DR. M. OVEREND

ENGINEERING TRIPOS IB

2011

PAPER 2 - STRUCTURES

SOLUTIONS.

VERSION : mot

1) a) $s - m = b + r - Dj$

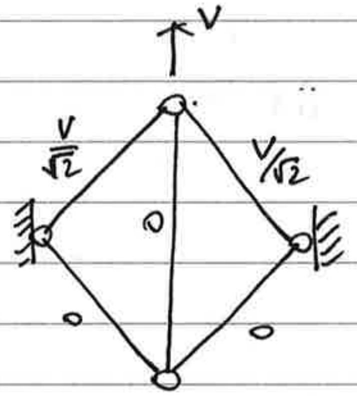
$s - 0 = 5 + (2\times2) - (2\times4)$

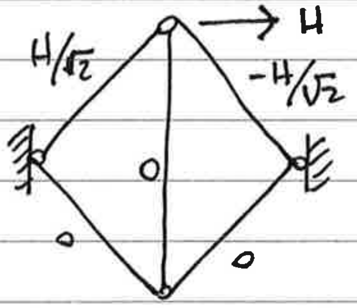
$\therefore \underline{\underline{s = 1}}$

ALTERNATIVELY REMOVING ONE MEMBER SAY MEMBER I OR MEMBER II WILL MAKE STRUCTURE STATICALLY DETERMINATE

$\therefore s = 1$

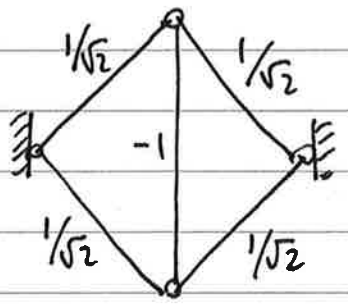
b) i) PARTICULAR SOLUTION WITH $t_{II} = 0$





$$\underset{\sim}{t_0} = \frac{V}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\\0\end{bmatrix} + \frac{H}{\sqrt{2}}\begin{bmatrix}1\\0\\-1\\0\\0\end{bmatrix}$$

STATE OF SELF STRESS $(t_I = -1)$



$$\underset{\sim}{s} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-\sqrt{2}\\1\\1\\1\end{bmatrix}$$

ELASTIC SOLUTION $\underset{\sim}{t} = \underset{\sim}{t_0} + x\underset{\sim}{s}$

FLEXIBILITY MATRIX $\underset{\sim}{F} = \frac{\sqrt{2}L}{AE}\begin{bmatrix} 1 & & & & \\ & \sqrt{2} & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$

BAR EXTENSIONS $\underset{\sim}{e} = \underset{\sim\sim}{Ft} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\frac{VL}{AE} + \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}\frac{HL}{AE} + \begin{bmatrix} 1 \\ -2 \\ 1 \\ 1 \\ 1 \end{bmatrix}\frac{xL}{AE}$

COMPATIBILITY FOR $\underset{\sim}{e} \Rightarrow \underset{\sim}{e}.\underset{\sim}{s} = 0$

$\therefore \frac{1}{\sqrt{2}}.\frac{L}{AE}\left[2V + (4+2\sqrt{2})x\right] = 0$

$x = \frac{-1}{2+\sqrt{2}} = -0.29V$

$\therefore \underset{\sim}{t} = \frac{V}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \frac{H}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} - \frac{0.29V}{\sqrt{2}}\begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$\underset{\sim}{t} = \frac{V}{\sqrt{2}}\begin{bmatrix} 0.71 \\ 0.41 \\ 0.71 \\ -0.29 \\ -0.29 \end{bmatrix} + \frac{H}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

$t_{I} = 0.5V + 0.71H$
$t_{II} = 0.29V$
$t_{III} = 0.5V - 0.71H$
$t_{IV} = -0.21V$
$t_{V} = -0.21V$

b) ii) HORIZONTAL DISPLACEMENT (δ_H) $H=1$; $V=0$

$$\therefore \underset{\sim}{t}_H = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \frac{1}{\sqrt{2}}$$

$$\therefore \delta_H = \underset{\sim}{e} \cdot \underset{\sim}{t}_H = \frac{HL}{AE} \cdot \frac{1}{\sqrt{2}} \cdot 2$$

$$\delta_H = \frac{\sqrt{2}HL}{AE}$$

VERTICAL DISPLACEMENT (δ_V) $V=1$; $H=0$

$$\therefore t_V = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \frac{1}{\sqrt{2}}$$

$$\therefore \delta_V = \underset{\sim}{e} \cdot \underset{\sim}{t}_V = \frac{VL}{AE} \cdot \frac{1}{\sqrt{2}} \cdot 2 + \frac{xL}{AE} \cdot \frac{1}{\sqrt{2}} \cdot 2$$

SUB $x = -0.29V$

$$\therefore \delta_V = \frac{VL}{AE}$$

1B/2011/2/2

↻ ABOUT B FOR BC :

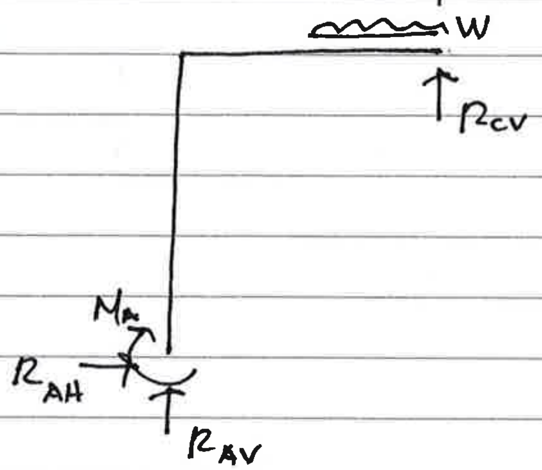
$$-M_B - R_{CV} L + W . \frac{3L}{4} = 0$$

