Introductory business economics, Dr M Kitson
Civil and structural engineering, Dr A Al-Tabbaa
Mechanical engineering, Dr M Sutcliffe
Aerothermal engineering, Prof P G Tucker
Electrical engineering, Prof W I Milne
Information engineering, Prof R Cipolla
Engineering for the life sciences, Dr M Lengyel
Manufacturing, management and design, Dr T H W Minshall

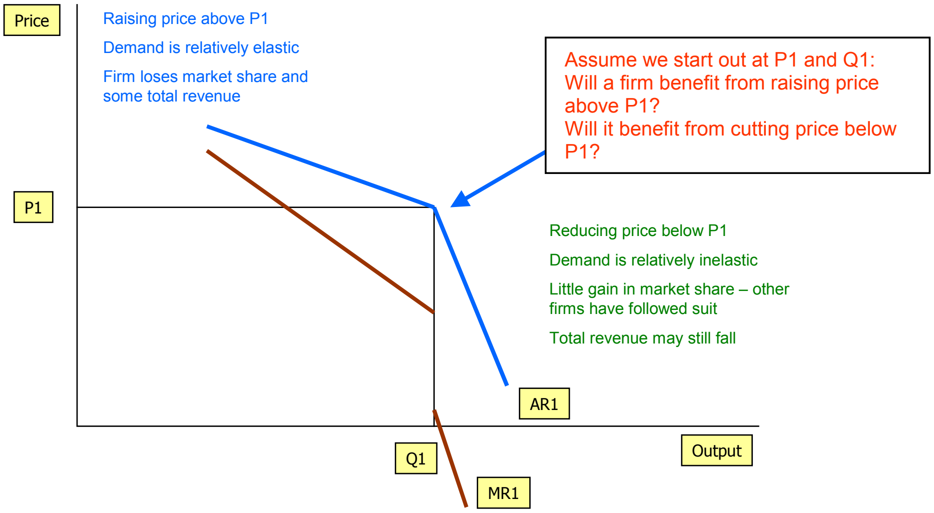
2011

Paper 8
A Introductory Business Economics

Answer not more than **one** question from this section.

1 (a) Describe one model of oligopoly. [5]

The most common answer is likely to be the kinked demand theory. As shown in the diagram below, firms believe that their competitors would follow them if they were to reduce their price below the prevailing market price, and that it is therefore not possible to gain much in the way of extra sales by reducing price (i.e. demand is relatively inelastic below the market price). Also, as shown below, firms believe that their competitors would not follow them if they were to raise their price above the prevailing market price (i.e. demand is relatively elastic above the market price).



Some candidates may describe the Cournot Model (with simultaneous output decisions) or the Bertrand Model (simultaneous Price Decisions). Due credit will be given to candidates describing these or any other appropriate models.

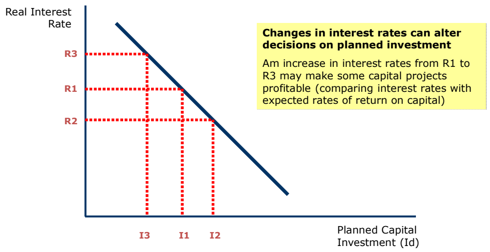
(b) Explain the concept of comparative advantage. [6]

The theory of comparative advantage explain why economies may improve their welfare by trading with each other even if one country has an absolute cost advantage. Candidates may illustrate this with a simple two-country and two-good example (as discussed in lectures). Some candidates may also outline some of the limitations of comparative advantage.

(c) What impact would the following have on the level of investment in the macroeconomy:

i a rise in the rate of interest; [3]

As shown in the diagram below, an increase in the interest rate will decrease the amount of autonomous investment in the macro economy. This is because the downward sloping marginal efficiency of capital curve indicates that there are fewer profitable investment opportunities at higher interest rates.



Changes in interest rates can alter decisions on planned investment

Am increase in interest rates from R1 to R3 may make some capital projects profitable (comparing interest rates with expected rates of return on capital)

ii new economic forecasts that suggest a slowdown in the rate of economic growth; [3]

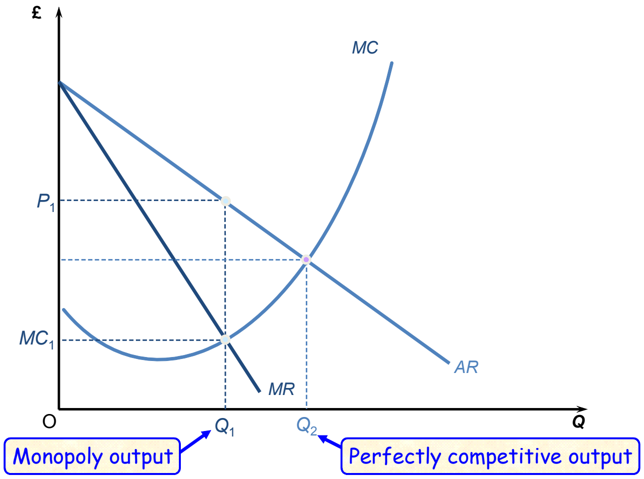
Such forecasts of lower level of economic activity than previously expected are likely to lead to a fall in investment as indicated by the Accelerator model. Net capital investment is the amount by which the required capital stock changes. It follows that the amount of investment depends on the level of expected economic activity. If this is reduced then the desired size of the capital stock needs to be reduced and investment will fall.

iii a decline in the willingness of banks to lend. [3]

This is likely to lead to fall in investment as it will act as a liquidity constraint on the investment plans of firms. Of course, many firms will be able to use their own resources to invest but many firms depend on external finance which will be in restricted supply.

2 (a) Using an appropriate diagram, explain the profit maximising position for a monopolist. [5]

Profits are maximised when the next unit produced and sold, adds as much to total revenue as it does to total cost. Profit maximisation occurs when marginal revenue = marginal cost (MR=MC). If MR exceeds MC profit can be increased by increasing production, if MC exceeds MR profit can be increased by cutting back on production. For monopolist, this is shown below.



(b) Why is it perfect competition normally considered to be more beneficial than monopoly? [5]

A monopolist can take the market demand curve as its own demand curve and can enjoy some power over the setting of price or output – as shown in the diagram above a lower level of output is produced and consumed at a higher price compared to the position with perfect competition. Thus the standard case against a monopoly is that these businesses can earn abnormal profits at the expense of economic efficiency. The monopolist is extracting a price from consumers that is above the cost of resources used in making the product. Consumers' needs and wants are not being satisfied, as the product is being under-consumed.

(c) Using the Keynesian consumption function model, explain the potential impact of increasing indirect taxes (such as VAT) on the level of aggregate demand. [5]

In the Keynesian case, current consumption (C) is determined by real current personal disposable income (Y). An increase in indirect taxes (such as VAT) should lead to an increase in prices and hence a fall in real current income. This will lead to a fall in consumption and thus a reduction in aggregate demand. This may lead to a fall in output or inflation (or possibly both) depending on the level of economic activity in the economy.

(d) Using the life cycle model or the permanent income model, explain the potential impact of increasing indirect taxes (such as VAT) on the level of aggregate demand. [5]

The life cycle model is based on the notion that consumption is determined by long-run or normal income. Thus, temporary income changes will have less impact on current consumption than permanent income changes. Thus the impact of the tax increase will depend on whether it is considered to be temporary (with little impact on consumption) or permanent (when it will lead to fall in consumption). Candidates may comment on the limitations of the life cycle model – for instance, it assumes perfect foresight and no liquidity constraints.

Q. 1 Solution

Page 1 of 4

(a) Short term conditions

- Use undrained shear strength of clay to calculate horizontal total stress, σ_h

active $\sigma_h = \sigma_v - 2S_u$

(σ_v = total vertical stress
S_u = undrained shear strength)

passive $\sigma_h = \sigma_v + 2S_u$

- For sand, calculate effective horizontal stress (σ_h') and add water pressure (u) total horizontal stress σ_h

$$\sigma_h = \sigma_h' + u$$

$$\sigma_h' = K_a \sigma_v'$$ (σ_v' = vertical effective stress)

$$K_a = \frac{1 - \sin\phi'}{1 + \sin\phi'}$$ (for a smooth wall)

- $\phi' = 35° \Rightarrow K_a = \frac{1 - \sin 35°}{1 + \sin 35°} = 0.27$

Backfilled side (active pressures)

At ground surface $\sigma_v = \sigma_v' = 50 \text{ kN/m}^2$

$\sigma_h = \sigma_h' = K_a \sigma_v' = 0.27 \times 50 = 13.5 \text{ kN/m}^2$

At 5m depth (to water level) $\sigma_v = 50 + 5 \times 17 = 135 \text{ kN/m}^2$

$u = 0, \therefore \sigma_v' = \sigma_v \; ; \; \sigma_h = \sigma_h'$

$\sigma_h = \sigma_h' = 0.27 \times 135 = 36.5 \text{ kN/m}^2$

At 7m depth (sand/clay interface)

At bottom of sand

$\sigma_v = 50 + (5\times17) + (2\times20) = 175$ kN/m^2

$u = 2\times10 = 20$ kN/m^2

$\therefore \sigma_v{}' = 155$ kN/m^2

$\sigma_h{}' = 0.27 \times 155 = 42$ kN/m^2

$\therefore \sigma_h = \sigma_h{}' + u = 42 + 20 = \underline{62 \text{ kN/m}^2}$

At top of clay

$\sigma_v = 175$ kN/m^2

$\sigma_h = \sigma_v - 2S_u = 175 - (2\times60) = \underline{55 \text{ kN/m}^2}$

At 10m depth

$\sigma_v = 50 + (5\times17) + (2\times20) + (3\times20)$

$= 235$ kN/m^2

$\sigma_h = \sigma_v - 2S_u = 235 - (2\times60) = \underline{115 \text{ kN/m}^2}$

Opposite side (passive pressures)

At 7m below top of tunnel (at river bed level)

$\sigma_h = \sigma_v = u = 2\times10 = \underline{20 \text{ kN/m}^2}$

$\sigma_h = \sigma_v + 2S_u = 20 + (2\times60) = \underline{140 \text{kN/m}^2}$

At 10m below top of tunnel

$\sigma_v = 20 + (3\times20) = 80$ kN/m^2

$\sigma_h = 80 + (2\times60) = \underline{200 \text{ kN/m}^2}$

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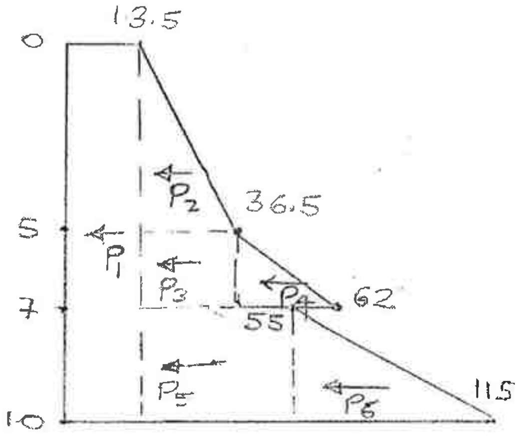
Backfilled side : total horizontal stresses
(active pressures)



$P_1 = 10 \times 13.5 = 135$ kN/m ~~97.5~~

$P_2 = \frac{1}{2} \times 5(\overset{36.5}{\cancel{46.2}} - \overset{13.5}{\cancel{26}}) = 65.5$ kN/m

$P_3 = 2 \times (36.5 - 13.5) = \cancel{23}\ 46$ kN/m

$P_4 = \frac{1}{2} \times 2(62 - 36.5) = 25.5$ kN/m

$P_5 = 3 \times (55 - \overset{13.5}{\cancel{20}}) = \cancel{105}\ 124.5$ kN/m

$P_6 = \frac{1}{2} \times 3(115 - 55) = 90$ kN/m

$\Sigma(P_1 \dots P_6) = \underline{\cancel{444}}$ kN/m

478.5

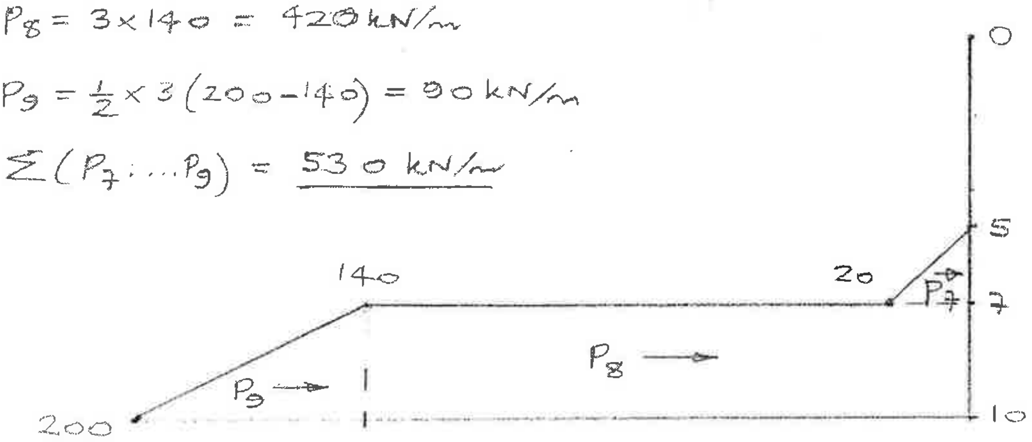
Opposite side : total horizontal stresses
(passive pressures)

$P_7 = \frac{1}{2} \times 2 \times 20 = 20$ kN/m

$P_8 = 3 \times 140 = 420$ kN/m

$P_9 = \frac{1}{2} \times 3(200 - 140) = 90$ kN/m

$\Sigma(P_7 \dots P_9) = \underline{530 \text{ kN/m}}$



(b) Factor of safety $= \dfrac{\text{total passive force}}{\text{total active force}}$

(ignoring friction on tunnel base)

$= \dfrac{530}{444} = \underline{1.1\cancel{9}}$

This is very low. The main reason for the low factor of safety is the 50 kN/m² surcharge.

