ENGINEERING TRIPOS PART IB

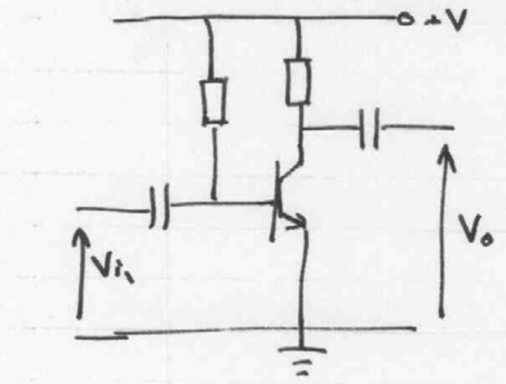
Wednesday 8 June 2011 2 to 4

Paper 5 - SOLUTIONS

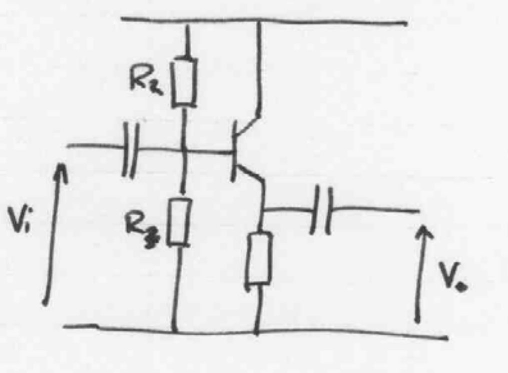
ELECTRICAL ENGINEERING

SECTION A

1 (a) Circuits for the common emitter amplifier and emitter follower amplifier are shown below:



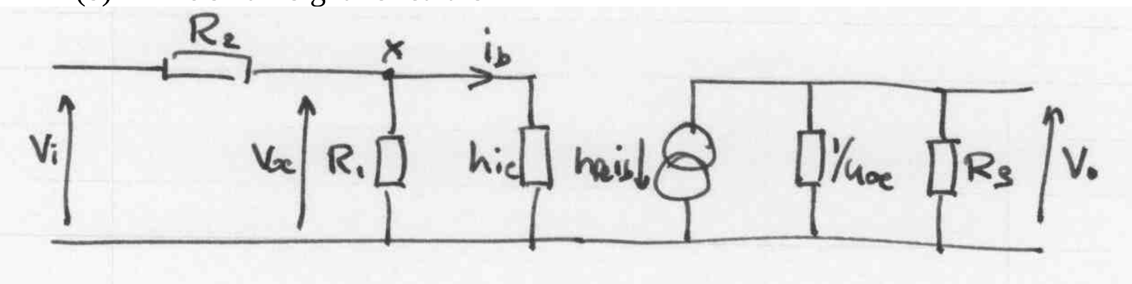
Common Emitter Amplifier



Emitter follower amplifier

In the common emitter amplifier, the emitter is held at 0 V and the output is connected to the collector, whilst in the emitter follower amplifier, the output is connected to the emitter, which ash a resistance between it and earth across which an output voltage can be developed. The emitter follower amplifier has a gain of ~1, and is used as a buffer. The common emitter amplifier has a large, negative gain, and so can be used as a simple, single stage, inverting, Class A amplifier. [6]

(b) The small-signal circuit is



At the output,

$$V_o = -i_b h_{fe}\left(R_3 || {}^1/_{h_{oe}}\right) \quad (1)$$

Performing a nodal voltage analysis at X,

$$\sum I_{out} = 0 = \frac{V_x - V_i}{R_2} + \frac{V_x}{R_1} + i_b$$

However, in practice $R_1 >> h_{ie}$, so i_b flows through R_2. Hence,

$$V_i = i_b(R_2 + h_{ie}) \qquad (2)$$

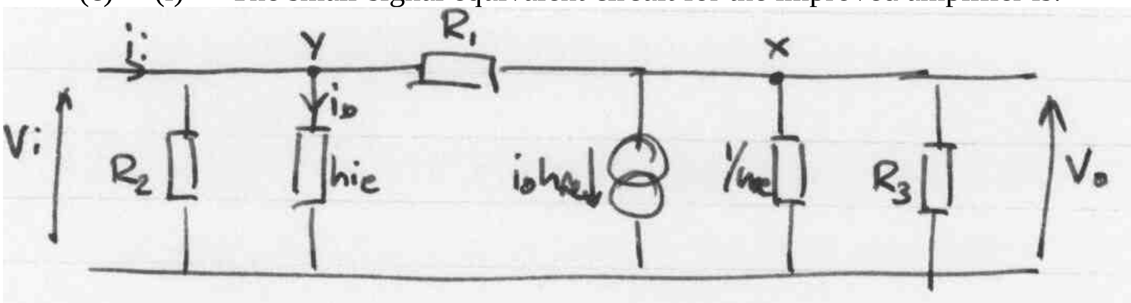
Dividing (1) by (2) gives

$$\frac{V_o}{V_i} = \frac{-h_{fe}\left(R_3 || {}^{1}/_{h_{oe}}\right)}{(R_2 + h_{ie})}$$