∴ SUB FOR M_B :

$$R_{CV} L = \frac{3WL}{4} - \frac{7WL}{256}$$

$$\therefore R_{CV} = \underline{\underline{\frac{185W}{256}}} \quad (= 0.72W)$$

CONSIDER FREE BODY DIAGRAM



VERTICAL EQUILIBRIUM :

$$R_{AV} = W - \frac{185W}{256}$$

$$R_{AV} = \underline{\underline{\frac{71W}{256}}}$$

ALTERNATIVELY:
BY INSPECTION $M_A = M_B$
$$= \underline{\underline{\frac{7WL}{256}}}$$

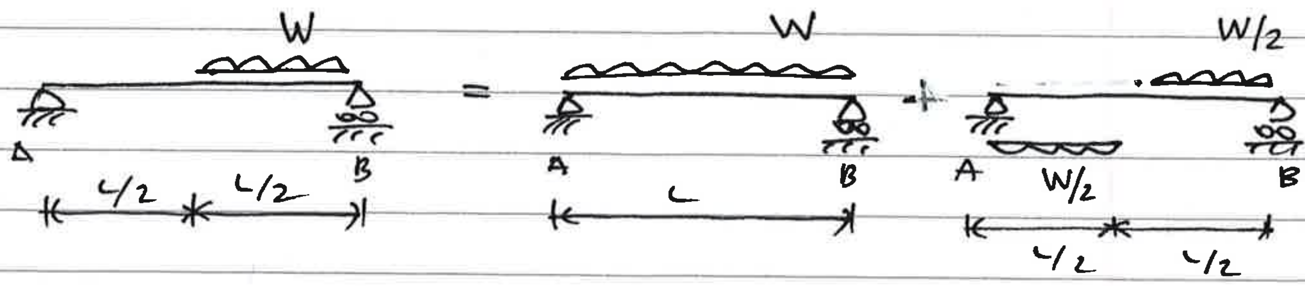
↻ ABOUT A:

$$M_A = W . \frac{3L}{4} - R_{CV} . L$$

$$= \left(\frac{192 - 185}{256}\right) WL$$

$$= \underline{\underline{\frac{7 WL}{256}}}$$

2a)



$$\theta_A = \theta_{A1} + \theta_{A2}$$

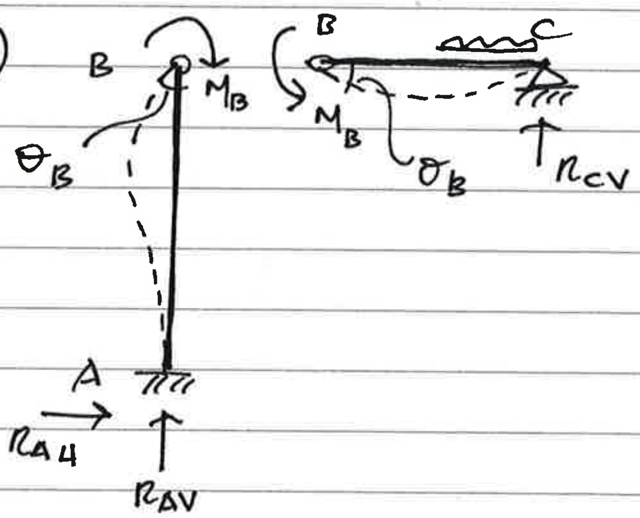
$$= WL^2/24EI + \left(-\frac{W}{2}\cdot\left(\frac{L}{2}\right)^2/24EI\right)$$

$$= \frac{WL^2}{EI}\left(\frac{1}{24} - \frac{1}{192}\right)$$

$$= \frac{7WL^2}{192EI}$$

b) i) INTRODUCING PIN AT B OR REMOVING ROLLER SUPPORT AT C MAKES STRUCTURE DETERMINATE ∴ ONE REDUNDANCY.

b) ii)



INTRODUCING PIN AT B

$$AB: \quad \theta_B = M_BL/EI$$

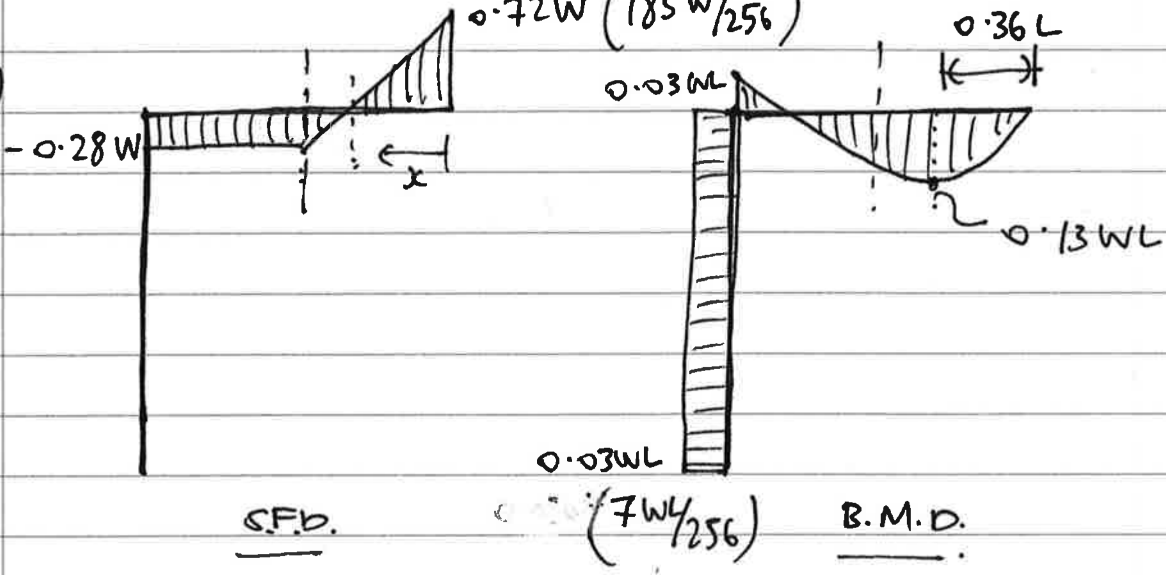
$$BC: \quad \theta_B = -\frac{M_BL}{3EI} + \frac{7WL^2}{192EI}$$