Calculations	Job No	Page No	Revision
		4	

Revision	Suffix					
	Date					

Date	Initials	Checked	Job	Drawing No

Page 4 of 4

(c) Softening of clay on backfilled side to long term conditions

- horizontal stress only changes in the clay

At 7m depth (sand/clay interface)

At top of clay

$\sigma_v = 175$ kN/m^2

$u = 2 \times 10 = 20$ kN/m^2

$\therefore \sigma_v' = \sigma_v - u = 175 - 20 = 155$ kN/m^2

For clay, $\phi' = 20°$ $K_a = \frac{1 - \sin 20}{1 + \sin 20} = 0.49$

$\therefore \sigma_h' = K_a \sigma_v' = 0.49 \times 155 = 76$ kN/m^2

$u = 20$ kN/m^2

$\therefore \sigma_h = 76 + 20 = \underline{96 \text{ kN/m}^2}$

At 10m depth

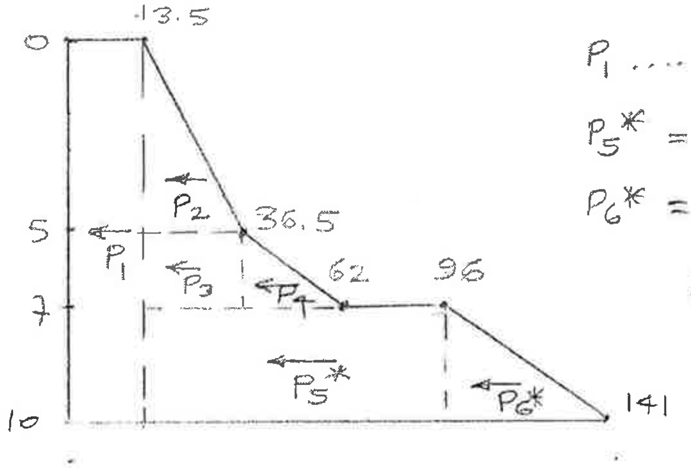
$\sigma_v = 235$ kN/m^2

$u = 5 \times 10 = 50$ kN/m^2

$\therefore \sigma_v' = \sigma_v - u = 235 - 50 = 185$ kN/m^2

$\sigma_h' = K_a \sigma_v' = 0.49 \times 185 = 91$ kN/m^2

$\sigma_h = \sigma_h' + u = 91 + 50 = \underline{141 \text{ kN/m}^2}$



$P_1 \dots P_4$ same as before

$P_5^* = 3 \times (96 - \overset{13.5}{\cancel{20}}) = \overset{247.5}{\cancel{228}}$ kN/m

$P_6^* = \frac{1}{2} \times 3 (141 - 96) = 67.5$ kN/m

$\Sigma P_1 + P_2 + P_3 + P_4 + P_5^* + P_6^* = \underline{\overset{579}{\cancel{545}} \text{ kN/m}}$

Factor of safety $= \frac{530}{545} = \underline{\underline{0.97}}$

The softening of the clay would cause failure, although the Factor of Safety was already low (see p. 3).

Q2. Solutions

(a)(i) A tunnel's stability in clay is determined by its stability ratio, N, defined as

$$N = (\sigma_v - \sigma_t)/s_u$$

where σ_v = total vertical pressure at tunnel axis level

σ_v = tunnel support pressure (if any, = 0 if open face)

s_u = undrained strength of the clay at tunnel axis level

If the value of N is less than about *5* the tunnel face will be stable.

In City A, s_u is 200 kPa at a depth of 20m, so that N = 2 for an open face tunnel (assuming unit weight of 20 kN/m^3). In City B, s_u is 25kPa at 20m so that N = 16.

Hence it is safe to construct an open face tunnel beneath City A but not for City B. A closed face tunnelling machine – either an earth pressure balance machine or a slurry shield – will be needed for City B. [4]

(ii) Masonry buildings are particularly susceptible to differential settlement and cracking is associated with tensile strain. Buildings subjected to hogging deformation are more susceptible than those subject to sagging, because the tensile strains tend to be induced in the top of the building whereas in the sagging zone they are in the foundations. A settlement trough induced at or near the ground surface by a tunnel being constructed is Gaussian in shape: this means that the building directly above the tunnel centreline can be only in a sagging mode, whereas to one side it is more likely to be in a hogging mode. [3]

(iii) Compensation grouting involves injection of grout into the ground between the tunnel and the building foundation in a controlled manner. The grout is injected from tube-à-manchettes (TAM's) which are installed in the ground before tunnelling, usually from an adjacent shaft. Instrumentation is installed on the building (levelling and/or electrolevels) and in the ground (extensometers) to monitor settlement and ground movements, and the grout is injected in response to the measurements. The principal aim is to reduce the potential differential settlement of the building, thereby limiting damage. [3]

(iv) (a) Segmental linings. These are commonly used for lining circular tunnels, constructed with tunnelling machines. The segments are usually made out of pre-cast concrete, but sometimes from SGI (Spheroidal Graphite Iron). These could be used for the tunnels beneath City A and City B.
Advantages: made in factory under carefully controlled conditions, relatively easy to handle, erected within tunnelling machine, robust, very rare for collapse to occur. *Disadvantages*: usually only OK for circular tunnels, therefore lack of flexibility on shape, difficult to vary thickness

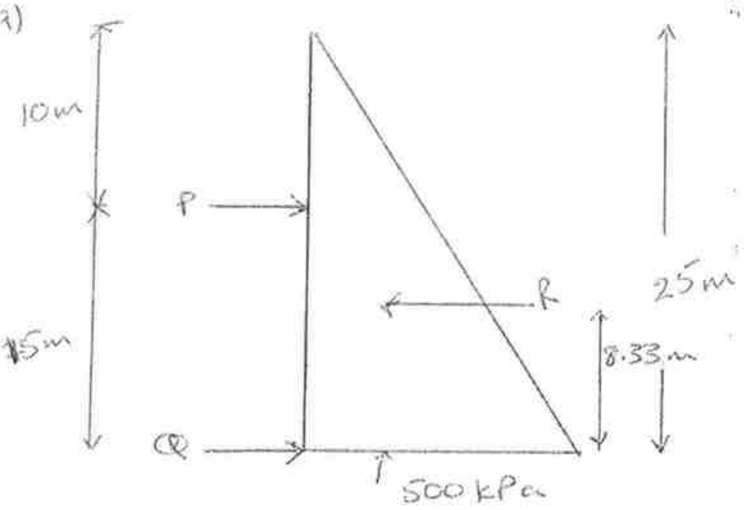
(b) Sprayed concrete linings. Sometimes known as NATM (New Austrian Tunnelling Method). Concrete sprayed onto excavated soil surface, accelerators added, hardens rapidly, usually with light reinforcement mesh or with steel fibres added. Could only be used in tunnels beneath City A (stiff clay, open face tunnelling). *Advantages*: very versatile, can easily change thickness, excavated shape. Disadvantages: needs careful quality control, susceptible to poor workmanship, collapse of tunnels has occurred (eg Heathrow, 1994).

[6]

(b)(i) If tunnelling in clays, the permeability is low enough for there to be no time for drainage (unless tunnelling is halted) and therefore the undrained strength (see answer to (a)(i)) is often high enough to ensure temporary stability of the tunnel face – as is the case for City A. However, if tunnelling in sands and gravels below the water table, as beneath part of City B, the water will flow into the face, causing collapse and de-stabilising the tunnel.

Potential problems in tunnelling below the water table in sands and gravels in City B can be overcome by (a) lowering the water table by pumping from wells installed for the purpose (b) injecting grout into the ground in advance of tunnelling – usually chemical grouts – to reduce the permeability (c) using compressed air in the tunnel – all of (a), (b) and (c) enable open face tunnelling to proceed – or (d) closed face tunnelling machines, either slurry machines or earth pressure balance machines. [4]

3 (a)



Resultant force on the wall is

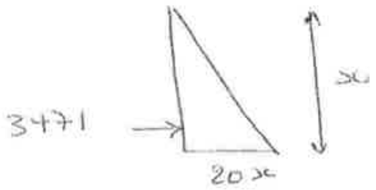
$$R = 500 \times \frac{1}{2} \times 25 \times 10^3 N = 6250 \text{ kN/m}$$

Taking moments about the base

$$6250 \times 8.33 = P \times 15 \Rightarrow P = 3471 \text{ kN/m}.$$

$$\therefore Q = R - P = 2779 \text{ kN/m}.$$

∴ At depth x below prop ($x > 10$m).



$$\text{Shear force} = \frac{1}{2} \times 20x \times x - 3471$$

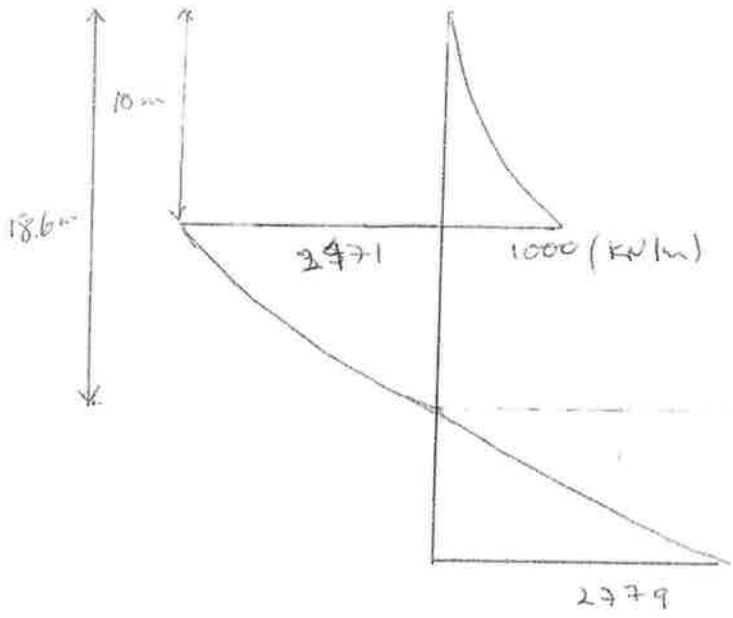
$$\therefore SF = 0 \text{ when } 10x^2 = 3471$$

$$\Rightarrow x = 18.6 \text{ m}$$

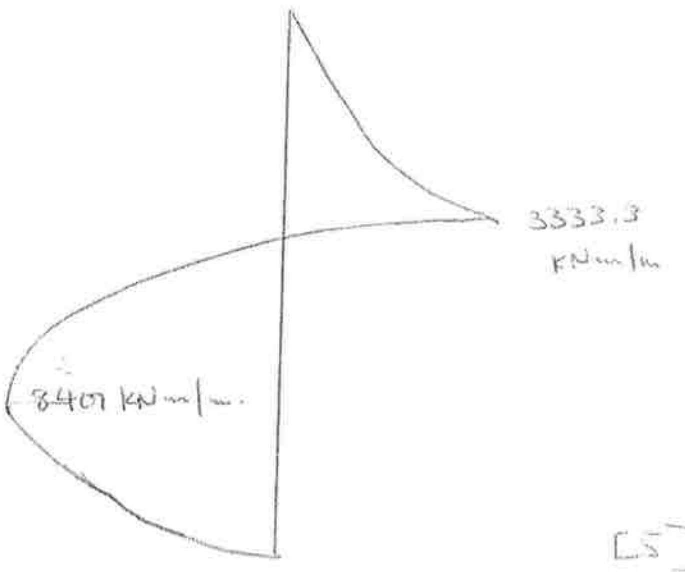
$$\text{Bending moment} = \frac{1}{2} . 20x . x . \frac{x}{3} - 3471(x - 10)$$

$$= \frac{10x^3}{3} - 3471x + 34710$$

Shear Force Diagram



Bending Moment Diagram



[5]

(b) Cube strength, $f_{cu} = 40 \text{ N/mm}^2$.
Yield strength, $f_y = 460 \text{ N/mm}^2$

From datasheet

$$M \leqslant 0.15 f_{cu} b' d^2$$

$$8401 \times 10^6 = 0.15 \times 40 \times 1000 \times d^2$$

$$\Rightarrow d = 1183 \text{ mm} \simeq 1185 \text{ mm}.$$

This is the effective depth

$\therefore$ Actual wall thickness will be greater to allow for c/c etc.

To find steel required at prop position.

Guess $x = 0.5$ and from Data Sheet.

$$M = 0.87 f_y A_s d \left(1 - \frac{x}{2}\right)$$

$$3333.3 \times 10^6 = 0.87 \times 460 \times A_s \times 1185 \times \frac{3}{4}$$

$$\Rightarrow A_s = 9372 \text{ mm}^2/\text{m}$$

But $x = 2.175 \dfrac{f_y}{f_{cu}} \dfrac{A_s}{bd}$ (Data Sheet).

$$\Rightarrow x = 0.198$$

Calculate new $A_s = 7801 \text{ mm}^2/\text{m}$.