Calculating values, gives

$$\frac{V_o}{V_i} = \frac{-80(10\text{k}||8333)}{5\text{k}+1.5\text{k}} = \boxed{-55.9}$$ [6]

(c) (i) The small-signal equivalent circuit for the improved amplifier is:



Performing a nodal voltage analysis at X:

$$\sum I_{out} = 0 = i_b h_{fe} + \frac{V_o - V_i}{R_1} + \frac{V_o}{\left({}^{1}/_{h_{oe}} || R_3\right)}$$

We know that $V_i = i_b h_{fe}$, so

$$0 = \frac{V_i}{h_{ie}} h_{fe} + \frac{V_o - V_i}{R_1} + \frac{V_o}{\left({}^{1}/_{h_{oe}} || R_3\right)}$$

Collecting terms,

$$V_o\left(\frac{1}{R_1} + h_{oe} + \frac{1}{R_3}\right) = V_i\left(\frac{1}{R_1} + \frac{h_{fe}}{h_{ie}}\right)$$

Therefore,

$$\frac{V_o}{V_i} = \frac{\frac{1}{R_1} + \frac{h_{fe}}{h_{ie}}}{\frac{1}{R_1} + h_{oe} + \frac{1}{R_3}} = \frac{\frac{1}{50\text{k}} - \frac{80}{1.5\text{k}}}{\frac{1}{50\text{k}} + 120\mu + \frac{1}{10\text{k}}} = \boxed{-222}$$ [4]

(ii) Performing a nodal voltage analysis at Y,

$$\sum I_{out} = 0 = -i_i + \frac{V_i}{R_2} + \frac{V_i}{h_{ie}} + \frac{V_i - V_o}{R_1}$$

However, if V_o / V_i is the gain, G, then

$$i_i = V_i\left(\frac{1}{R_2} + \frac{1}{h_{ie}} + \frac{1-G}{R_1}\right)$$

Therefore,

$$Z_{in} = \left(\frac{1}{R_2} + \frac{1}{h_{ie}} + \frac{1-G}{R_1}\right)^{-1} = \left(\frac{1}{5\text{k}} + \frac{1}{1.5\text{k}} + \frac{1-(-222)}{50\text{k}}\right)^{-1} = \boxed{188\ \Omega}$$ [2]

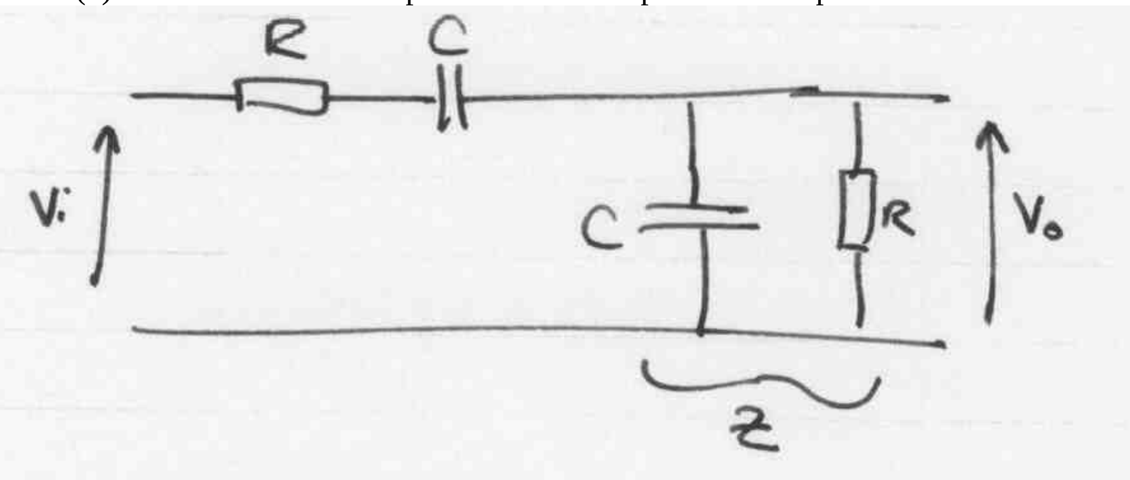
(iii) The resistor R_1 has introduced negative feedback into the circuit, and this will have the effect of stabilising the circuit. [2]

2 (a) The closed loop gain, *G*, for a circuit consisting of an amplifier with open loop gain, *A*, and for which a fraction, *B*, of the output is fed back to be subtracted from the input is

$$G = \frac{A}{1+AB}$$

Stable oscillations require a lop gain of 1 with no phase shift. [4]

(b) First consider the input circuit to the operational amplifier:



Where

$$Z = Z_C || Z_R = \frac{R}{1+\mathrm{j}\omega RC}$$

The feedback fraction is then

$$B = \frac{V_o}{V_i}$$

$$B = \frac{Z}{Z+R+{}^{1}/_{\mathrm{j}\omega C}}$$

$$B = \frac{\frac{R}{1+\mathrm{j}\omega RC}}{\frac{R}{1+\mathrm{j}\omega RC}+R+{}^{1}/_{\mathrm{j}\omega C}}$$

$$B = \frac{1}{3+j\left(\omega RC-{}^{1}/_{\omega RC}\right)}$$ [6]

(c) For stable oscillations, the imaginary part of *B* is zero, so *B* = 1/3. Therefore, if *AB* = 1 for stable oscillations, then *A* = 3. We have a non-inverting amplifier, so

$$A = 1 + \frac{R_1}{R_2}$$

$$3 = 1 + \frac{R_1}{5\mathrm{k}}$$

$\boxed{R_1 = 10\ \mathrm{k\Omega}}$ [3]

(d) For the imaginary part to be zero, we require

$\omega RC = \frac{1}{\omega RC}$

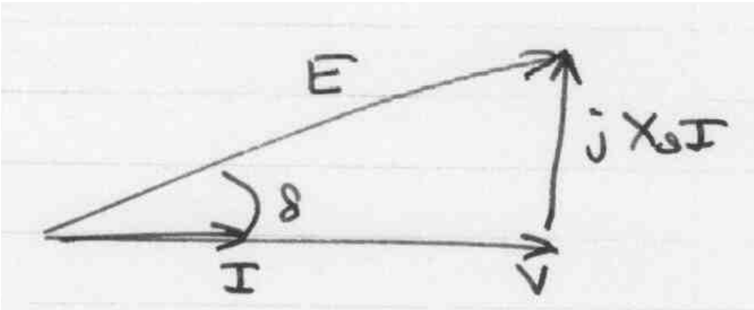
$C = \frac{1}{\omega R} = \frac{1}{2\pi.15\mathrm{k}.10\mathrm{k}} = \boxed{1.06\ nF}$ [3]

(e) The output impedance should be much less than the load, $R_1 + R_2 = 15\mathrm{k\Omega}$. Therefore, the output impedance must be less than 1.5 kΩ, and preferably below 150 Ω. Input impedance should be much greater than R, so at least 50 kΩ and preferably over 500 kΩ. [4]

SECTION B

3 (a) The two conditions that must be satisfied if a *synchronous a.c. generator* is to produce steady torque are: (i) that the number of poles on the rotor and the stator must be the same in order that a net torque is exerted on the rotor at any moment in time; and (ii) that the angular velocity of the rotor and stator fields must be the same (ω_s/p) in order that a constant torque is exerted which is non-zero when averaged over time. [Examiner's Note: many students started discussing the asynchronous motor in their answer]. [4]

(b) If the power factor ($\cos\varphi$) is unity, then there is no phase difference between V and I, so we have the following situation:



Therefore, given that we have a three-phase system [Examiner's Note: many students did not allow for the fact that this is three-phase], the line current and the phase current are same, and given by

$$I_l = I_{ph} = \frac{P}{\sqrt{3}V_l \cos\varphi} = \frac{650\times10^6}{\sqrt{3}.22\times10^3.1} = 17.06 \text{ kA}$$

Therefore,

$$X_s I = 17.06 \times 10^3.0.5 = 8529 \text{ V}$$

Hence, from the figure above, we can calculate the generated excitation voltage as

$$E = \left(V_{ph}^2 + (X_s I)^2\right)^{1/2} = \left\{\left(\frac{22\times10^3}{\sqrt{3}}\right)^2 + 8529^2\right\}^{1/2} = \boxed{15.3\ kV}$$

The load angle is then

$$\delta = \tan^{-1}\left(\frac{X_s I}{V_{ph}}\right) = \tan^{-1}\left(\frac{8529.\sqrt{3}}{22\times10^3}\right) = \boxed{33.9°}$$ [5]

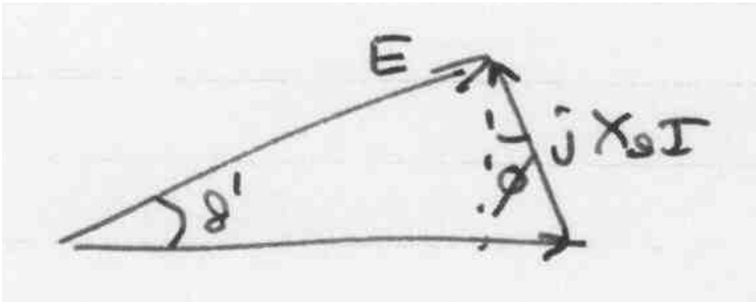
(c) If the excitation has been reduced by 12%, then the new excitation is

$$E' = 0.88E = 0.88.15.3 \times 10^3 = 13.46 \text{ kV}$$

The power factor will no longer be unity necessarily [Examiner's Note: although many students incorrectly assumed that it was], so the best we can do for the moment is to calculate

$$I_l' \cos\varphi = \frac{P}{\sqrt{3}V_l} = \frac{500\times10^6}{\sqrt{3}.22\times10^3} = 13122 \text{ A} \qquad \text{Eqn. 1}$$

