10}\text{in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



Design Stresses

Cube strength for concrete f_{cu}. At failure in bending, stress in concrete $= 0.4f_{cu}$ over whole area of concrete in compression.

Tensile yield stress of steel f_y. At failure in bending, stress in bars in tension $=$ $0.87f_y$, stress in bars in compression $= 0.75f_y$.

Design Equations

Moment capacity of singly reinforced beam

$$M \leq 0.15\, f_{cu} bd^2$$
$$M = 0.87 f_y A_s d(1 - x/2)$$
$$x = 2.175(f_y / f_{cu}).(A_s / bd) \quad (\leq 0.5 \text{ to avoid over reinforcement})$$

Moment capacity of doubly reinforced beam

$$M = 0.15 f_{cu} bd^2 + 0.75 f_y A'_s (d - d')$$
$$0.87 f_y A_s = 0.75 f_y A'_s + 0.2 f_{cu} bd$$

Shear capacity of all beams

Total shear capacity $V = (v_c + v_s)bd$

Where, $v_c = 0.68(100\,A_s/bd)^{0.33}.(400/d)^{0.25}$ $\quad (N/mm^2)$

and $v_s = 0.87 f_y\, A_{sq}/(bs)$

in which s = shear link spacing, A_{sq} is total area of all shear bars in a link and A_s is the total area of effective longitudinal *tension* steel at the section.

Standard bar sizes

Diameter (mm) 6 8 10 12 16 20 25 32 40
50
Area (mm^2) 28 50 78 113 201 314 491 804 1256
1963

Available steel types

Deformed high yield steel $f_y = 460 \quad N/mm^2$

Plain mild steel $f_y = 250 \quad N/mm^2$

Lap and anchorage lengths 40 bar diameters

Density of reinforced concrete: $24\ kN/m^3$

Reinforcement areas per metre width

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540
Areas calculated to 3 significant figures according to B.S.I recommendations										

Part IB Data Sheet: Electrical Engineering (2 pages)

Transistor Design

Gauss's Theorem

$\varepsilon_o\varepsilon_r E_1 - \varepsilon_o\varepsilon_r E_2$ = charge per unit area enclosed between upper surface 1 and lower surface 2.

FET Design Summary

- $\tau_t \rightarrow$ switching time as 1st approx. (scattering limited transit time).
- $\tau_{eff} = \tau_t + R_{load}C_{eff(output)} \rightarrow$ switching time as 2nd approx.
- $L = v_s \tau_t$ (source-drain spacing).
- $I_{sat} = e N v_s W d_s = e N W L d_s/\tau_t$
- Aspect ratio W/L (technology?).
- $(1/2)eN(d_s)^2/\varepsilon_o\varepsilon_r$ = (Max Gate Voltage)
- $E'_{peak} = eN d_s/\varepsilon_o\varepsilon_r < E_{breakdown}$
- Minimum Drain Source Voltage $\sim E_s L$ (E_s is the field required to reach limiting velocities).

Mutual Conductance

$g_{mo} \sim I_{sat}/V_{gate(max)}$

Mutual conductance reduces with frequency as $g_m(\omega) \approx g_{mo}/(1 + j\omega\tau_t)$;

$$v_{out} = g_m(\omega)\, R(1 + j\omega RC_{eff(out)}) \approx g_{mo}R/[1 + j\omega(\tau_t + RC_{eff(out)})]$$
$$= g_{mo}R/[1 + j\omega\tau_{eff}]$$

Capacitances for FET

Parallel Plate Capacitance: $\varepsilon_o\varepsilon_r$ Area/spacing

Used for rough estimates of parasitic capacitance.

Effective Capacitances for FET

$C_{eff(out)} \rightarrow C_{gate/drain} + C_{drain/source} + C_{load}$;

$C_{eff(in)} \rightarrow MC_{gate/drain} + C_{gate/source\,(proximity)} + C_{gate/source\,(electronic)}$;

$C_{electronic} = g_{mo}\tau_t$; $M = (1 + g_{mo}R_{load})$.

Time Constants for FET

$\mu = e\tau/m^*$ relates mean free time τ and mobility.

Transit time τ_t over distance L and scattering limited velocity v_s are related by $\tau_t = L/v_s$.

$v_{out} \approx g_{mo}R/[1 + j\omega\tau_{eff}] = g_{mo}R/[1 + j\omega/(2\pi f_t)]$

$1/(2\pi f_\tau) = \tau_t + RC_{eff(out)} = \tau_{eff}$ The transition frequency is f_t.

10% to 90% rise time is $T = 2.2\,\tau_{eff} = (2.2/2\pi)(1/f_t) = 0.35/f_t$.

Tunnel Barrier Design Summary Sheet

Schrodinger's Equation

Complex Wave $\Psi = A\ \exp(-j2\pi f t + j2\pi x/\lambda) = A\exp(-j\omega t)\exp(jkx)$;
<momentum> $\Psi = p\Psi = (h/\lambda)\Psi = -j(h/2\pi)\partial\Psi/\partial x$;

<Total energy> $\Psi = E\Psi = hf\Psi = j(h/2\pi)\partial\Psi/\partial t$
$(h/2\pi) \to \hbar$; $h = 6.625\times10^{-34}$ J/s.

Schrodinger's equation:-

$$E\Psi = (1/2m)[-j\hbar\partial/\partial x]^2\Psi + e\phi\Psi$$

Tunneling (Rectangular barrier $\varepsilon\phi$)

Propogating waves outside barrier with incident kinetic energy $U_{incident} = (\hbar k_i)^2/2m^*$

Evanescent waves inside barrier: $-(\hbar k)^2/2m^* = (\hbar k_i)^2/2m^* - e\phi$

Technology Design Summary

Diffusion

Constant Surface Concentration:

$$C(x,t) = C_s\ \mathrm{erfc}\left[\frac{x}{2(Dt)^{1/2}}\right]$$

Constant Total Dopant:

$$C(x,t) = \frac{S}{(\pi Dt)^{1/2}}\exp\left[\frac{-x^2}{4Dt}\right]$$

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13.3.06