(Could iterate further, but stop here).

40 mm bars = 1256.6 mm^2 $\therefore$ need 6.2 /m

50 mm bars = 1963.0 mm^2 $\therefore$ need 3.97/m.

$\therefore$ Say 50mm bars @ 250 mm centres $\Rightarrow$ 4 bars.

At maximum moment position.

$M = 8401$ kNm/m, Choose $x = 0.5$.

$A_s = 23619.7 \text{ mm}^2/\text{m}$

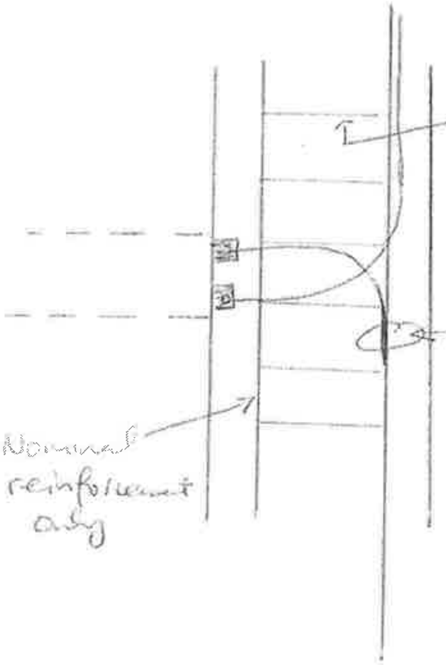
(No need to iterate as d chosen to give balanced section/load)

Say 50mm bars @ 75mm centres. [7]

(c) No real benefits of doubly reinforcing the wall. It would need to be applied over significant length. However, the trench could be narrower so that less material would need to be encountered, so there could be benefits depending on relative costs of

Excavation and Steel. [4].

(d)



Shear reinforcement on link bars - Not designed here but could be needed.

To give total effective area of 7801 mm²/m

Nominal reinforcement only

[4].

Engineering Tripos Part IB, 2011
Crib for Paper 8, Part C - Mechanics, Materials and Design

Michael Sutcliffe, Digby Symons, Tim Flack June 2011

6 (a) Typical elements :

Travel, totalling perhaps 50% – particularly air travel which would take up a big proportion
- cars

Goods (aka stuff) – a significant component with lots of embodied energy
Electrical – but phone chargers a minor concern
Heating – especially in the UK. Well worth insulating and reducing temperature
- baths etc consume significant energy

Food – another significant component, including transport costs. Meat more embodied energy than vegetables

(b) Need to use some measure of impact, e.g. CO_2 or energy. Material, Manufacture, Use and Disposal all use energy. All of tower, nacelle, foundation and blades are significant costs. For off-shore transmission is also important. Most of the energy cost is in material production. Transport probably not a major element. Recycling (e.g. of steel in tower and copper in cabling) can be on the plus side. These are energy costs. On the credit side the generator produces energy. The payback period can be found from LCA to identify when the generator has produced enough energy to outweigh the energy costs, including end-of-life disposal. Typically of the order of 6 months to a year.

(c) Embodied energy = $24 \times 150 + 1.9 \times 800 + 3 \times 7 \times 21 = 7430$ GJ

Assume that transport and recycling contributions are negligible (could also assume that steel is recycled) and that the capacity factor is 100%

Payback period = $7430/(10^{-3}\times60\times60\times24\times365) = 0.24$ years

For a realistic capacity factor of e.g. 30% payback period = 0.78 years.

(d) DFIGs operate as variable speed generators over a wide range of wind speeds, thereby allowing the system to always operate at optimal tip-speed ratio and hence maximize the power extracted from the wind.

A fractionally-rated power electronic converter is required, which reduces cost and increases reliability.

DFIGs are able to contribute to the demand for reactive power by altering the phase of the rotor injected voltage.

(e) (i) Phase voltage is $6.6/\surd 3 = 3.81$ kV (star-connected)
From equivalent circuit:

$I=V/((R_1+R_2'/s) + j(X_1+X_2')) = 3810/((0.7+0.6/(-0.02))+j(1.1+1.5)) = 130\angle -175^0$

Synchronous speed $\omega_s = \omega/p = 2\pi f/p = 100\pi/9 = 34.9$ rads^{-1}.
Actual speed $\omega_r = (1-s)\omega_s = (1-(-0.02))\omega_s = 1.03\times 78.5 = 35.6$ rads^{-1}

$T=3I_2'^2R_2'/(s\omega_s) = 3\times 130^2\times 0.6/(-0.02\times 34.9) = -43.6$ kNm

(ii) $P = 3VI\cos\varphi = 3\times 3810\times 130\cos(-175^0) = -1.48$ MW
$Q = 3VI\sin\varphi = 3\times 3810\times 130\sin(-175^0) = -129.5$ kVAr

$P_{loss} = 3I^2(R_1 + R_2') = 3\times 130^2\times(0.7+0.6) = 65.9$ kW

Input mechanical power = Output electrical power + Power losses = 1480 + 65.9 = 1.55 MW
Check: Input mechanical power = $T\omega_r = 43600\times 35.6 = 1.55$ MW

$\eta = P_{out(elec)}/P_{in(mech)} = 1.48/1.55 = 95.5$ %

Examiners' comments

Q6 Discussion, life cycle analysis (LCA) and electrical

(a) and (b) fine. Few problems in (c) with the main issues failure to account for the number of blades and to comment on the likely load factor knocking down the nominal rating. (d) Many candidates knew that the DFIG offers an efficient way of obtaining variable speed operation for the system, and hence maximum power extraction from the wind, but few were able to elaborate on its other advantages. (e) (i) Common mistakes were to forget that the phase voltage is given by the line voltage/√3, and so overestimated the current by √3. Another common error was to forget that the slip is negative for a generator.

Q7 Epicyclic gear

(a) Most candidates were able to correctly use the coefficient of performance in a power equation to obtain the rotor swept area and hence blade radius. With the tip speed ratio given most then straightforwardly obtained the turbine angular velocity and then the torque. (b) Usually well answered although some candidates didn't read the description of turbine/gearbox/generator layout carefully enough. (c) Most candidates wrote down or sketched something sensible for this final part but there were very few complete answers. Some derived the limiting contact force due to bending stress on a tooth, some calculated the kinematics of the epicyclic, but not many were able to do both correctly and then go on to provide sensible tooth numbers and overall gearbox size.

Q8 Vibration analysis

Only attempted by a few. The first part was a discussion about vibration, with some sensible points. Part (b) was meant to be a simple single degree-of-freedom (DOF) question but tripped up the majority, who failed to do a rigorous mechanics analysis and hence tended to be out by a factor of L. Part (b)(i) The better students knew what to do, though did not get the right answer, while the poor students were not able to make much impact on the question. In retrospect the setter of the question was over-optimistic in believing that students would be comfortable with this part of the 1A course.

7)

Given data: Incident wind speed $V_o = 10\,m/s$
Turbine power $P = 2\times10^6\,W$
Tip speed ratio $\lambda = 8$
Coefficient of performance $C_p = 0.4$
Air density $\rho = 1.225\,kg/m^3$

Power $P = \frac{1}{2}\rho A V_o^3 C_p$ where $A = \pi R^2$ is swept area of turbine

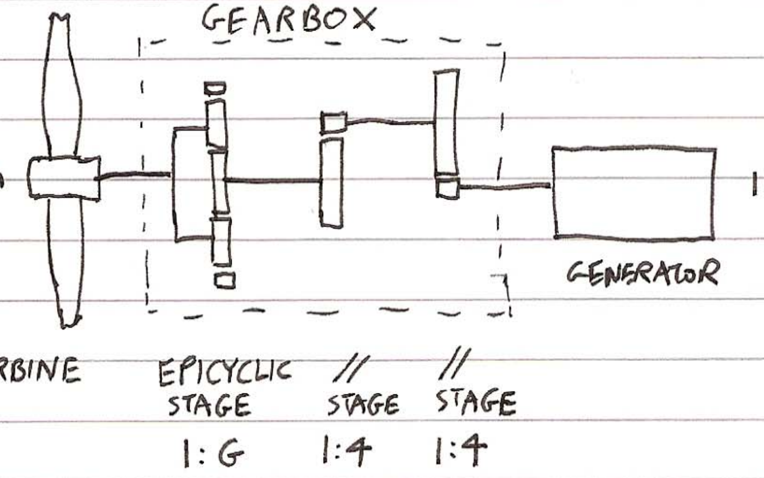
$$\therefore A = \frac{2P}{\rho V_o^3 C_p} = \frac{2\times 2\times10^6}{1.225\times10^3\times0.4} = 8163\,m^2$$

(a) $\therefore$ Blade radius $R = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{8163}{\pi}} = 51.0\,m$

Tip speed ratio $\lambda = \frac{\omega R}{V_o}$ $\therefore$ $\omega_{in} = \frac{\lambda V_o}{R} = \frac{8\times10}{51}$
$= 1.57\,rad/s = 15.0\,rpm$

Power $P = T_{in}\,\omega_{in}$ $\therefore$ $T_{in} = \frac{P}{\omega_{in}} = \frac{2\times10^6}{1.57} = 1.27\times10^6\,Nm$
input torque to gearbox

(b) $\omega_{in} = 15\,rpm$



1500 rpm = ω_{gen}

$$\frac{\omega_{gen}}{\omega_{in}} = \frac{1500}{15} = 100 = G\times4\times4 \quad \therefore \quad G = 6.25$$

Epicyclic gear ratio

7) (cont.)

(b) (cont.)

For epicyclic stage : INPUT speed $\omega_{in} = 15 \text{ rpm} = 1.57 \text{ rad/}$

torque $T_{in} = 1.27 \times 10^6 \text{ Nm}$

Assume no losses $\therefore P_{in} = P_{out}$

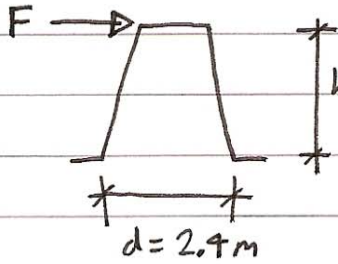
thus : OUTPUT speed $\omega_{out} = 15 \times 6 = 93.75 \text{ rpm}$

$= 9.82 \text{ rad/s}$

torque $T_{out} = \frac{T_{in}}{G} = \frac{1.27 \times 10^6}{6.25}$

$= 204{,}000 \text{ Nm}$

(c) Permitted bending stress in tooth $= \sigma_b = 400 \text{ MPa}$



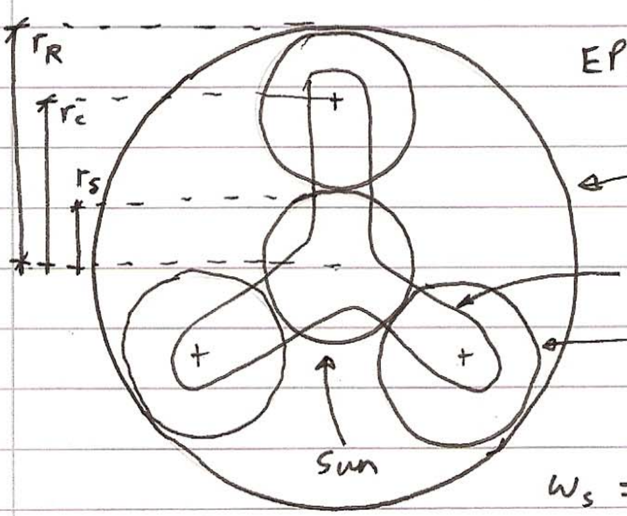
$$\sigma_b = \frac{Fh}{bd^2/6}$$

module $m = 20\text{mm}$ width $b = 50\text{mm}$

$$\therefore F = \frac{\sigma_b b d^2}{6h} = \frac{\sigma_b b (2.4m)^2}{6 \times 2m} = \frac{\sigma_b b m}{2.083}$$

$$= 400 \times 10^6 \times 50 \times 10^{-3} \times 20 \times 10^{-3} / 2.083 = 192{,}000 \text{ N}$$

allowable contact force



EPICYCLIC STAGE

Ring gear, stationary $\therefore \omega_R = 0$

Carrier $\omega_c = \omega_{in}$

Planet

Sun

$\omega_s = \omega_{out}$

7) (cont.)

(c) (cont.)