The current is low leading the voltage in the system, and so



It is clear from the figure that

$$E' \sin\delta' = X_s I' \cos\varphi'$$

and so

$$\delta' = \sin^{-1}\left(\frac{X_S I\prime \cos\varphi\prime}{E\prime}\right) = \sin^{-1}\left(\frac{0.5.13122}{13.46\times10^3}\right) = \boxed{29.2°}$$

Furthermore, using the cosine rule,

$$(X_s I')^2 = {E'}^2 + V^2 - 2E'V\cos\delta'$$

$$(X_s I')^2 = 13460^2 + \frac{(22\times10^3)^2}{3} - 2.13460.\frac{22\times10^3}{\sqrt{3}}\cos 29.9° = 44.03\times10^6 \text{ V}^2$$

$$X_s I' = 6635 \text{ V}$$

Therefore

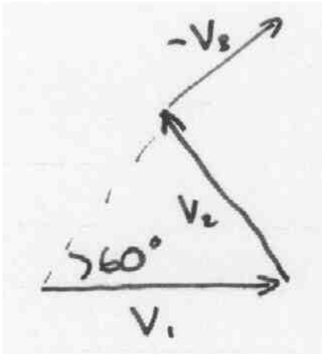
$$I' = \frac{6635}{0.5} = 13271 \text{ A}$$

By combining this with Eqn. 1, we can find the power factor as

$$\cos\varphi' = \frac{I_l\prime \cos\varphi\prime}{I_l\prime} = \frac{13122}{13271} = \boxed{0.99}$$ [7]

(d) In a grid connected power distribution system which consists of a number of synchronous a.c. generators that are connected through a network of transformers to a number of loads, the scaling in the *per-unit* system means that all of the generators are considered as having a per-unit voltage of 1. Furthermore, because they are all in phase with the grid (being synchronous), then they all are in phase with each other. Finally, they all have a common earth. Therefore, they all effectively act in parallel and may be combined as a single generator. [4]

4 (a) (i) The situation that we have is



Taking components of V_1 and V_2 in the direction of $-V_3$,

$$V_1 \text{ component} = |V_1|\cos 60° = 0.5|V_1|$$

$$V_2 \text{ component} = |V_2|\cos 60° = 0.5|V_2|$$

There is no net component perpendicular to $-V_3$. Also, as we have a balanced load, $|V_1| = |V_2| = |V_3| = V$, and therefore

$$|V_1 + V_2 - V_3| = |V_3| + 0.5|V_1| + 0.5|V_2| = \boxed{2V}$$ [4]

(ii) In the series case,

$$P_{max} = \frac{2V}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} = VI_{max}.$$

Whereas in the three-phase case,

$$P_{max} = 3\frac{V}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} = \frac{3VI_{max}}{2}$$

Therefore, power is reduced from $\frac{3VI_{max}}{2}$ to VI_{max}, or <u>a factor of ⅓ reduction</u>. [3]

(b) (i) We begin by calculating the impedance per phase of factory 1 as

$$Z_{ph1} = R + j\omega L = 2 + j.2\pi.50.9.55 \times 10^3 = (2 + 3j)\ \Omega$$

The power factor for factory 1 is therefore

$$\cos\varphi_1 = \cos\left\{\tan^{-1}\left(\frac{3}{2}\right)\right\} = 0.55 \text{ lagging}$$

Also, the line current drawn by factory 1 is

$$I_{l1} = \left|\frac{V_l}{\sqrt{3}Z_{ph1}}\right| = \left|\frac{11\times10^3}{\sqrt{3}(2+3j)}\right| = 1761.4\ \text{A}$$

The real and reactive power for factory 1 can now be calculated

$$P_1 = \sqrt{3}V_l I_{l1}\cos\varphi_1 = \sqrt{3}.11 \times 10^3 \times 1761.4 \times 0.55 = 18.46\ \text{MW}$$

$$Q_1 = \sqrt{3}V_l I_{l1}\sin\varphi_1 = \sqrt{3}.11 \times 10^3 \times 1761.4 \times \sin\left(\tan^{-1}\left(\frac{3}{2}\right)\right)$$

$$Q_1 = 27.92\ \text{MVAR}$$

[Examiner's Note: many candidates got MW and kW muddled up!]