COMPATIBILITY OF ROTATIONS AT B:

$$M_BL/EI = -M_BL/3EI + 7WL^2/192EI$$

$$\therefore \quad M_B = \frac{21WL}{768} = \frac{7WL}{256}$$

biii)



S.F.D.   B.M.D.

S.F. = 0

$$\frac{185}{256} W = x \cdot \frac{W}{(L/2)}$$

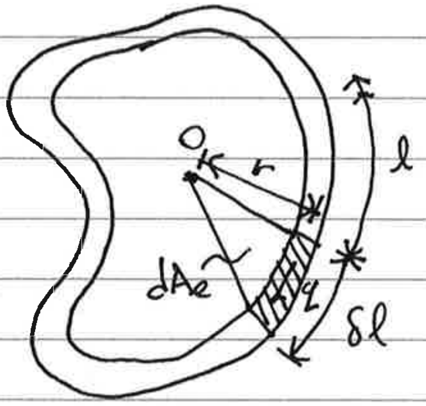
$\therefore \ x = 0{\cdot}36L$

MAX. B.M. @ SF = 0

$$= \frac{0{\cdot}72W \times 0{\cdot}36L}{2}$$

$$= 0{\cdot}13WL$$

3 a) CONSIDER AN ARBITRARY SECTION WITH ORIGIN O:



FORCE ON ELEMENT = $q\,\delta l$

MOMENTS ABOUT O:

$$dT = q\,r\,dl$$

BUT $r\,dl = 2\,dA_e$

$$\therefore \quad dT = 2q\,dA_e$$

INTEGRATE AROUND SECTION:

$$T = \oint dT = \oint 2q\,dA_e$$

q is CONSTANT ALONG ALL l

$$\therefore \quad T = 2qA_e$$

$$q = T/2A_e$$

b) LONGITUDINAL STRESSES DUE TO BENDING: $\sigma = My/I$

AT A $\quad y = 247.5$ mm

$I = \pi r^3 t =$

$M = 20 \times 10^3 \times 3000$ Nmm

$$\therefore \quad \sigma = \frac{20 \times 10^3 \times 3000 \times 247.5}{(247.5)^3 \times \pi \times 5} = 62.4\ \text{N/mm}^2$$

AT B $\quad y = 0 \quad \therefore \quad \sigma = 0$

LONGITUDINAL STRESSES DUE TO PRESSURE $= pr/2t$

AT A = AT B $= \dfrac{1.5\times10^6 \times 0.5}{2\times0.005}$

$= 75\times10^6\ N/m^2$

$= 75\ N/mm^2$

$\therefore$ TOTAL LONGITUDINAL STRESSES : AT A $= 137.4\ N/mm^2$

AT B $= 75\ N/mm^2$

HOOP STRESSES (DUE TO PRESSURE) $= pr/t = 150\ N/mm^2$

$\therefore$ TOTAL HOOP STRESSES : AT A $= 150\ N/mm^2$

AT B $= 150\ N/mm^2$

b ii) SHEAR STRESS DUE TO SHEAR FORCE $\tau_s = \dfrac{SA\bar{y}}{I.2t}$

$A\bar{y} = 2tr^2$ (MECH. DATA BOOK or $\int_0^{\pi} tr^2 \sin\theta\, d\theta$)

$\therefore \tau_s = \dfrac{20\times10^3 \times \cancel{2}t\cancel{r^2}}{\pi r^{\cancel{3}} t} \times \dfrac{1}{\cancel{2}t}$

$= 5.1\ N/mm^2$ AT B ($= 0$ AT A).

SHEAR FORCE DUE TO TORQUE $\tau_T = q/t = \dfrac{T}{2A_e t}$

(τ_T AT A $= \tau_T$ AT B)

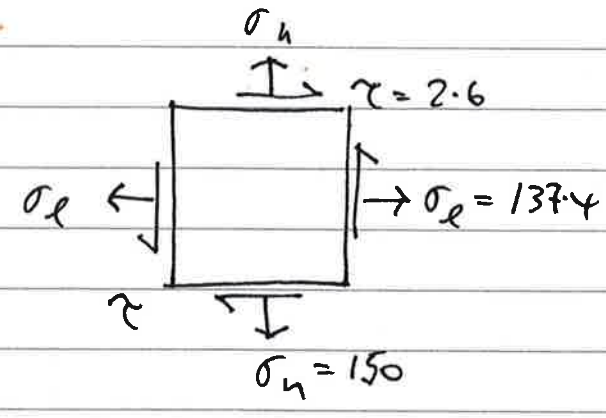
$\therefore \tau_T = \dfrac{20\times10^3 \times 250}{2\times\pi\times247.5^2\times5}$