$r_c - r_s = r_p$ Planet radius

$2r_p + r_s = r_R$ Ring gear radius

$\therefore\ r_p = \frac{1}{2}(r_R - r_s)$ and $r_c = r_p + r_s = \frac{1}{2}(r_R - r_s) + r_s = \frac{1}{2}(r_R + r_s$

Since ring gear stationary $\omega_c r_c = \frac{1}{2}\omega_s r_s$

$\therefore\ \frac{\omega_s}{\omega_c} = \frac{2r_c}{r_s} = \frac{r_R + r_s}{r_s} = 1 + \frac{r_R}{r_s} = 1 + \frac{N_R}{N_s}$ (no. of teeth)

Since $\frac{\omega_s}{\omega_c} = \frac{\omega_{out}}{\omega_{in}} = G = 6.25 \quad \therefore \frac{N_R}{N_s} = 6.25 - 1 = 5.25$

Torque on output sun gear is shared between 3 contacts hence

$$T_{out} = T_s = 3Fr_s$$

$$\therefore\ r_s = \frac{T_{out}}{3F} = \frac{204{,}000}{3 \times 192{,}000} = 0.354\,m$$

and $\therefore$ the number of teeth on the sun gear

$$N_s \geqslant \frac{d_s}{m} = \frac{2r_s}{m} = \frac{2 \times 0.354}{20 \times 10^{-3}} = 35.4$$

hence choose: $N_s = 40$ teeth (sun gear)

$N_R = 5.25\,N_s = 210$ teeth (ring gear)

$N_p = \frac{1}{2}(N_R - N_s) = 85$ teeth (planet gear)

in order to achieve integer tooth numbers

Diameter of gearbox $> 2r_R = 2 \times 5.25\,r_s = 3.72\,m$

$\therefore\ \simeq 4\,m$

$\left(\text{or} \quad = \underset{\nwarrow\ \text{module}}{m}N = 20 \times 10^{-3} \times 210 = 4.2\,m\right)$

⑥

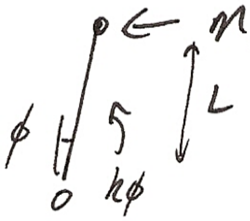
8(a) Driving frequency - 1P or 3P passing frequencies
- need to consider passing through resonance

Sources - unsteady wind loading (turbulence)
- wind shear gives 1P & 3P loading on tower
- self weight (1P loading on blades)
- out-of-balance in mass & pitch.

weight at tower particularly bad for tower mode.

(b)(i)

Neglect gravity

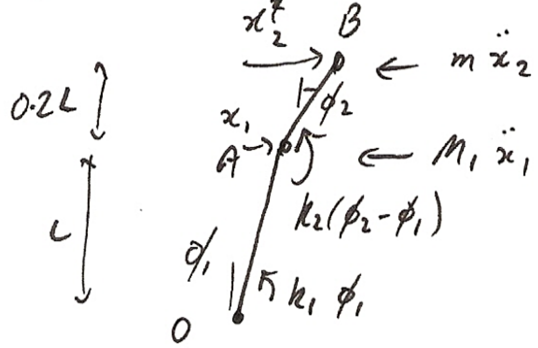


$\leftarrow M_1 L\ddot{\phi}$ (D'Alembert) $M_1 = 0.24M + M_H = 150$ Tonne

Moments about O $\circlearrowleft$ $k\phi + M_1 L^2\ddot{\phi} = 0$

$\Rightarrow$ SHM with $f = \frac{1}{2\pi}\sqrt{\frac{k}{M_1 L^2}} = 0.64\,Hz$

(ii)



$\phi_1 = x_1/L \qquad \phi_2 = \frac{x_2 - x_1}{0.2L}$

$M_O \circlearrowleft \; k_1\frac{x_1}{L} + M_1\ddot{x}_1 L + m\ddot{x}_2\, 1.2L = 0$

M on AB AT A $\circlearrowleft$ $k_2\left(\frac{x_2-x_1}{0.2L} - \frac{x_1}{L}\right) + m\ddot{x}_2\, 0.2L = 0$

In matrix form $\begin{pmatrix} M_1 L & 1.2mL \\ 0 & 0.2mL \end{pmatrix}\begin{pmatrix}\ddot{x}_1 \\ \ddot{x}_2\end{pmatrix} + \begin{pmatrix} k_1/L & 0 \\ -\frac{6k_2}{L} & \frac{5k_2}{L}\end{pmatrix}\begin{pmatrix}x_1 \\ x_2\end{pmatrix} = 0$

In matrix form $\underline{m}\,\ddot{\underline{x}} + \underline{k}\,\underline{x} = 0 \Rightarrow \ddot{\underline{x}} + \underline{m}^{-1}\underline{k}\,\underline{x} = 0$

$\rightarrow$ Harmonic solns given by $|\underline{m}^{-1}\underline{k} - \lambda \underline{I}| = 0$

$\underline{M} = \underset{kg}{10^3} L \begin{pmatrix} 150 & 12 \\ 0 & 2 \end{pmatrix}, \; \underline{m}^{-1} = \frac{1}{10^3 L}\begin{pmatrix} 2 & -12 \\ 0 & 150\end{pmatrix}\frac{1}{300}\;\frac{1}{kg}$

$\underline{k} = \frac{1}{L}\begin{pmatrix} 6\times10^9 & 0 \\ -120\times10^6 & 100\times10^6\end{pmatrix}$ Nm (b)(ii) cont $\underline{m}^{-1}\underline{k} = \frac{1}{50^2 \, 3\times10^5} \begin{pmatrix} 2 & -12 \\ 0 & 150 \end{pmatrix} 10^9 \begin{pmatrix} 6 & 0 \\ -0.12 & 0.1 \end{pmatrix} = \frac{4}{3} \begin{pmatrix} 13.44 & -1.2 \\ -18 & 15 \end{pmatrix} \frac{N}{kg}$ $[s^{-2}]$

$$|m^{-1}k - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 17.92-\lambda & -1.6 \\ -24 & 20-\lambda \end{vmatrix} = 0$$

$$(17.92-\lambda)(20-\lambda) - 24\times1.6 = 0 \Rightarrow \lambda^2 - 37.92\lambda + 320 = 0$$

$$\lambda = \frac{37.92 \pm \sqrt{37.92^2 - 4.320}}{2} = 12.68, 25.26 \ s^{-1}$$

lowest freq $f = \frac{1}{2\pi}\sqrt{\lambda} = 0.57 Hz$ [11% drop due to added mass]

b (iii) At rotational speed of 30 rpm $1P = \frac{30}{60} = 0.5 Hz$

$3P = 1.5 Hz$

So these frequencies are close to 0.57 Hz and likely to be troublesome. Need to control speed carefully and measure/model vibration accurately.

9) (a) $\eta_p = \dfrac{Power\ to\ aircraft}{Mech.\ power\ to\ jet} = \dfrac{V\dot{m}\left(V_j - V\right)}{0.5V\dot{m}\left(V_j^2 - V^2\right)} = \dfrac{2V}{V_j + V}$

$$\eta_{th} = \frac{Mech.\ power\ to\ jet}{Rate\ of\ \ fuel\ energy\ given}$$

The overall efficiency (η_o) is the overall effectiveness of converting fuel energy to <u>useful</u> mechanical energy i.e. $\eta_o = \eta_p \eta_{th}$. [6]

(b)

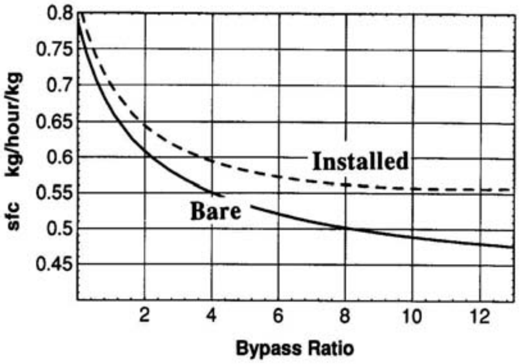
i) $\eta_p = 2V/\left(V_j + V\right)$ and civil aircraft need a high η_p on economic grounds i.e. $V_j \approx V$. To achieve thrust a high mass flow is needed. This leads to high bypass ratio (BPR) engines.

Military aircraft need compact engines that give high speed aircraft i.e. high V_j. Efficiency is a secondary concern. The high V_j. allows a lower mass flow rate through the engine with still high thrust. This leads to the low BPR or even pure jet engine.

ii) For civil aircraft BPR = 5 – 11. For military BPR <=1.

iii) Increased nacelle drag, cost, installation problems, higher landing gear, fan transportation problems, higher fan tip speed, airport facility problems.

iv)



[7]

c)

i) Increasing the turbine inlet temperature makes the core turbine pressure ratio smaller than the core compressor. This increases the power available to the low pressure turbine. It also increases the thermal efficiency.

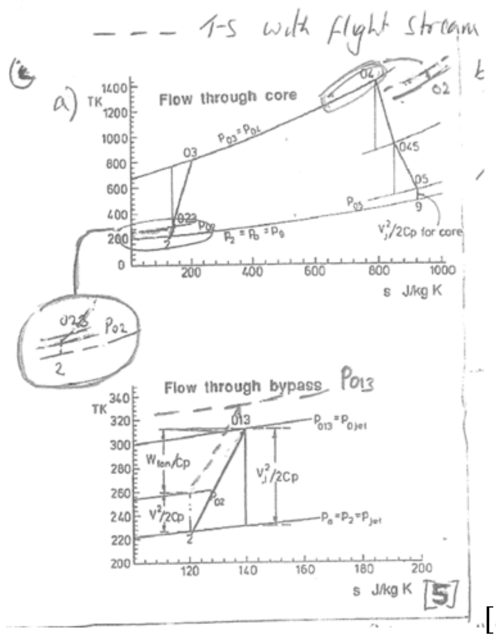
ii) Using cooling air taken from the compressor ultimately released through film cooling holes on the blade surface. Special materials – single crystal blades. Impingement cooling, ribbed passages internal to blades.

iii) At take off – where ambient temperature is relatively high and the engine working hard.

iv) At the top of climb – where the engine is sill working hard to give climb but the ambient temperature lower.

[7]

10)



a) [5]

b) $\eta_c = \dfrac{T_{03s} - T_{023}}{T_{03} - T_{023}}, \quad \dfrac{T_{03s}}{T_{023}} = \left(\dfrac{P_{03}}{P_{023}}\right)^{\frac{\gamma-1}{\gamma}}, \; T_{03} = T_{023}\left\{\dfrac{1}{\eta_c}\left[\left(\dfrac{P_{03}}{P_{023}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] + 1\right\}$

$$T_{03} = 345\left\{\frac{1}{0.9}\left[(25)^{\frac{0.4}{1.4}} - 1\right] + 1\right\} = 923\,K, \; p_{03} = 25p_{023} = 25(170) = 4250\,kPa$$

$$Work_T = Work_C, \; c_p(T_{03} - T_{023}) = c_p(T_{04} - T_{045}), \quad T_{045} = T_{04} + (T_{03} - T_{023}),$$

$$T_{045} = 1650 + (345 - 923) = 1072K$$

$$\eta_T = \frac{T_{04} - T_{045}}{T_{04} - T_{045s}},$$

$$\left(\frac{P_{04}}{P_{045}}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_{04}}{T_{045s}}, T_{045s} = T_{04} - (T_{04} - T_{045})/\eta_t = 1650 - (1650 - 1072)/0.93 = 1028K$$

$$P_{045} = P_{04}\left(\frac{T_{045s}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}}, p_{04} \approx p_{03}, P_{045} = 4250\left(\frac{1003}{1650}\right)^{\frac{1.4}{0.4}} = 812kPa$$ [7]

c) $T_{05s} = T_{045}\left(\dfrac{P_{05}}{P_{045}}\right)^{\frac{\gamma-1}{\gamma}} = 1072\left(\dfrac{140}{812}\right)^{0.286} = 648K, \quad \eta_T = \dfrac{T_{045} - T_{05}}{T_{045} - T_{05s}} = \dfrac{1072 - 680}{1072 - 648} = 0.925$

which is a reasonable value.[3]

d) Isentropic nozzle $T_{05} = T_{09}, \; \text{SFEE} \rightarrow \mathrm{c_p}T_{05} = c_pT_9 + \frac{1}{2}V_{jc}^2$

$$V_{jc} = \sqrt{2c_p(T_{05} - T_9)}, \text{Assuming} \quad T_9 = T_{05}(p_5/p_{05})^{\gamma-1/\gamma}$$

$$V_{jc} = \sqrt{2(1005)(688 - 619)} = 349 \text{ m/s}$$

[1]

e) $F_{net} = \dot{m}\left(V_j - V\right), \dot{m} = \frac{12 \times 10^3}{349} = 34.4 \text{ kg/s}$ [1]

f) $\left(\frac{\dot{m}\sqrt{c_p T_{02}}}{P_{02} A}\right)_{Ground} = \left(\frac{\dot{m}\sqrt{c_p T_{02}}}{P_{02} A}\right)_{Flight}$

$$T_{02} = T_a\left[1 + \frac{(\gamma - 1)}{2} M^2\right] = 216.7\left[1 + (0.2)0.85^2\right] = 248K$$

$$p_{02} = p_a\left[1 + \frac{(\gamma - 1)}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}} = 19.7 \times 10^3\left[1 + (0.2)0.85^2\right]^{3.5} = 31.6kPa$$

$$\dot{m}_{Flight} = \dot{m}_{Ground}\left(\frac{\sqrt{T_{02}}}{P_{02}}\right)_{Ground}\left(\frac{p_{02}}{\sqrt{T_{02}}}\right)_{Flight} = 34.4\,\frac{\sqrt{293}}{101}\,\frac{31.6}{\sqrt{248}} = 11.7 \text{ kg/s}$$

[3]

11) a) $dp = -\rho g dz$, $\rho = p/RT$, $T = T_o - kz$

$$\int \frac{1}{p} dp = \int -\frac{g}{R(T_o - kz)} dz, \quad \ln p = \frac{g}{kR}\left[\ln(T_o - kz)\right] + c \qquad (1)$$

Boundary conditions are at z=0, p = p_o. Hence,

$$c = \ln p_o - \frac{g}{kR} \ln T_o \qquad (2)$$

Combining equations (1) and (2) gives

$$\frac{p}{p_o} = \left(1 - \frac{kz}{T_o}\right)^{g/kR}$$

$$c_1 = g/kR = 9.81/\left(\left(6.5\times10^{-3}\right)287.3\right) = 5.25 \; c_2 = k/T_o = 6.5\times10^{-3}/(288.15) = 2.25\times10^{-5}$$

So $\frac{p}{p_o} = (1 - c_2 z)^{c1} = \left(\frac{T}{T_o}\right)^{c_1}$ [6]

b) $dp = -\rho g dz = -\frac{p}{RT_t} g dz, \quad \int \frac{dp}{p} = -\frac{g}{RT_t}\int dz$, $\ln(p) = -\frac{gz}{RT_t} + c$

Constants of integration are z = z_t, p = p_t.