For factory 2, things are little simpler as the impedances are in parallel, and so the real and reactive power for factory 2 can calculated directly,

$$P_2 = \frac{3V_l^2}{R} = \frac{3\left(11\times10^3\right)^2}{15} = 24.2\ \text{MW}$$
$$Q_2 = \frac{3V_l^2}{X} = -3V_l^2\omega C = 3(11\times10^3)^2 \times 2\pi \times 50 \times 7.1 \times 10^6$$
$$Q_2 = -0.81\ \text{MVAR}$$

Therefore, the total real and reactive power and apparent power for the two factories together are

$$P_{tot} = P_1 + P_2 = 18.46\ \text{MW}+24.2\ \text{MW}=42.6\ \text{MW}$$
$$Q_{tot} = Q_1 + Q_2 = 27.92\ \text{MVAR} - 0.81\ \text{MVAR}=27.1\ \text{MVAR}$$
$$S_{tot} = (P_{tot}^2 + Q_{tot}^2)^{1/2} = 50.5\ \text{MVA}$$

[Examiner's Note: many students tried to add together S for the two factories. Pythagoras' Theorem does not allow this – you must calculate P_{tot} and Q_{tot} and then use the result to calculate S_{tot}].

We can now calculate the desired values as

$$\cos\varphi = \cos\left\{\tan^{-1}\left(\frac{Q_{tot}}{P_{tot}}\right)\right\} = \boxed{0.84\ \text{lagging}}$$
$$I_l = \frac{S_{tot}}{\sqrt{3}V_l} = \frac{50.5\times10^6}{\sqrt{3}.11\times10^3} = 2650\ \text{A}$$

[Examiner's Note: Many candidates forgot to give an answer for either the line current or power factor]

(ii) With the overall power factor of the two factories corrected to PF'=0.95 lagging (and remembering that power factor correction does not affect real power consumed)

$$Q'_{tot} = P_{tot}\tan\{\cos^{-1}PF'\} = 42.6\tan(\cos^{-1}0.95) = 14.0\ \text{MVAR}$$

The change in Q due to the capacitors is then

$$\Delta Q = Q'_{tot} - Q_{tot} = 14.0 - 27.1 = -13.1\ \text{MVAR}$$

This is across all three phases, and so

$$\Delta Q = -3V_{ph}^2.\omega C$$
$$\therefore C = \frac{-\Delta Q}{3.2\pi f V_{ph}^2} = \frac{13.1\times10^6}{3.2\pi.50.\left(\frac{11\times10^3}{\sqrt{3}}\right)^2} = \boxed{345\ \mu\text{F}}$$ [5]

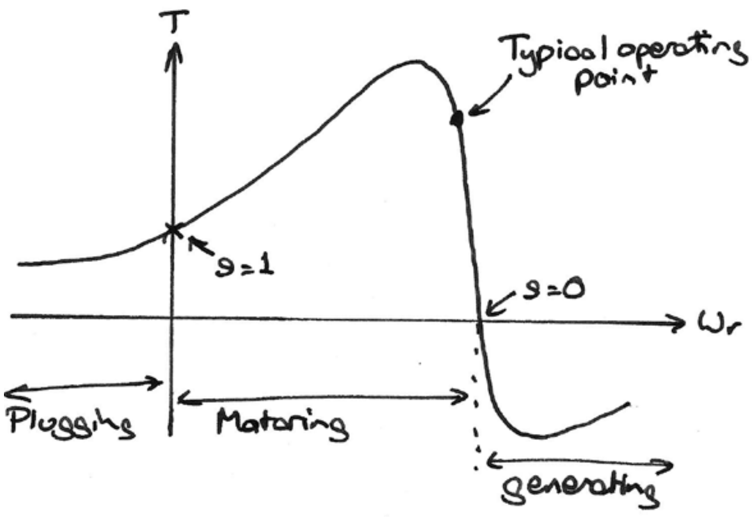
(iii) The new total line current is

$$I_l' = \frac{P_{tot}}{\sqrt{3}V_l \cos\varphi\prime} = \frac{42.6\times10^6}{\sqrt{3}.11\times10^3.0.95} = 2345 \text{ A}$$

Remembering that there is power dissipation across all three phases in the real component of the impedance, we have the change in power dissipation in the line as being

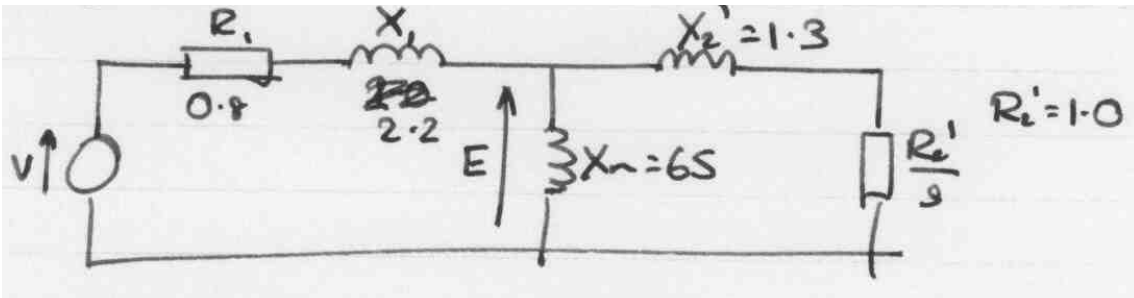
$$\Delta P_{line} = 3\left(I_l^2 - {I_l'}^2\right).R_l = 3(2650^2 - 2345^2).0.5 = \boxed{2.3\text{ MW}}$$