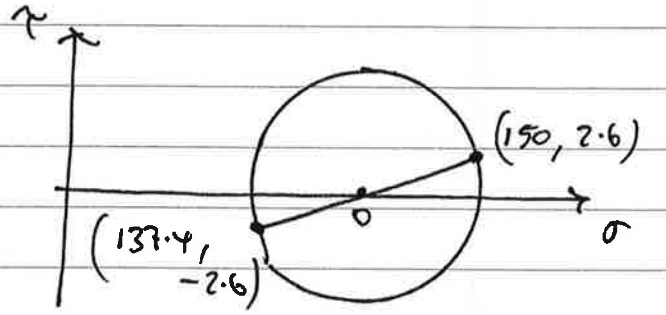
$= 2.6\ N/mm^2$

$\therefore$ TOTAL SHEAR STRESSES AT A $= 2.6\ N/mm^2$

AT B $= 7.7\ N/mm^2$

b iii) AT A:





$$\sigma_o = (150 + 137.4)/2 = 143.7\ \mathrm{N/mm^2}$$

$$\therefore \quad \sigma_1 = \left(\sqrt{\left(\tfrac{150}{2} - \tfrac{137.4}{2}\right)^2 + 2.6^2}\right) + 143.7$$

$$= 150.5\ \mathrm{N/mm^2}$$

$$\sigma_2 = 136.9\ \mathrm{N/mm^2}$$

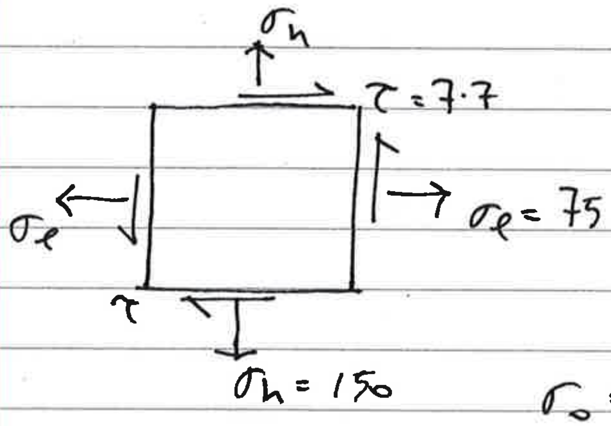
Von Mises : $(\sigma_1 - \sigma_2)^2 + \sigma_1^2 + \sigma_2^2 = 2Y^2$

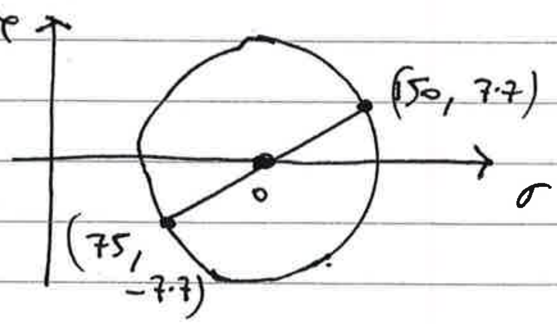
$$\therefore \quad Y = 144.2\ \mathrm{N/mm^2}$$

$$\therefore \quad \lambda = 275/144.2$$

$$\underline{\underline{\lambda = 1.91}}$$

AT B:





$$\sigma_o = (150 + 75)/2 = 112.5\ \mathrm{N/mm^2}$$

$$\therefore \quad \sigma_1 = \left(\sqrt{\left(\tfrac{150}{2} - \tfrac{75}{2}\right)^2 + 7.7^2}\right) + 112.5$$

$$= 150.8\ \mathrm{N/mm^2}$$

$$\sigma_2 = 74.2\ \mathrm{N/mm^2}$$

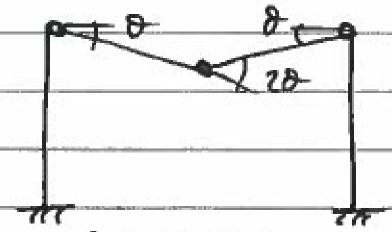
From Von Mises $\quad Y = 130.6$

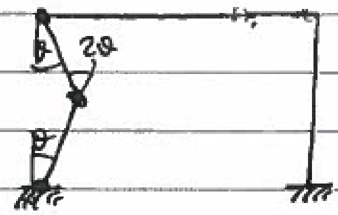
$$\therefore \quad \underline{\underline{\lambda = 2.1}}$$

1B/2011/4/1

Crib corrected Apr. '16
S.P.T.

4 a)





(1) BEAM:

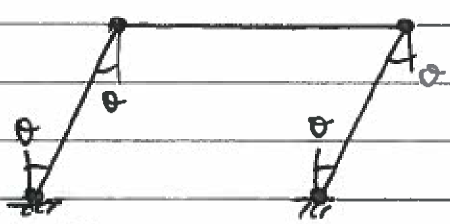
$VL\theta = 4M_p\theta$

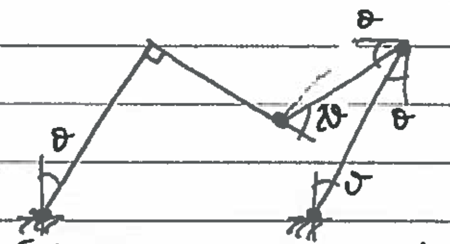
$\therefore V = 4M_p/L$

(2) COLUMN

$\cancel{2} H L\theta/4 = 4M_p\theta$

$\therefore H = \cancel{8M_p/L} = \frac{16M_p}{L}$





(3) SWAY

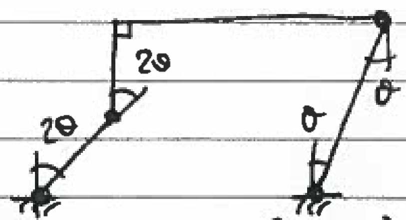
$H . \frac{L\theta}{2} = 4M_p\theta$

$\therefore H = 8M_p/L$

(4) COMBINED (B+S)