$c = \ln(p_t) + \frac{gz_t}{RT_t}$, hence $\frac{p}{p_t} = e^{-g/RT_t (z - z_t)}$

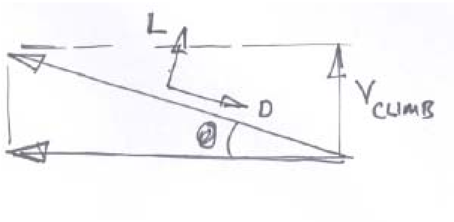
$c_3 = -\frac{g}{RT_t} = \frac{-9.81}{(287.3)(216.65)} = -1.58\times10^{-4}$, $c_4 = z_t = 11\times10^3$

$\frac{p}{p_t} = e^{c_3(z - z_t)}$ [3]

c) $C_L = \frac{L}{0.5\rho V^2 A}, \quad A = \frac{L}{0.5\rho V^2 C_L}, \quad \rho = \frac{p}{RT} = \frac{1.013\times10^5}{(287.3)(288.15)} = 1.22$

$A = \frac{2(500{,}000)9.81}{0.8(1.22)\left(85^2\right)1.6} = 867 \; m^2$ [2]

d)



$\theta = \tan^{-1}\left(\frac{1.6}{256.7}\right) = 0.33^{o}, \quad L = mg\cos\theta, \cos\theta = 0.9999$ hence can ignore V_{climb}. Z = 9.45 km, M = 0.85. $C_L = \frac{L}{0.5\rho V^2 A}$

$$p = 1.013\times10^{5}\left[1-\left(2.25\times10^{-5}\right)9.45\times10^{3}\right]^{5.25} = 0.285\,bar$$

$$T = 288.15-\left(6.5\times10^{-3}\right)z = 226.725K$$

$$\rho = p/RT = 0.285\times10^{5}/\left[(226.725)287.3\right] = 0.437$$

$$V = M\sqrt{\gamma RT} = 0.85\sqrt{1.4(287.3)226.7} = 256.7\,m/s$$

A = 867 m^2 (Part (c))

$$C_L = \frac{L}{0.5\rho V^2 A} = \frac{2\left(4.9\times10^{6}\right)}{0.437(256.7)^2 867} = 0.39$$

[4]

e)

$$F_N - D = mg\sin\theta, \frac{F_N}{mg} = \sin\theta + \frac{D}{L}$$

$$F_N = mg\,(\sin\theta + 0.05) = 4.9\times10^{6}(\sin 0.33 + 0.05) = 0.27\times10^{6}N$$

Ignoring the sinθ term gives F_N = 0.245 x 10^6 N. Hence, thurst over 10% greater with margin for climb included. [2]

f) New weight = (0.55)4.9x10^6 = 7 x 10^6

New C_L = 0.39, T = 216.65K, M = 0.85,

$$V = 0.85\sqrt{1.4(287)216.65} = 250.8 \text{ m/s}$$

$$C_L = \frac{L}{0.5\rho V^2 A} \rightarrow \rho = \frac{2L}{C_L V^2 A} = \frac{2\left(2.7\times10^{6}\right)}{0.39\left(250.8^2\right)867} = 0.25$$

$$p = \rho RT = 0.25(287.3)(216.65) = 0.151 \text{ bar}$$

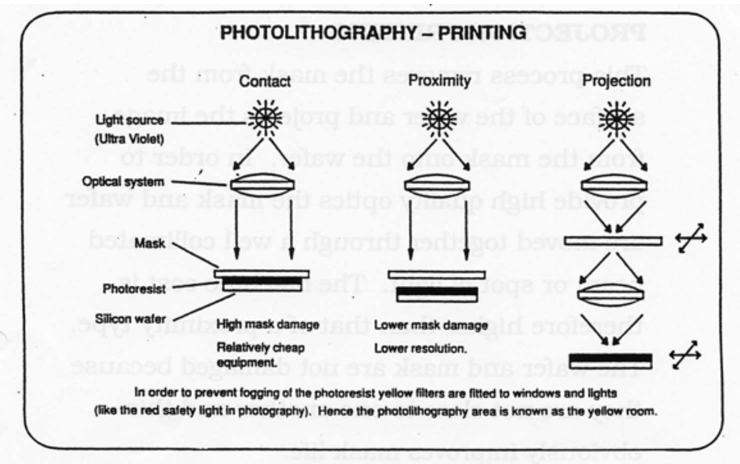
$$\ln\left(\frac{p}{p_t}\right) = c_3(z-z_t), z = \frac{1}{c_3}\ln\left(\frac{p}{p_t}\right) + z_t = \frac{-1}{1.58\times10^{-4}}\ln\frac{\left(0.151\times10^{5}\right)}{\left(0.226\times10^{5}\right)} + 11\times10^{3} = 13.5 \text{ km}$$

[3]

Pt IB Paper 8 20ll – Cribs

12 (a) Class 1 Clean Room – should not contain more than 1 particle of 0.5 micron diameter (or larger) in 1 cubic foot of air. (Some books say 0.2 micron). (2)

(b)



In order to prevent fogging of the photoresist yellow filters are fitted to windows and lights (like the red safety light in photography). Hence the photolithography area is known as the yellow room.

Proximity/Contact Printing is characterised by getting the mask as close as possible to the surface of the wafer to minimise errors due to dispersion of the light beam and vibration.

In some cases the mask may actually touch the wafer (contact printing). Such machines are relatively cheap but both wafer and mask will suffer some mechanical damage during exposure.

Projection Printing

This process removes the mask from the surface of the wafer and projects the image from the mask onto the wafer. In order to provide high quality optics the mask and wafer are moved together through a well collimated beam or spot of light. The machine cost is therefore higher than that of a proximity type. The wafer and mask are not damaged because they are not close to one another and this obviously improves the mask life.

For the 0.2 micron required, a projection process is best. (6)

(c) The optical elements in most modern projection printers are so perfect that their imaging characteristics are dominated by diffraction effects rather than by lens aberration – diffraction limited systems. The resolution of a diffraction limited printer is roughly 0.5 (λ/NA) where NA is the numerical aperture of the projection optics and λ is the wavelength. The depth of focus of the system is approx. given by $\frac{\lambda}{(\mathrm{NA})^2}$, i.e. a high resolution (large NA) is achieved at the expense of depth of focus.

$$\text{Resolution} \sim 0.5\left(\frac{\lambda}{NA}\right) \quad \therefore \text{ for 200 nm source}$$

$$.2 \text{ micron} \sim \frac{.5\times 200\times 10^{-9}}{NA}$$

$$\text{NA} \sim \frac{.5\times 200\times 10^{-3}}{.2}$$

$$\text{NA} \sim 500 \times 10^{-3}$$

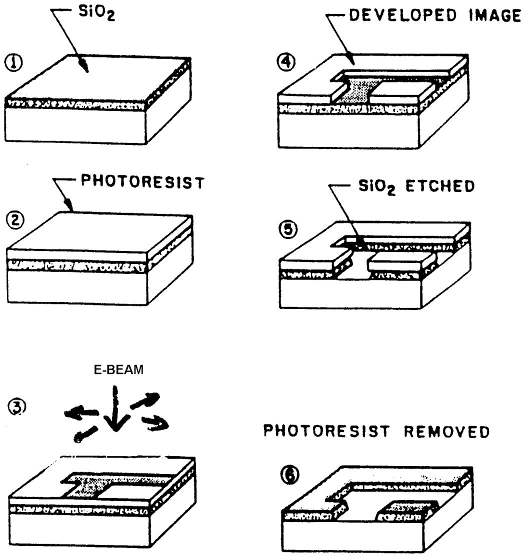
$$\text{Depth of focus} \sim \frac{\lambda}{(NA)^2} = \frac{200\times 10^{-9}}{(.5\times .5)}$$

$$= 0.8 \text{ micron}$$

∴ Depth of focus is insufficient to accommodate variation in wafer flatness, ∴ would have to use a step and repeat process. (5)

(d) Electron beam writing

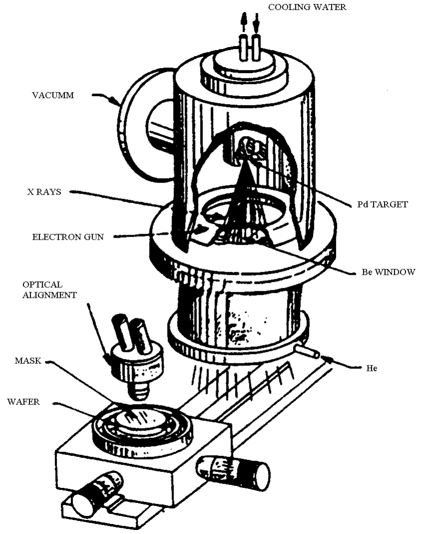
Electron beam direct writing of a wafer is a technique where the photoresist is exposed by a beam of electrons that is steered to produce the pattern needed. No mask is used. Consequently the machine is very versatile and multiple designs can be mixed on one wafer. Used for fast turn round of new designs etc. **but** no good for mass production.



Details of the lithographic transfer process using the e-beam.

X-Ray Lithography

This is an extension of optical proximity printing in which the exposing wavelength is in the 4-50A range. Because x-ray optical elements are not yet available, x-ray lithography is limited to shadow printing. System is as shown below.



The mask and wafer are separated by about 40 microns. The full wafer is exposed in about 1 minute. The primary reason for developing x-ray lithography is the possibility of achieving high resolution and high through-put at the same time. Also since x-rays are not absorbed by dirt with low atomic number, dirt on the mask does not print as a defective pattern on the resist. Masks however are still a problem. (4)

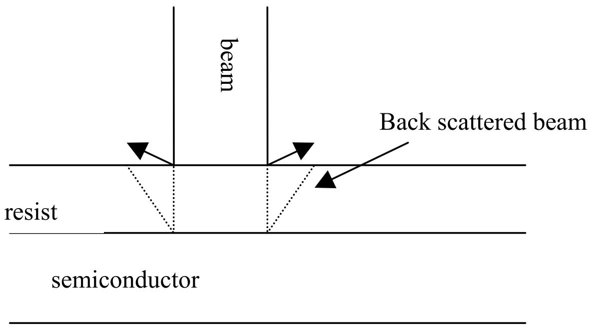
(e) e-beam

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.625\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.6\times10^{-19}\times40\times10^{13}}}$$

$$\lambda = \sim 6\times10^{-12}m$$

However back scattering will limit resolution. (3)

e.g.



13 (a) $t = L/v$, $v= \mu E$, so $t = L/\mu E$ (3)

(b) no field at bottom of layer, using Gauss' law, displacement at top of layer under gate electrode is

$D = N.e.x$

$E = Nex/\varepsilon$

$V = \int E.dx = -\int N.e.x.dx/\varepsilon = \frac{1}{2} Ne.d^2/ \varepsilon$ (6)

(c) scaling means $L' = L/k$, $E' = E$, $V' = V/k$ where k is factor of ~2 per 2 years.

$t/t' = L/L'$, transit time $= 10^{-7}.65/10^4 =$ 650 ps.

This assumes mobility is same, as it is occurring at the same electric field.

(i) $V' = V/k = 65/10^4 = 6.5$ mV. This shows that voltage scaling did not occur. (3)

(ii) from $E = N.e.x./\varepsilon$, if E = constant, x scales as $x' = x/k$, then N scales as $N' = N.k$. (4)

$N= (10^4/65).10^{20} = 1.6\text{x}10^{22}\ \text{m}^{-3}$.

(iii) Lithography is the standard limit to further scaling. Materials are the main practical limit, as making the devices with scaled quantities is not easy. (4)

14 (a) evidence = electron diffraction, electron microscopy, as expressed as DeBroglie waves and his wavelength. Quantum theory of atom, electronic orbitals, bonding, etc. (4)

(b) Schrödinger equation is

$$(-\frac{\hbar^2}{2m}\nabla^2 + V)\psi = E\psi$$

for a trial wave function of $\psi=\exp(i\beta x)$, we get $E = \hbar^2 \beta^2/2m^*$. Using $k=\beta=2\pi/\lambda$, this gives $E = h^2/2m\lambda^2$ (3)

(c) From (a), taking $E = 0$, $J = J_0 \exp(-2\beta.d) = J_0 \exp(-2[2.m^*V/\hbar^2]^{1/2} d)$, where $\beta = 2.Vm^*/\hbar^2$ (3)

(d) For SiO_2, $V = 3.5$ eV and $m^* = 0.5$, so

$\beta = (2 \times 0.5 \times 0.9.10^{-30} \times 3.5 \times 1.6.10^{-19})^{1/2} / (1.1\ 10^{-34}) = 6.45.10^{9}\ m^{-1}$

As $d = 1.5\ 10^{-9}$ m, so $J = 2.10^{13}$. $\exp(-2 \times 6.45 \times 1.5) =$ **7.9×10^4 A/m^2**. (3)

(e) $C = \dfrac{\varepsilon_0 KA}{d}$ or $\dfrac{C}{A} = \dfrac{\varepsilon_0 K}{d}$

hence $d_{HfO2} = d_{SiO2}.(K/3.9) = 1.5\ (22/3.9)$

or $d = 8.46$ nm (3)

(f) for HfO_2, $V = 1{,}5$ eV, $m^* = 0.7$,

$\beta = (2 \times 0.7 \times 0.9.10^{-30}\ \ 1.5 \times 1.6.10^{-19})^{1/2} / 1.1.10^{-34} = 5.5.10^{-25}/1.1.10^{-34} = 5.10^{9}\ m^{-1}$

so, $2\beta d = 2 \times 8.46 \times 5 = 84.6$

$J = 2\text{x}10^{13} \times 1.8\ .10^{-37} = 3.6\text{x}10^{-24}$ A/m2.