5



The motor is usually operated close to, but just below maximum torque to the right of the peak. In this region, there is a linear variation in torque with rotation speed. Also, if the load increases, then as the rotation speed drops, the torque will rise in compensation, and so, as long as the maximum torque is not reached, the motor will not stall. [5]

(b) (i) The equivalent circuit for the motor is



The maximum torque is produced when power dissipation is maximised in $\frac{R_2'}{s}$, so using the maximum power transfer theorem, this is when $\left|\frac{R_2'}{s}\right|$ is equal to the magnitude of the impedance of the circuit looking back in from $\frac{R_2'}{s}$. Hence,

$$\left|\frac{R_2'}{s}\right| = \left|\mathrm{j}X_2' + \left(\mathrm{j}X_m||(R_1 + \mathrm{j}X_1)\right)\right|$$

$$\left|\frac{R_2'}{s}\right| = \left|\mathrm{j}1.3 + \left(\mathrm{j}65||(0.8 + \mathrm{j}2.2)\right)\right|$$

$$\left|\frac{R_2'}{s}\right| = |0.75+\mathrm{j}3.44| = 3.517$$

Therefore,

$$s = \frac{R_2'}{3.517} = \frac{1.0}{3.517} = 0.284$$

The rotor speed is therefore

$$\omega_r = (1-s)\omega_s = (1-s)\frac{\omega}{p} = (1-0.284)\frac{2\pi.50}{3} = 74.9 \text{ rad s}^{-1}$$

$$N_r = \frac{\omega_r}{2\pi}.60 = \boxed{716 \text{ rpm}}$$

We need to calculate the current in the rotor, I_2'. To help with this, let us define

$$Z = \mathrm{j}X_m || \left(\mathrm{j}X_2' + \frac{R_2'}{s}\right) = \mathrm{j}65||(\mathrm{j}1.3 + 3.517) = 3.37 + \mathrm{j}1.45$$

Hence,

$$E = V_{ph}.\frac{Z}{Z+R_1+jX_1} = \frac{415}{\sqrt{3}}.\frac{3.37+\mathrm{j}1.45}{3.37+\mathrm{j}1.45+0.8+\mathrm{j}2.2} = (151.0 - \mathrm{j}48.7)\text{ V}$$

$$|E| = 158.8 \text{ V}$$

We can now calculate the current in the rotor (remembering to include phase differences) as

$$I_2' = \frac{E}{\frac{R_2'}{s}+\mathrm{j}X_2'} = \frac{151.0-\mathrm{j}48.7}{3.517+\mathrm{j}1.3} = 33.27 - \mathrm{j}26.15$$

$$\therefore |I_2'| = 42.3 \text{ A}$$

We can now calculate the torque from the equation in the Data Book as

$$T = \frac{3}{\omega_s}|I_2'|\frac{R_2'}{s} = \frac{3}{104.7}42.3^2.3.517 = \boxed{180.3 \text{ N m}}$$

(ii) To calculate the motor output power and efficiency at maximum torque, we must first determine how much of the torque calculated in part (i) is lost due to friction and windage losses as

$$T_{loss} = \frac{P_{loss}}{\omega_r} = \frac{260}{74.9} = 3.47 \text{ N m}$$

Therefore the output torque is

$$T_{out} = T - T_{loss} = 180.3 - 3.47 = 176.8 \text{ N m}$$

The output power is then

$$P_{out} = T_{out}\omega_r = 176.8 \times 74.9 = \boxed{13240 \text{ W}}$$

To get the efficiency, we need to work out the electrical power in to the system. For this, we must determine the input line current, which is

$$I_l = \frac{V_{ph}}{Z_{in}} = \frac{V_{ph}}{R_1+\mathrm{j}X_1+Z} = \frac{415/\sqrt{3}}{0.8+\mathrm{j}2.2+3.37+\mathrm{j}1.45} = 32.5 - \mathrm{j}28.5$$

$$\therefore |I_l| = 43.2 \text{ A}$$

Also, the power factor may be calculated as

$$\cos\varphi = \cos\left\{\tan^{-1}\left(\frac{\Im(I_l)}{\Re(I_l)}\right)\right\} = \cos\left\{\tan^{-1}\left(\frac{28.5}{32.5}\right)\right\} = \cos 41.2° = 0.75$$