$H . \frac{L\theta}{2} + VL\theta = 6M_p\theta$

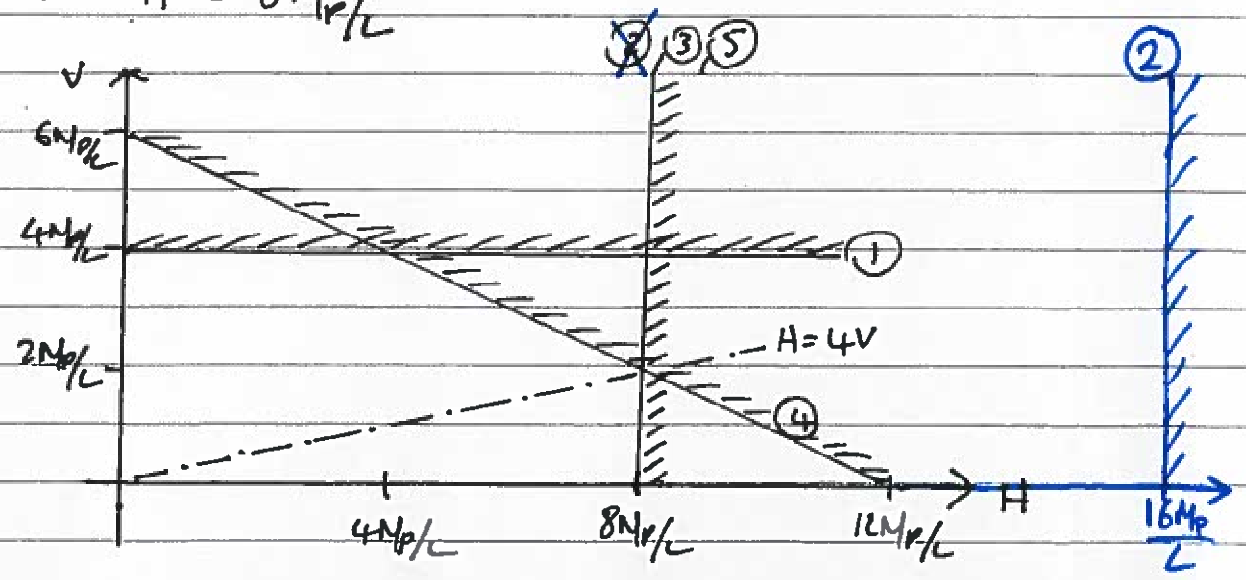
$\therefore H + 2V = 12M_p/L$



(5) COMBINED (C+S)

$H . L\theta . \frac{3}{4} = 6M_p\theta$

$\therefore H = 8M_p/L$



b) WHEN $H = 4V$ → ②, ③, ④ AND ⑤ ALL CRITICAL

$\therefore$ $H = 8M_p/L$

$V = 2M_p/L$

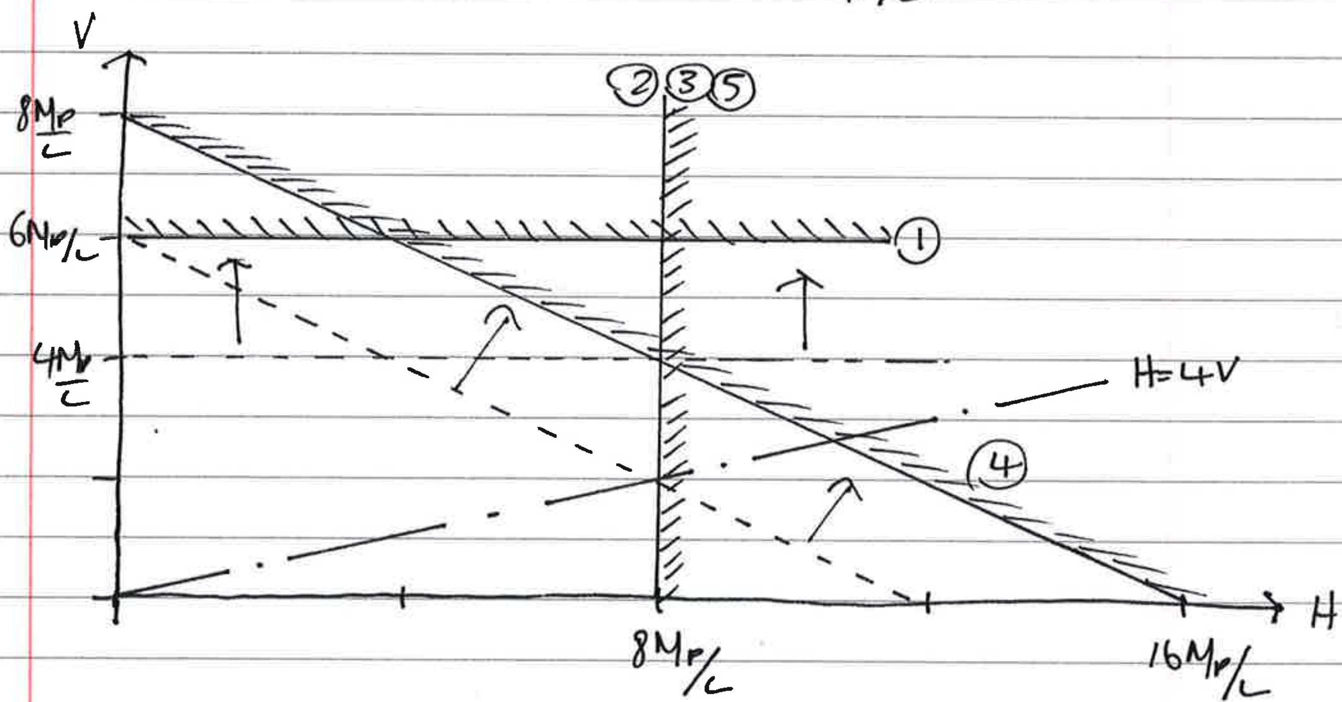
(c) MODIFICATION AFFECTS CENTRAL HINGE IN BEAM ∴ CHANGES TO BEAM MECH ① AND COMBINED MECH. ④.