This is many orders of magnitude smaller than for the SiO2 case, showing why the replacement is made.

(4)

Thus the term current varies with the factor $(V'm^{*}')^{1/2}K$. To minimise J, maximise this term, that is maximise V', m*' and K of the new oxide.

PtIB Paper 8 2011

Numerical Answers

12 (c) 0.5, 0.8 micron

(e) ~ 6×10^{-12}m

13 (c) (i) 650 ps

(ii) 6.5mV, ~1.6×10^{22}m^{-3}

14 (d) ~7.85×10^{4}A/m^{2}

(e) 8.46 nm

(f) ~3.6×10^{-24}A/m^{2}

Part IB Paper 8 – Information Engineering

15. (a) (i) Functions which move pixels around include;

Resizing the image
Rotating the image
Morphing the image

All use interpolation to achieve sub-pixel moves.

(ii) Functions which map pixels include:

Conversion from one colour space to another
Colour correction (eg for tungsten lighting)
Lighting correction (eg poor foreground illumination)
– can use a user-defined mapping or histogram equalisation.

(iii) Functions which perform filtering include:

Smoothing / Denoising by lowpass filtering;
Edge ~~enhancement~~ sharpening / De-blurring by highpass filtering;
Adaptive enhancement by edge-adaptive filtering (lowpass or highpass).

15 (b) Bi-linear interpolation estimates the intensity of the image at an arbitrary point $x_{p,q}$ by a linear combination of the image intensities at the four pixels, nearest to (p,q), which are

$$\begin{array}{cc} x_{a,c} & x_{a,d} \\ x_{b,c} & x_{b,d} \end{array} \quad \text{where} \quad \begin{array}{c} a \leq p \leq b \\ c \leq q \leq d \end{array}$$

In 1-D: $$x_p = \frac{(b-p)x_a + (p-a)x_b}{(b-a)}$$

So in 2-D:

$$x_{p,q} = \frac{(d-q)\left[(b-p)x_{a,c} + (p-a)x_{b,c}\right] + (q-c)\left[(b-p)x_{a,d} + (p-a)x_{b,d}\right]}{(b-a)(d-c)}$$

Interpolation is required by functions which move pixels around, when the location, from which a given pixel in the output image comes, is not at an exact pixel position in the input image. For example, image rotation requires pixels to be moved by varying amounts, according to how far they are from the centre of rotation.

15(c) The RGB space stores the amplitudes of the red, green and blue components of each pixel separately in the 3 image planes. Changes to the brightness (luminance) of a pixel require all 3 RGB components to be modified.

In YUV space, the luminance is the Y component and U & V represent colour difference components ($U \propto (B-Y)$ and $V \propto (R-Y)$). Hence the luminance may be changed (or just measured) using only the Y component. By scaling the U & V components up or down, it is possible to adjust the strength (saturation) of the colours. However if Y is varied, the colour saturation will also vary unless U & V are scaled by the same amount as Y, since the relationships are linear.

To avoid this drawback, we can work in HSV space, where V is the value (approximate luminance), S is the colour saturation, and H is the hue. This is a non-linear mapping which gives independent control of the 3 main aspects of luminance and colour, but it is complicated to convert back to RGB from HSV.

15(d) Lowpass filtering is used to apply smoothing to image pixel data. This has the desirable effect of reducing noise, although it has the undesirable effect of blurring sharp edges and fine detail.

Highpass filtering gives greater gain to high frequencies and tends to enhance edges and fine detail, although it also amplifies any noise. If the gain of the filter tends to zero at low frequencies, then it can be used as an edge/detail detector, whereas if the gain tends to one at low frequencies, ~~the~~ it acts as an edge enhancement process.

Spatially adaptive filters may be designed to provide lowpass filtering (and hence denoising) in regions of the image that are not close to edges or fine detail, where the noise would be most noticable, and to provide edge enhancement close to edges or detail. It is even possible to perform lowpass filtering in a direction parallel to an edge and highpass in the normal direction, all at the same location to achieve denoising and edge ~~enhance~~ sharpening in the same place.

~~#~~15(d) (cont.)

A 1-D lowpass filter has an impulse response

$$h(x) = K e^{-x^2/2\sigma^2}$$

This may be applied to the rows of an image.

A similar filter in the vertical direction is

$$h(y) = K e^{-y^2/2\sigma^2}$$

The 2-D impulse response, after applying $h(x)$ to the rows of an image, and $h(y)$ to the columns of the result is

$$h(x,y) = K^2 e^{-x^2/2\sigma^2} \cdot e^{-y^2/2\sigma^2}$$

$$= K^2 e^{-(x^2+y^2)/2\sigma^2} = K^2 e^{-r^2/2\sigma^2}$$

where $r = \sqrt{x^2+y^2}$

Hence the filter is isotropic about the origin as its response depends only on r, the radius.

This gives an isotropic lowpass filter.

To get an isotropic highpass filter, we simply subtract the lowpass filtered image from the original image. Note that separate highpass filters in x & y will not give an isotropic overall response, as the lowpass Gaussian function is the only function that has this property.

Q16

a) (i) Smoothing ≡ low pass filtering needed

— reduce effect of additive noise before differentiation

— select spatial frequency of objects of interest (large σ means large objects).

(ii) larger σ = lower low-pass cut-off frequency for filter.

(iii) 2D Gaussian is used $G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp - \left\{ {x^2+y^2}/{2\sigma^2} \right\}$

(b). $$S(x,y) = \sum_{-n}^{n}\sum_{-n}^{n} G_\sigma(i,j)\, I(x-i, y-j)$$

$$= \sum_{-n}^{n}\sum_{-n}^{n} g_\sigma(i)\, g_\sigma(j)\, I(x-i, y-i)$$

where $g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$

$$S(x,y) = g_\sigma(x) * g_\sigma(y) * I(x,y)$$

Q16 (cont)

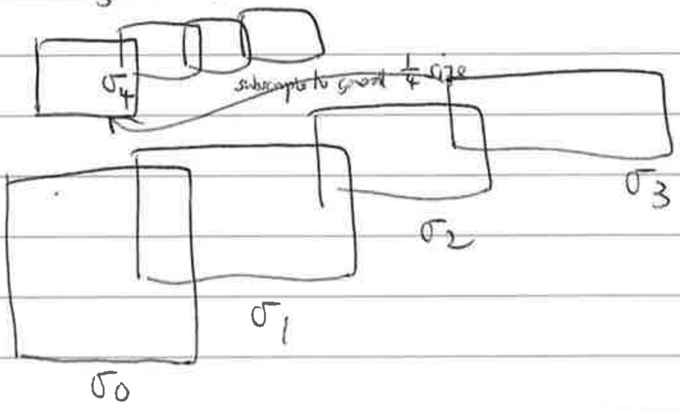
(c) Scale-space representation of image $L(x,y,t) = G(x,y,t) * I(x,y)$

Sample scale-space for Gaussian Image Pyramid.

Within each octave convolve with $G(\sigma_i)$ where $\sigma_i = 2^{\frac{i}{s}} \sigma_0$
so that σ doubles after s convolutions
ie. blur for σ to 2σ.

The last Image is then sub-sampled to generate first image of new octave.

eg. $s = 3$ $\sigma_3 = 2\sigma_0$



All blurs are implemented as 1D blurs + incrementally to avoid large σ.

(d) Band-pass filtering is implemented as Laplacian of Gaussian and approximated as difference of gaussians.
Images as neighbours in octave are subtracted

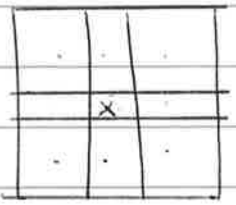
$$\nabla^2 G_\sigma * I(x,y) \simeq G_{k\sigma} * I(x,y) - G_\sigma * I(x,y)$$

$$\simeq G_{k\sigma}(x,y) * I(x,y) - G_\sigma(x,y) * I(x,y)$$

Blob-like features are labelled as max/min of Laplacian of gaussian response.

Q16 (e) Localize scale as max/min of $\nabla^2 G * I(x,y)$ response in (x,y,σ)

Check 9+9+8 neighbour value to see if max/min.



Location is (x,y) pixel.

Scale is σ in pyramid

Feature is encoded as 16x16 pixels at appropriate scale.

(f) Matching of interest pts in image mosaicing / 2D pattern recognition

Q17.

Part 1B paper 8 2011 Solutions

January 27, 2011

(a) Let $c = 1/\sqrt{2\pi}$

$$p(x_1|M_1) = \frac{1}{\sqrt{2\pi}} \exp\{-1/2\} = \frac{c}{\sqrt{e}}$$

$$p(x_2|M_1) = \frac{1}{\sqrt{2\pi}} \exp\{-9/2\} = ce^{-4.5}$$

$$p(x_1|M_2) = \frac{1}{\sqrt{2\pi}\sqrt{2}} \exp\{-0\} = \frac{c}{\sqrt{2}}$$

$$p(x_2|M_2) = \frac{1}{\sqrt{2\pi}\sqrt{2}} \exp\{-16/4\} = \frac{c}{\sqrt{2}} e^{-4}$$

Using this we see that $p(x_1|M_2) > p(x_1|M_1)$ and $p(x_2|M_2) > p(x_2|M_1)$, so both x_1 and x_2 are more probable under model M_2.

(b) The maximum likelihood Gaussian assuming iid data x_1 and x_2 has a mean of -1 and a variance of 4. This should be derived by writing down the log likelihood and maximising as a function of μ and σ.

(c) Using the results from (a), Bayes rule, and this equal prior we can show the following:

$$\begin{aligned} p(M_2|x_1, x_2) &= \frac{p(x_1, x_2|M_2)P(M_2)}{p(x_1, x_2|M_1)P(M_1) + p(x_1, x_2|M_2)P(M_2)} \\ &= \frac{p(x_1|M_2)p(x_2|M_2)}{p(x_1|M_1)p(x_2|M_1) + p(x_1|M_2)p(x_2|M_2)} \\ &= \frac{c/\sqrt{2}\, c/\sqrt{2}\, e^{-4}}{c/\sqrt{e}\, ce^{-4.5} + c/\sqrt{2}\, c/\sqrt{2}\, e^{-4}} \\ &= \frac{e^{-4}/2}{e^{-5} + e^{-4}/2} \\ &= \frac{e}{2+e} \end{aligned}$$

ENGINEERING TRIPOS   PART IB

XX 2011   xx to xx

Paper 8 Section G

Answer: CRIB

ENGINEERING FOR THE LIFE SCIENCES

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Write short notes on:

(i) intracellular vs. extracellular neuronal recordings;

Answer: Intracellular: Neuron cell bodies vary in size from 4 to 100 μm and to record from inside the cell a glass microelectrode is produced by an“electrode puller” in which you heat up the middle of a 1 mm glass tube and pulls ends apart at high velocity. This results in an electrodes with a very fine tip down to 0.1 μm. The electrodes are then filled with electrolytes and placed inside a cell body. This allows the membrane potential as well as spiking activity to be recorded. Extracellullar: Glass electrode or metal shaft electrodes with tip diameter of 3-10 μm are placed close to, but outside, of neurons. This allows only the spiking activity (and local field potential) to be recorded but one electrode can potentially record several different neurons at the same time. Compared to intracellular recording less stability is required.

(ii) optical neuronal imaging;

Answer: In optical imaging, a voltage-sensitive dye is (usually) used on the surface of an exposed region of the brain which changes its reflectance properties with electrical activity. Using special illumination and high-speed cameras it is possible to record the activity of a population of neurons.

(iii) implants in the visual cortex vs. implants in the retina.

Answer: Both implants tend to take a camera based input to directly activate neurons. Implanting electrodes in the visual cortex can be used to directly stimulate the neurons. However, coding in the visual cortex is complex and there is no simple mapping of features in the visual space onto features in the visual cortex making the encoding hard. In contrast by stimulating in the retina there is a simpler topographical mapping and therefore damage to photoreceptors is a more promising avenue.

[25%]

(b) In the context of the visual system:

(i) Describe briefly the receptive fields of the different cell types from retina to primary visual cortex.