The input power is then

$$P_{in} = \sqrt{3}V_l|I_l|\cos\varphi = \sqrt{3} \times 415 \times 43.2 \times 0.75 = 23290\ \mathrm{W}$$

The efficiency is

$$\eta = \frac{P_{out}}{P_{in}} = \frac{13240}{23290} = \boxed{57\%}$$ [6]

(c) Slip rings can be used to add resistance to R_2'. In this way, the speed at which maximum torque is produced can be varied so that torque is maximised at any operating speed in the motoring regime. This includes when the motor is starting up (slip is unity) when a high torque is requed to overcome the high friction present when starting motion. [3]

SECTION C

6 (a) The pointing vector gives the instantaneous power density (per unit area) in an electromagnetic wave as its magnitude and the direction is the direction of energy transmission. It is defined as $\boldsymbol{N} = \boldsymbol{E} \times \boldsymbol{H}$. Hence, average power is $|\boldsymbol{N}|/_2$. [3]

(b) (i) Using the speed of light in free space, the wavelength is

$$\lambda = \frac{c}{f} = \frac{2.998\times10^8}{642\times10^6} = \boxed{0.467\text{ m}}$$ [2]

(ii) The effective surface area of a wave propagating uniformly in all directions a distance of 20 km away from the Sandy Heath transmitter is

$$A = 4\pi r^2 = 4\pi.20000^2 = 5.02 \times 10^9\text{ m}^2$$

Therefore, accounting for the gain, G, the intensity (power per unit area), which is equal to $|\boldsymbol{N}|$, in the wave is

$$|\boldsymbol{N}| = \frac{P}{A}.G = \frac{20\times10^3}{5.02\times10^9}.10 = 3.97 \times 10^{-5}\text{ W m}^{-2}$$

Now, impedance is defined in terms of electric and magnetic field strength as

$$\eta = \frac{\boldsymbol{E}}{\boldsymbol{H}} \qquad \text{Eqn. 1}$$

and, from the Poynting vector,

$$|\boldsymbol{N}| = \frac{|\boldsymbol{E}||\boldsymbol{H}|}{2} \qquad \text{Eqn. 2}$$

so by combining Eqn. 1 and Eqn. 2,

$$|\boldsymbol{E}|^2 = 2\eta|\boldsymbol{N}| = 2.377.8.51 \times 10^{-5}$$

$$\therefore |\boldsymbol{E}| = \boxed{0.17\text{ V m}^{-1}}$$

Finally, using Eqn. 1 again, we can determine the magnetic field strength as

$$|\boldsymbol{H}| = \frac{|\boldsymbol{E}|}{\eta} = \frac{0.17}{377} = \boxed{459\ \mu\text{A m}^{-1}}$$ [5]

(iii) The power input to the antenna is given by

$$P_{in} = |\boldsymbol{N}|A = 3.97 \times 10^{-5}.0.25 = 9.93 \times 10^{-6}\text{ W}$$

where A is the area of the antenna. If this is fed into a 50 Ω matched load, then this will produce a current of

$$P_{in} = \frac{1}{2}|I|^2 Z$$

$$|I|^2 = \frac{2P_{in}}{Z} = \frac{2.9.93\times10^{-6}}{50} = 3.97 \times 10^{-7}\text{ A}^2$$

$$I_{rms} = \frac{|I|}{\sqrt{2}} = \sqrt{\frac{3.97\times10^{-7}}{2}} = \boxed{0.46\text{ mA}}$$ [5]

(iv) The signal inside the building will be decreased by the reflection that takes place at both the outside and inside surface of the brick wall [Examiner's Note: many candidates only considered the reflection from the outside surface]. To calculate this, we need to know the impedance of the wall, which is

$$\eta_{wall} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{4\pi\times10^{-7}.1}{8.854\times10^{-12}.4.2}} = 184\ \Omega$$

The reflection coefficient at the two interfaces squared will allow the power transmitted to be worked out. This is given by

$$\rho^2_{air-wall} = \left(\frac{184-377}{184+377}\right)^2 = 0.188 = \left(\frac{377-184}{377+184}\right)^2 = \rho^2_{wall-air}$$

Therefore, the fraction of power transmitted at each interface is

$$1-\rho^2 = 0.882$$

and the total fraction of power transmitted through both interfaces is 0.882^2, which is 0.778.