(1) BEAM : $VL\theta = 6M_p\theta$

$\therefore V = 6M_p/L$

(4) COMBINED : $HL\theta/2 + VL\theta = 8M_p\theta$

$\therefore H + 2V = 16M_p/L$



COLLAPSE LOAD FOR MECH. ① AND MECH ④ INCREASES, BUT ②, ③, ⑤ ARE UNCHANGED

∴ V IS UNCHANGED $= 2M_p/L$

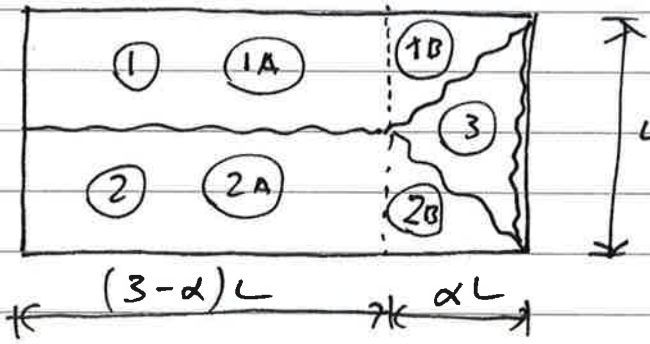
5a) 1. YIELD LINES DIVIDE SLAB INTO RIGID PLANAR REGIONS WHERE EACH REGION ROTATES ABOUT ITS UNIQUE AXIS.

2. YIELD LINES ARE STRAIGHT

3. A YIELD LINE BETWEEN TWO REGIONS PASSES THROUGH THE INTERSECTION OF THE AXES OF ROTATION OF THE TWO REGIONS.

b) SEE ATTACHED.

c) i)



YIELD LINE 1-2 DISPLACES VERTICALLY BY δ.

EXTERNAL WORK.

LOAD ON 1A $= (3-\alpha)\frac{L^2 w}{2}$; MEAN DISPL. $= \delta/2$

" " 1B $= \frac{\alpha L^2}{4} w$; " " $= \delta/3$

" " 3 $= \frac{\alpha L^2}{2} w$; " " $= \delta/3$

$$\therefore W.D. = 2\left[(3-\alpha)\frac{L^2 w}{2}\cdot\frac{\delta}{2} + \frac{\alpha L^2 w}{4}\cdot\frac{\delta}{3}\right] + \frac{\alpha L^2 w}{2}\cdot\frac{\delta}{3}$$

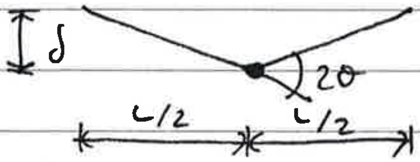
$$= \frac{3}{2}L^2 w\delta - \frac{\alpha L^2 w\delta}{2} + \frac{\alpha L^2 w\delta}{6} + \frac{\alpha L^2 w\delta}{6}$$

$$= \frac{3}{2}L^2 w\delta - \frac{\alpha L^2 w\delta}{6}$$

$$= \frac{L^2 w\delta}{6}(9-\alpha)$$

INTERNAL ENERGY.

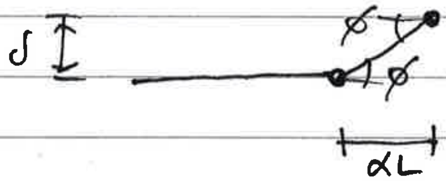
TRANSVERSELY: $= 3L \cdot 2m\theta$



BUT $\frac{\theta L}{2} = \delta \quad \therefore \quad \theta = \frac{2\delta}{L}$

$\therefore$ INTERNAL ENERGY $= 12m\delta$

LONGITUDINALLY $= L \cdot m\, 2\phi$



BUT $\phi\, \alpha L = \delta$

$\therefore \quad \phi = \delta/\alpha L$

$\therefore$ INTERNAL ENERGY $= 2m\delta/\alpha$

TOTAL INTERNAL ENERGY $= 12m\delta + 2m\delta/\alpha = 2m\delta\left(6 + \frac{1}{\alpha}\right)$

$\therefore$ WORK EQN: $2m\cancel{\delta}\left(6 + \frac{1}{\alpha}\right) = \frac{L^2 w\cancel{\delta}}{6}(9 - \alpha)$

$$\therefore \quad w = \frac{12m}{L^2}\left(\frac{6 + 1/\alpha}{9 - \alpha}\right)$$

c ii)

α	$wL^2/12m$
0·5	0·94
1·0	0·875
1·5	0·889
1·1	0·875.
1·05	0·875.