Answer: Photoreceptors respond to spots of light. Ganglion cells have roughly circular receptive fields from a few minutes of arc at fovea, to few degrees at periphery. On-centre ganglion cells have an excitatory receptive field centre and inhibitory surround thereby respond optimally to differential illumination. Off-centre ganglion cells have an inhibitory receptive field centre and excitatory surround. There are also colour opponent cells which are

(cont.

either red- green or blue-yellow. Lateral geniculate nucleus neurons are similar to ganglion cells. In the visual cortex simple cells respond to either a line or edge detector in a particular location and complex cell respond to a line with a particular orientation moving in a particular direction.

(ii) Describe the spatial frequency theory of vision and evidence that supports the theory.

Answer: The theory proposes that the two-dimensional luminance of an image is represented as a combination of elements such as sinusoidal gratings of different spatial frequencies, amplitudes, phases and orientations. Therefore the neurons are effectively doing a two-dimensional Fourier analysis. Importantly the theory suggests that the different components are processed by separate channels. Evidence for this comes from fatiguing one particular frequency leads to a dominant perception of the other spatial frequencies such as in the spatial frequency aftereffect. In addition the processing of each frequency is independent of others and so if two different frequencies sinusoids are combined the detectability of the stimulus is independent of the phase of the combination. In addition, neurophysiological evidence from the cells in visual cortex show receptive field properties which are wavelet like (sinusoid windowed by a Gaussian) of varying spatial frequencies.

(iii) Describe the role of Bayesian processing in the perception of the speed and direction of object motion.

Answer: Bayes rule states that the posterior is proportional to the prior times the likelihood. As motion is detected by small receptive fields, due to the aperture problem the motion is ambiguous and the likelihood means many (or a set of) motions consistent with the sensory input. However, the prior over possible movement speeds is peaked at zero and can be approximated a Gaussian with a zero mean. Therefore the posterior tends to be biased towards inferences with slower speeds leading to illusory slower motion and even misperception of direction especially at low contrast when the prior will dominate.

(iv) What is colour constancy and how might the brain achieve it?

Answer: The frequency of light emanating from an object depends on two features. First, it depends on the illumination spectral power and second on the reflectance function of the surface. Colour constancy refers to the ability of the visual system to discount the illuminant and extract the reflectance function which is integral feature of the object. One mechanism which has been proposed is based on a double colour opponents cells which would have, for example, centre which is excitatory for green and excitatory for red with a surround that is inhibitory for red and excitatory for green.

[75%]

(TURN OVER

2 This question regards the sensitivity of the retina to single photons.

(a) Let $\mathrm{P}(n|\bar{n})$ denote the (Poisson) probability of n photons hitting the retina for a flash of light that on average results in $\bar{n}$ photons at the retina, and let K denote the minimum number of photons that need to reach the retina for us to see (or, more precisely, to report seeing). Using these quantities, provide the formula for $\mathrm{P}_{\mathrm{see}}$, the probability of seeing given a mean photon count at the retina $\bar{n}$.

Answer: $\mathrm{P}_{\mathrm{see}} = \sum_{n=K}^{\infty} \mathrm{P}(n|\bar{n})$ [10%]

(b) In practice, we usually know the intensity of the light emitted by a light source, denoted by I, but not the average number of photons that eventually reach the retina from it, $\bar{n}$. Nevertheless, there is a simple relationship between the two, up to some unknown constant(s): what is it?

Answer: It is a linear relationship: $\bar{n} = \alpha I$, where α is an unknown constant. [5%]

(c) In an experiment, we measure the fraction of trials in which subjects report seeing a brief flash of light, $\mathrm{P}_{\mathrm{see}}$, while we systematically vary the light intensity of the flash, I. Describe the qualitative aspects of the curve plotting $\mathrm{P}_{\mathrm{see}}$ against $\log I$, and explain how we can estimate K from it (as defined in (a)) especially given the problem with relating light source instensity to mean photon count at the retina (as described in (b))?

Answer: $\mathrm{P}_{\mathrm{see}}$ is a monotonically increasing, sigmoidal function of $\log I$ with $\mathrm{P}_{\mathrm{see}} \to 0$ with $\log I \to -\infty$ and $\mathrm{P}_{\mathrm{see}} \to 1$ with $\log I \to +\infty$. The shape of the curve (the slope of its middle portion) depends crucially on the threshold photon count K, but changing the unknown constant α just translates the curve along the x-axis. Thus, measuring the slope should give us information about K. [15%]

(d) In an experiment with human subjects it was indeed found that subjects responded as if they had a hard threshold, K, for reporting that they saw the stimulus. That is, the data suggest each subject always reported seeing the stimulus whenever at least K photons reached their retina, and they never reported seeing it otherwise. K was estimated to be around 5. Explain why this implies that individual photoreceptor cells generate reliable responses to single photons, and that summation of these responses along the visual pathway must be near ideal.

Answer: Since $K = 5$ is several orders of magnitude smaller than the number of (rod) photoreceptor cells in the retina, there is a negligible probability of double hits, ie. each photon is received by a

(cont.

different photoreceptor, or conversely each photoreceptor receives at most 1 photon. If either the individual photoreceptors were unreliably responding to single photons or their responses were summed up unreliably then we would see a soft rather than a hard threshold at $K = 5$, because sometimes 5 photons would be counted as more or less, and so the data could not be fitted with the hard threshold model. [25%]

(e) Does the activation of a receptor cell mean that a photon was received by it? Explain what the answer to this question implies about the optimal setting of K, defined as the minimal number of receptor activations above which seeing the flash should be reported, if one's goal is to maximise the number of correct answers in an experiment in which light is only flashed on some of the trials.

Answer: A receptor cell may also become active in the absence of photons ("dark noise"), so receptor cell activation does not necessarily mean that a photon was received by it. The level of dark noise determines the optimal setting of K to maximise the number of correct responses, because K must be high enough so that one doesn't get too many false positives due to spontaneous activation of receptor cells (and it also needs to be low enough so that one doesn't miss too many trials in which there was a flash.) [25%]

(f) The same subjects who participated in the experiment described in (d), also participated in another experiment that used the same light source. The new experiment also found subjects used a hard threshold, but in this case it was estimated to be at $K = 2$ rather than $K = 5$. Assuming that both experiments, including the analysis of data, were properly conducted, and that in both experiments subjects behaved optimally given responses in their retina, discuss potential reasons that could account for this discrepancy in K but also for the fact that a hard threshold was found in both experiments.

Answer: Since a hard threshold was found in both experiments, the responses of individual retinal photoreceptors to single photons and the summation of these responses had to be highly reliable in both cases. That is, if a photon reached a photoreceptor it reliably responded. The $K = 2$ result of the new experiment indicates that subjects can sense the activation of as much as 2 photoreceptors. However, in order to achieve optimal performance subjects may choose to use a higher threshold for reporting seeing the stimulus as the optimal threshold also depends on other factors that may have differed between the two experiments: the fraction of trials on which a light is flashed (the higher it is the lower the optimal K is), and the magnitude of rewards for guessing correctly that the light was flashed (hit) or not (correct rejection) in a trial (the more hits are rewarded relative to correct rejections the lower the optimal K is). [30%]

3 (a) What affects the imaging depth and resolutions in each dimension which can be achieved when imaging the eye using Optical Coherence Tomography (OCT)? Give approximate values for each. What are the advantages of spectral over time-domain OCT?

Answer: Optical Coherence Tomography (OCT) imaging depth is determined by the absorption coefficient for light, which varies with incident wavelength and typically allows imaging to 3 mm. In spectral OCT the depth is also limited by the frequency resolution of the CCD array used to record the intensity of the interference spectra.

Resolution is different in the lateral and the axial directions. In the lateral direction it is dominated by the lens power or numerical aperture and also the achievable spot size of the laser on the fundus. In an ideal situation the resolution could be up to 2 μm, but imaging through the lens of the eye introduces aberrations which increase this to closer to 15 μm.

The axial resolution is dominated by the shape of the laser pulse, and in particular the bandwidth of the laser. The higher the bandwidth, the shorter the coherence length and hence the better the resolution. The best OCT systems can achieve resolutions of better than 5 μm.

In time-domain OCT, only a single point is imaged at any one time, whereas in spectral OCT a set of points over a range of depths are imaged using a single laser pulse (or in more complex systems a small number of laser pulses). This can dramatically reduce the time taken to acquire an image of the fundus, which in turn means it is much more likely that the eye will have remained still, reducing motion blur. It also reduces the light intensity required for imaging. [20%]

(b) Given an ideal OCT system imaging in air, with no dispersion, where:

$E(\omega) = E_0(\omega)\, e^{j\omega t}$ is the laser pulse
$S_0(\omega) = |E_0(\omega)|^2$ is the input intensity spectrum
$r_s(l)$ is the amplitude reflectivity density function with depth l
l_s, l_m are round-trip distances to the scatterers s and mirror m
c is the speed of sound in all media

show that the intensity at the output of the interferometer is given by:

$$I(\omega) = S_0(\omega) + S_0(\omega)\left|\int_{-\infty}^{\infty} r_s(l_s)\, e^{j\frac{\omega}{c}l_s}\, \mathrm{d}l_s\right|^2 + 2\Re\left\{S_0(\omega)\int_{-\infty}^{\infty} r_s(l_s)\, e^{j\frac{\omega}{c}(l_m-l_s)}\, \mathrm{d}l_s\right\}$$

Answer: The electric field reflected from the (perfect) mirror $E_m(\omega)$ at the interferometer output is the input field delayed by the round-trip path length to the mirror:

$$E_m(\omega) = E_0(\omega)\, e^{j\left(\frac{\omega}{c}l_m - \omega t\right)}$$

Similarly, the electric field from *one* scatterer at l_s in the sample is this field multiplied by the scatterer reflectivity at that location $r_s(l_s)\mathrm{d}l_s$:

$$E_s(\omega) = r_s(l_s)\mathrm{d}l_s\, E_0(\omega)\, e^{j\left(\frac{\omega}{c}l_s - \omega t\right)}$$

(cont.

Hence the total electric field from all scatterers is this term integrated over depth:

$$\begin{aligned} E_s(\omega) &= E_0(\omega) \int_{-\infty}^{\infty} r_s(l_s)\, e^{j\left(\frac{\omega}{c} l_s - \omega t\right)}\, \mathrm{d}l_s \\ &= \left[E_0(\omega) \int_{-\infty}^{\infty} r_s(l_s)\, e^{j\frac{\omega}{c} l_s}\, \mathrm{d}l_s \right] e^{-j\omega t} \end{aligned}$$

The output of the interferometer is the sum of E_m and E_s. However, the photo diode responds to light *intensity* $I(\omega)$:

$$\begin{aligned} I(\omega) &= |E_m + E_s|^2 \\ &= |E_m|^2 + |E_s|^2 + 2\Re\{E_m E_s^*\} \\ &= |E_0(\omega)|^2 + \left| E_0(\omega) \int_{-\infty}^{\infty} r_s(l_s)\, e^{j\frac{\omega}{c} l_s}\, \mathrm{d}l_s \right|^2 + 2\Re\{E_m E_s^*\} \\ &= S_0(\omega) + S_0(\omega) \left| \int_{-\infty}^{\infty} r_s(l_s)\, e^{j\frac{\omega}{c} l_s}\, \mathrm{d}l_s \right|^2 + 2\Re\left\{ S_0(\omega) \int_{-\infty}^{\infty} r_s(l_s)\, e^{j\frac{\omega}{c}(l_m - l_s)}\, \mathrm{d}l_s \right\} \end{aligned}$$

which is the equation we are asked to prove. The first term is the power spectral density of the reflected signal, the second of the scattered signal, and the third is the interference term of interest. [30%]

(c) With reference to the equation in (b), explain how the amplitude reflectivity density function $r_s(l)$ can be recovered from the intensity $I(\omega)$ in the following systems:

(i) Time-domain OCT.

Answer: In time-domain OCT, r_s is estimated at a single depth l_s at a time, by slowly (in comparison to the speed of light) varying the reference mirror location l_m. This variation causes the third term in the equation above to oscillate with l_m, and hence exhibit an AC signal with time. Both the other terms in the equation are constant with time.

Hence the third term can be selected by high pass filtering the received intensity at a single photodiode. The *amplitude* of the third term is then recovered by rectifying this signal and low-pass filtering the result. The photodiode responds to light at all frequencies, effectively integrating I over ω. Hence the amplitude term I_{AC} (with some re-arrangement) is:

$$I_{AC}(\omega) = 2\Re\left\{ \int_{-\infty}^{\infty} r_s(l_s)\, e^{-j\frac{\omega_0}{c} l_s} \int_{-\infty}^{\infty} S_0(\omega_b)\, e^{-j\frac{\omega_b}{c}(l_m - l_s)}\, \mathrm{d}\omega_b\, \mathrm{d}l_s \right\}$$

Using the Wiener-Khinchin theorem, the inner integral can be represented as an auto-correlation function located at $l_s = l_m$, so this signal returns the integrated reflection r_s around the mirror location l_m. [25%]

(ii) Spectral OCT.

Answer: In spectral OCT, $I(\omega)$ is measured directly by using a diffraction grating to spread the intensity spectrum over a linear CCD array. The first and second terms can be removed by calibration of the system. The first term is the spectral power of the source, and can be measured by blocking the sample arm of the interferometer. The second term is the spectral power of the reflection from the sample alone and can be measured by blocking the reference mirror arm of the interferometer.