Therefore, the power received by the antenna inside the building is

$$P = 9.93\times10^{-6}.0.778 = 7.72\times10^{-6}$$

and from this, we can calculate the r.m.s current in the same way as for part (iii) to be

$$|I|^2 = \frac{2P}{Z} = \frac{2.7.72\times10^{-6}}{50} = 3.09\times10^{-7}\ \mathrm{A}^2$$

$$I_{rms} = \frac{|I|}{\sqrt{2}} = \sqrt{\frac{3.09\times10^{-7}}{2}} = \boxed{0.39\ \mathrm{mA}}$$ [5]

7 (a) Characteristic impedance is the ration of voltage to current at any point in a transmission line when there are no reflections present. [2]

(b) At the load, $V = V_F + V_B$ and $I = I_F + I_B$, where the subscripts F and B denote the forward and backwards components of voltage a current. Using the definition of characteristic impedance, we know that

$$Z_0 = \frac{V_F}{I_F} = -\frac{V_B}{I_B} \qquad \text{Eqn. 1}$$

Also, at the load,

$$Z_L = \frac{V}{I} = \frac{V_F+V_B}{I_F+I_B} = \frac{V_F+V_B}{\frac{V_F}{Z_0}-\frac{V_B}{Z_0}}$$

Therefore,

$$\frac{Z_L}{Z_0}.(V_F - V_B) = V_F + V_B$$

$$V_F\left(\frac{Z_L}{Z_0} - 1\right) = V_B\left(1 + \frac{Z_L}{Z_0}\right)$$

$$\therefore \rho_L = \frac{V_B}{V_F} = \boxed{\frac{Z_L-Z_0}{Z_L+Z_0}}$$ [4]

(c) (i) The voltage that is input into the coaxial cable from the power supply is

$$V_{in} = V.\frac{Z_0}{Z_0+Z_p} = 12.\frac{75}{75+15} = 10 \text{ V}$$

where Z_p is the internal resistance of the power supply. The power input is then

$$P_{in} = \frac{V_{in}^2}{Z_0} = \frac{10^2}{75} = 1.33 \text{ W}$$

The voltage reflection coefficient at the load is

$$\rho = \frac{Z_L-Z_0}{Z_L+Z_0} = \frac{200-75}{200+75} = 0.455$$

The reflected power is therefore

$$P_{ref} = P_{in}\rho^2 = 1.33 \times 0.455^2 = 1.33 \times 0.207 = \boxed{0.275 \text{ W}}$$ [4]

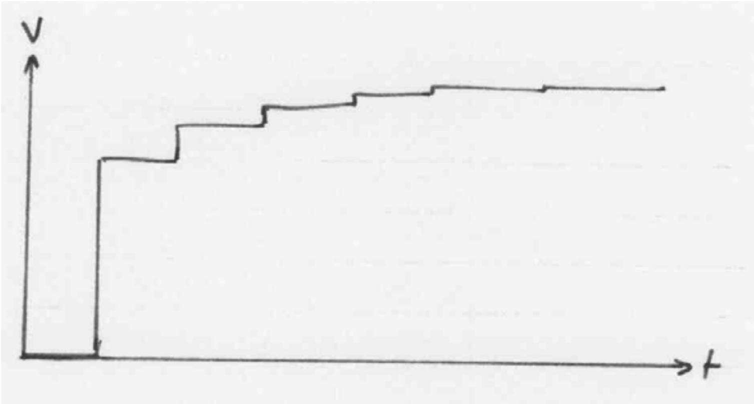
(ii) At the input to the circuit,

$$\rho = \frac{Z_p-Z_0}{Z_p+Z_0} = \frac{15-75}{15+75} = -\frac{2}{3}$$

$$\rho^2 = 0.444$$

To get to less than 1% of input power, we need 0.207×0.444×0.207×0.444. Therefore, two complete circuits are required, or the wave travels from end to end of the 50 m cable four times. This is a total distance of 200 m at a velocity of 6×10^7 m s^{-1}. Hence, the time is <u>3.33 μs</u>. [6]

(iii) The voltage across the load resistor will vary with time after the switch is closed as follows



The key features are that the time between voltage steps is constant and that the voltage is tending asymptotically towards a constant, non-zero, value. [4]

AJF
June 2011

ENGINEERING TRIPOS PART IB

Wednesday 8 June 2011 2 to 4

Paper 5 – NUMERICAL SOLUTIONS

ELECTRICAL ENGINEERING

1 (b) Gain = -55.9
(c) (i) Gain = -222
(ii) Z_{in} = 188 Ω

2 (c) R_1 = 10 kΩ
(d) C = 1.06 nF

3 (b) E = 15.3 kV and δ = 33.9°
(c) δ' = 29.2°, I = 13271 A and the power factor is 0.99

4 (a) (ii) The power is reduced by a factor of one third
(b) (i) The power factor is 0.84 lagging and the line current is 2650 A
(ii) C = 345 μF
(iii) The change in power is 2.3 MW

5 (b) (i) T_{max} = 180.3 Nm and N_r = 716 rpm
(ii) Efficiency is 57%

6 (b) (i) λ = 0.467 m
(ii) The electric field strength is 0.17 V m^{-1} and the magnetic field strength is 459 μA m^{-1}
(iii) I_{rms} = 0.46 mA
(iv) I_{rms} = 0.39 mA

7 (c) (i) Reflected power is 0.275 W
(ii) Time is 3.33 μs