$\therefore$ CHOOSE $\alpha = 0{\cdot}875 \Rightarrow \quad w = \frac{10{\cdot}5m}{L^2}$

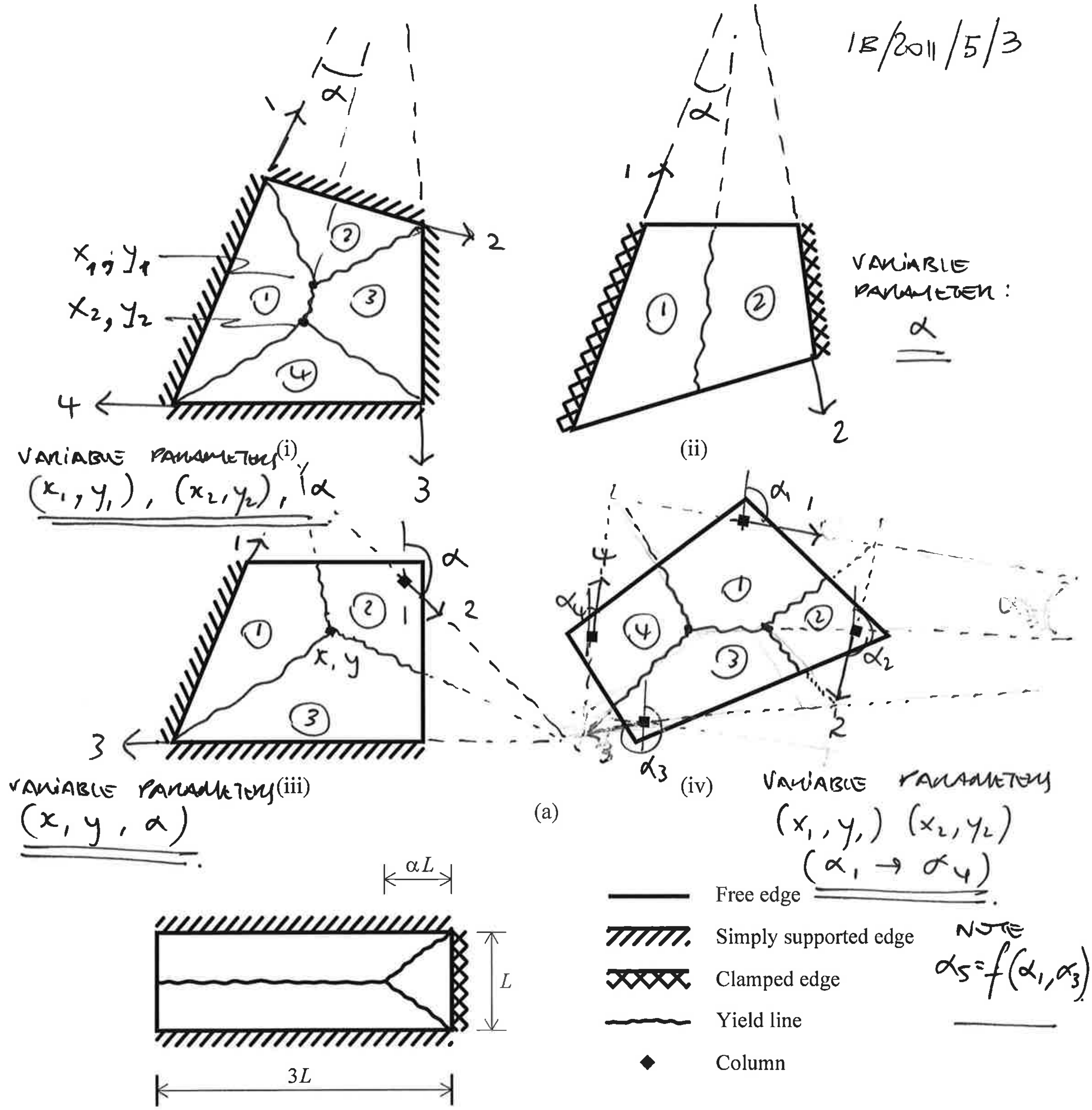
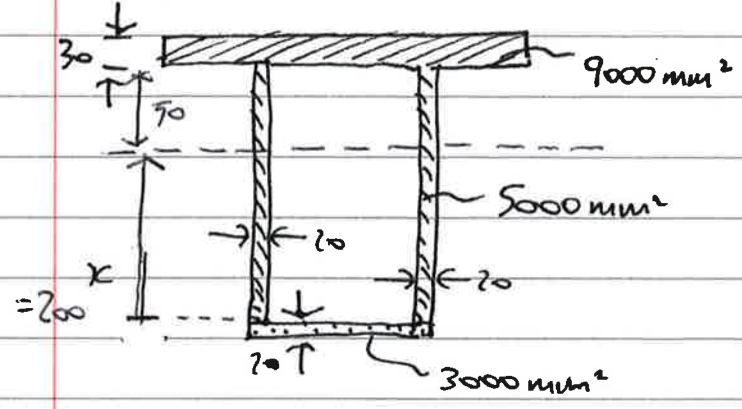

1B/2011/5/3
(i)
VARIABLE PARAMETERS
(x1, y1), (x2, y2), α
(ii)
VARIABLE PARAMETER: α
(iii)
VARIABLE PARAMETERS
(x, y, α)
(iv)
VARIABLE PARAMETERS
(x1, y1) (x2, y2)
(α1 → α4)
(a)
αL
L
3L
(b)
Free edge
Simply supported edge
Clamped edge
Yield line
Column
NOTE
α5 = f(α1, α3)

Fig. 6

6 a) FOR PLASTIC MOMENT CAPACITY, NEUTRAL AXIS IS EQUAL AREA AXIS.



$$\frac{A}{2} = \frac{1}{2}\left(9000 + 2(5000) + 3000\right)$$

$$= 11,000 \text{ mm}^2$$

ASSUME x IS IN WEBS:

$$3000 + 2(20x) = 11,000$$

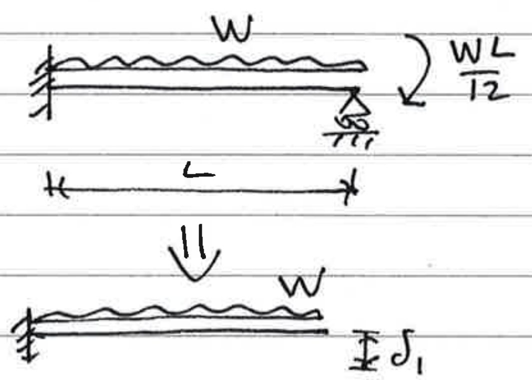
$$\therefore \quad x = 200\text{mm}.$$

$$M_p = \sum \sigma_y |A\bar{y}|$$

$$= 275\left[(3000 \times 210) + 2(200 \times 20 \times 100) + 2(50 \times 20 \times 25) + (9000 \times 85)\right]$$

$$= \underline{\underline{568 \text{ kNm}}}$$

b)

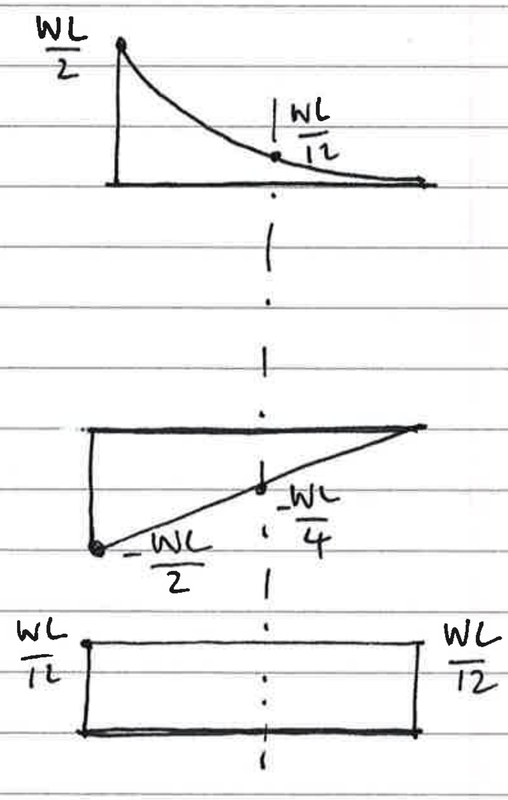


+

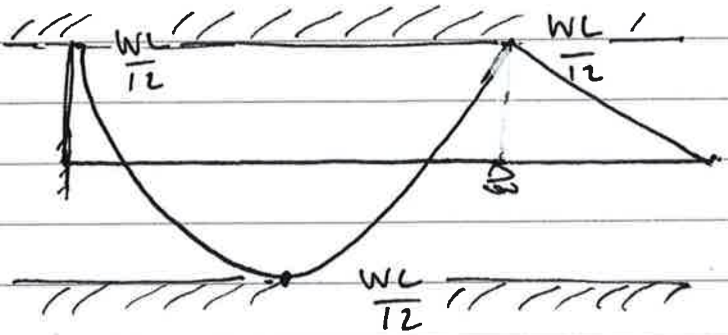
$$\frac{PL^3}{3EI} = \underbrace{\frac{WL^3}{8EI}}_{\delta_1} + \underbrace{\frac{WL}{12} \cdot \frac{L^2}{2EI}}_{\delta_2} \quad \therefore \quad P = W/2$$

+

LOWER BOUND.



OVERALL B.M.D :



$\therefore$ FOR OPTIMUM USE $\frac{WL}{12} + \frac{WL}{12} = 2M_p$

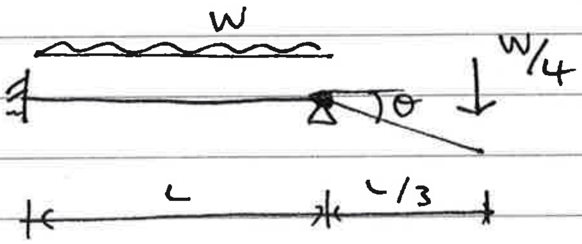
$W = 12M_p/L$

$= 12 \times 568/6$

$\underline{\underline{W = 1136 \text{ kN}}}$

(c) UPPER BOUND.

CANTILEVER MECHANISM.:

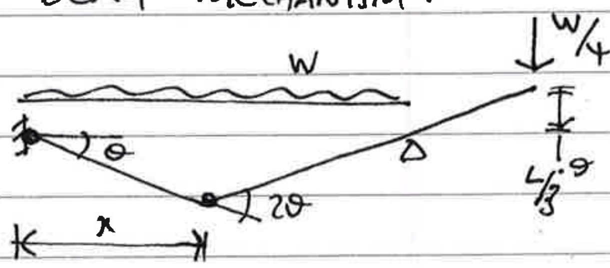


$\frac{W}{4} \cdot \frac{L\theta}{3} = M_p\theta$

$\therefore W = 12M_p/L$

$= \underline{\underline{1136 \text{ kN}}}$

BEAM MECHANISM :



ASSUME $x = L/2$

$\therefore W\frac{L}{4}\theta - \frac{W}{4}\frac{L}{3}\theta = 3M_p\theta$

$\frac{WL\theta}{6} = 3M_p\theta$

$\therefore W = 18M_p/L$ ⊛

(d) ⊛ COULD IDENTIFY ~~E~~XACT POSITION OF HINGE BY SETTING UP IN TERMS OF x AND FINDING MIN. OF dW_{MECH}/dx, BUT CANTILEVER MECH. MATCHES LOWER BOUND $\therefore$ THIS (1136 kN) IS TRUE COLLAPSE LOAD.