(TURN OVER for continuation of Question 3

This leaves the third term which is measured directly. The integral can be expressed as a Fourier Transform in $\frac{l_s-l_m}{c}$ of r_s, which means we can recover l_s by an inverse Fourier Transform of the received (calibrated) spectrum, having divided this by the source spectrum S_0:

$$r_s(\tfrac{l_s-l_m}{c}) = \mathscr{F}^{-1}\left\{\frac{I_{\text{cal}}(\omega)}{S_0(\omega)}\right\}$$

[25%]

END OF PAPER

2011 IB Paper 8, Section H

21 (a) Explain, illustrating your answer with examples, what is meant by:

(i) Radical innovation;

(ii) Incremental innovation.

[4]

(b) Describe the differences in the characteristics of a start-up firm and a large, long-established firm.

[4]

(c) Discuss, illustrating your answer with examples, the challenges of managing innovation in a large, long-established firm.

[12]

22 (a) When designing any production system, there is always a trade-off between the volume and variety of products that can be made. Sketch a diagram that illustrates the trade-off between volume and variety.

[4]

(b) Describe, illustrating your answer with examples, the characteristics of each of the following four types of production system:

(i) Projects;

(ii) Job shop;

(iii) Batch production; and

(iv) Mass production.

[8]

(c) Many start-up firms cannot afford to develop their own production systems and choose instead to form partnerships with larger, well-resourced firms. Discuss some of the challenges that might be faced by start-up firms when they try to set-up and manage partnerships with larger firms.

[8]

23 In discussion with two of your colleagues, you have developed an idea for a novel, low-cost electronic anti-theft alarm for bicycles.

(a) Discuss how you could identify:

(i) Who the potential buyers of this product might be; and

(ii) What the competitors for this product might be.

[12]

(b) Describe the types of Intellectual Property (IP) that would be relevant for this product. For each type of IP, identify the issues that would need to be considered in protecting that IP. [8]

Crib:

21 (a) Explain, illustrating your answer with examples, what is meant by:

(i) Radical innovation;

(ii) Incremental innovation.

[4]

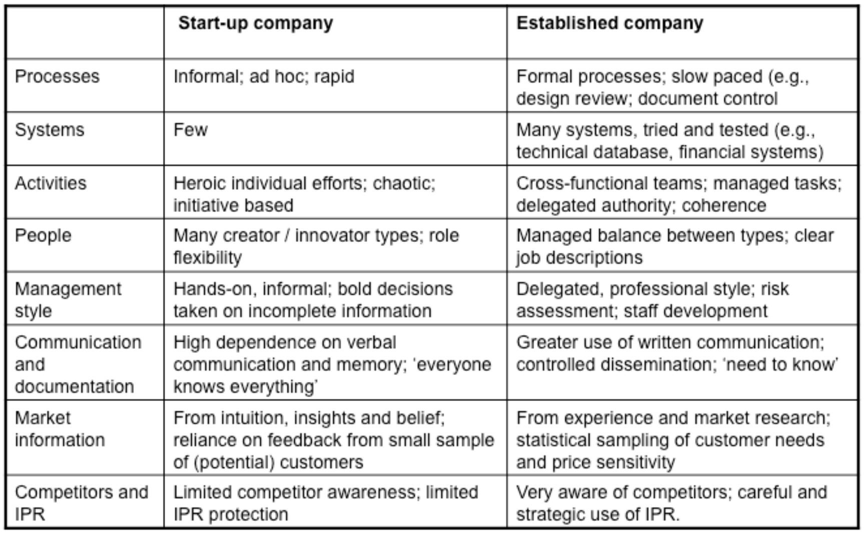
Radical innovation = Significant changes to products, services or processes: 'do what we do differently'. Example = petrol to electric engines for cars

Incremental innovation = Small improvements to existing products, services or processes: 'doing what we do but better'. Example = increasing size of LCD screens

(b) Describe the differences in the characteristics of a start-up firm and a large, long-established firm.

[4]

A good answer would draw upon several of the issues raised in this table:



	Start-up company	**Established company**
Processes	Informal; ad hoc; rapid	Formal processes; slow paced (e.g., design review; document control
Systems	Few	Many systems, tried and tested (e.g., technical database, financial systems)
Activities	Heroic individual efforts; chaotic; initiative based	Cross-functional teams; managed tasks; delegated authority; coherence
People	Many creator / innovator types; role flexibility	Managed balance between types; clear job descriptions
Management style	Hands-on, informal; bold decisions taken on incomplete information	Delegated, professional style; risk assessment; staff development
Communication and documentation	High dependence on verbal communication and memory; 'everyone knows everything'	Greater use of written communication; controlled dissemination; 'need to know'
Market information	From intuition, insights and belief; reliance on feedback from small sample of (potential) customers	From experience and market research; statistical sampling of customer needs and price sensitivity
Competitors and IPR	Limited competitor awareness; limited IPR protection	Very aware of competitors; careful and strategic use of IPR.

(c) Discuss, illustrating your answer with examples, the challenges of managing innovation in a large, long-established firm.

[12]

Innovation is one way in which firms can grow, either through

– Sustaining SOM (share of market) in a growing market

– Increasing SOM in a flat market

– Both approaches require sustaining innovation

Another way - Create a new market/category and grow it

– Radical innovation (or disruptive innovation)

There are generic challenges in managing innovation, and these can grouped into five areas: Innovation strategy: Idea generation: Selection and prioritization: Implementation: People and organization.

Established companies need both types of innovation to survive. Problem is that large firms have specific challenges that may stop them innovating:

Manufacturing plants

- *Huge assets which need to be kept full*

Keeping ahead of the competition

- *Sustaining: faster better cheaper*

Technology Platforms

- *What if the next innovation doesn't require current mfg plants?*
- *What if the next innovation has a lower profit margin*

Multiple sub-businesses

- *Multiple technology platforms?*
- *Each business may have different potential for growth*
- *Want to leverage resources across multiple businesses*

Speed of decision making

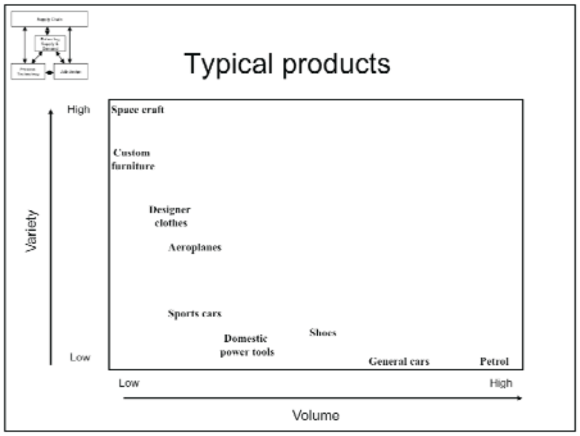
- *Lots of stakeholders*
- *Corporate processes*
- *Multiple locations*
- *Small companies can often move faster*

There are specific ways in which large companies can overcome these challenges:

- *Roadmapping – discipline*
- *Outsourcing manufacturing*
- *Cross-functional business decision teams*
- *Formal New Product Development Process*
- *Balanced R&D portfolio*
- *Spinouts*
- *R&D organisation*
- *Open Innovation model*

22 (a) When designing any production system, there is always a trade-off between the volume and variety of products that can be made. Sketch a diagram that illustrates the trade-off between volume and variety.

[4]



Typical products

(b) Describe, illustrating your answer with examples, the characteristics of each of the following four types of production system:

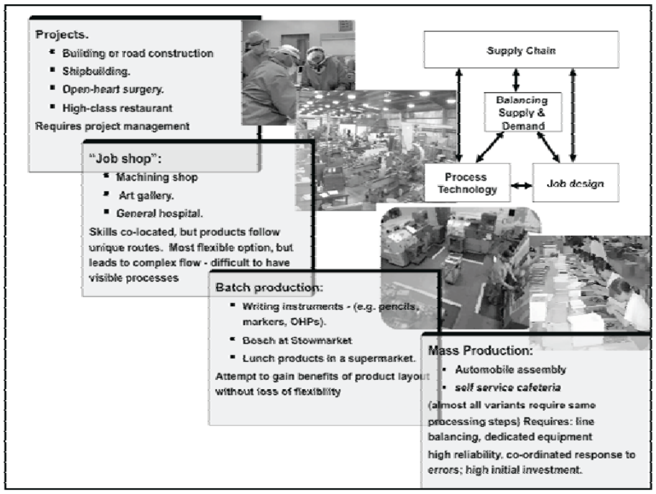
(i) Projects;

(ii) Job shop;

(iii) Batch production; and

(iv) Mass production.

[8]



Projects.
- Building or road construction
- Shipbuilding.
- Open-heart surgery.
- High-class restaurant

Requires project management

"Job shop":
- Machining shop
- Art gallery.
- General hospital.

Skills co-located, but products follow unique routes. Most flexible option, but leads to complex flow - difficult to have visible processes

Batch production:
- Writing instruments - (e.g. pencils, markers, OHPs).
- Bosch at Stowmarket
- Lunch products in a supermarket.

Attempt to gain benefits of product layout without loss of flexibility

Mass Production:
- Automobile assembly
- *self service cafeteria*

(almost all variants require same processing steps) Requires: line balancing, dedicated equipment

high reliability, co-ordinated response to errors; high initial investment.

The slide shows four main strategies for designing the production system. Generally the level of investment required to start each one – and hence the risk (that the money invested won't be returned through sales) – increases as the curve moves to the right. Most innovative products are thus introduced in smaller volumes – made using the more general skills and equipment of a 'job shop' – which allows variety/change to be introduced more cheaply while the product becomes established in the market. As *sales increase, the design will be fixed, and a move to a more dedicated production system becomes possible. Dyson is a good example of this – initially making small batches, then starting a small factory in the UK, and recently moving to a dedicated high volume factory in Malaysia.*

(c) Many start-up firms cannot afford to develop their own production systems and choose instead to form partnerships with larger, well-resourced firms. Discuss some of the challenges that might be faced by start-up firms when they try to set-up and manage partnerships with larger firms.

[8]

Answers should cover points selected from the following:

Start-up perspective	***Large firm perspective***
How to get in? *For large companies, the complexity and scale of operations may mean that even their own staff may not be able to help a start-up contact the right people.*	***Paranoia over IP and NDAs:*** *Start-ups are often reluctant to reveal details of their technology without a non-disclosure agreement (NDA). What they may fail to see is that somewhere within the large company IP may already be owned in this area.*
Who to talk to? *What the start-up really wants to know is: Who is the decision maker? Who influences them? Who will be working on implementing the partnership?*	***Brand abuse:*** *Start-ups are often very keen to promote relationships with established players as it may be seen to confer credibility. They may use the partner's brand in inappropriate ways in pursuit of this.*
Transfer of responsibility. *The transfer of responsibility from the large firm's R&D to their legal and procurement departments can change and disrupt the flow of the negotiations.*	***Technology, product or solution?*** *The gap between technology demonstrator to fully-supported product can often be quite significant and start-ups may not appreciate the time and cost involved in moving between the two.*
Slow decision cycles. *It is often very hard for large firms to make decisions at 'start-up speed', due to their complexity, size, and multiple layers of management.*	***Different functions:*** *Even when there is enthusiasm from R&D within the large firm, the transfer to operations (and 'collision' with procurement systems) can be problematic.*
Power imbalance. *The large firm may abuse its position by drawing-out negotiations and to attempt to prevent discussions with competitors.*	***Resource constraint & financial stability:*** *Start-ups need to be prepared to be subject to due-diligence checks to give potential partners confidence in their viability.*
Not understanding start-ups*. Demands made of start-ups by large firms sometimes show a lack of awareness of how a start-up operates.*	***Culture:*** *Start-ups may be run by individuals impatient for progress but unwilling to be governed by schedule and discipline dictated by the larger firm.*

23 In discussion with two of your colleagues, you have developed an idea for a novel, low-cost electronic anti-theft alarm for bicycles.

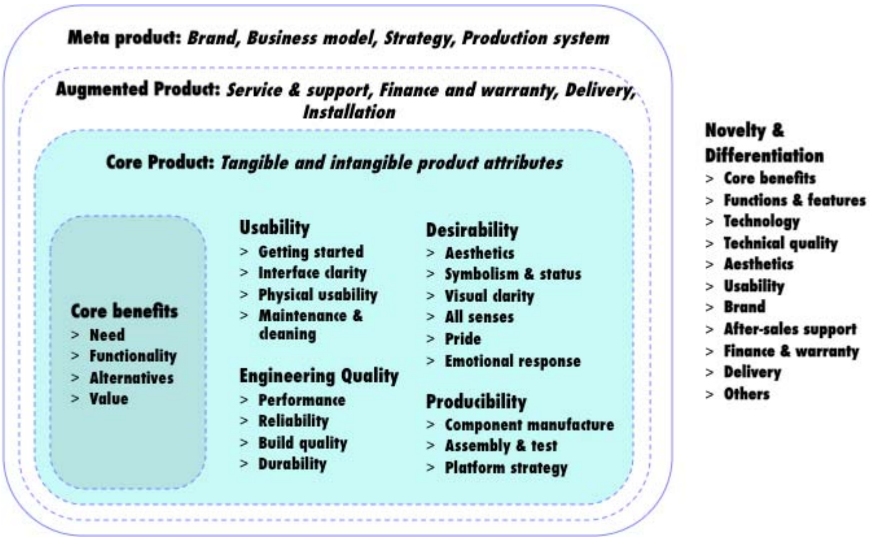
(a) Discuss how you could identify:

(i) Who the potential buyers of this product might be; and

(ii) What the competitors for this product might be.

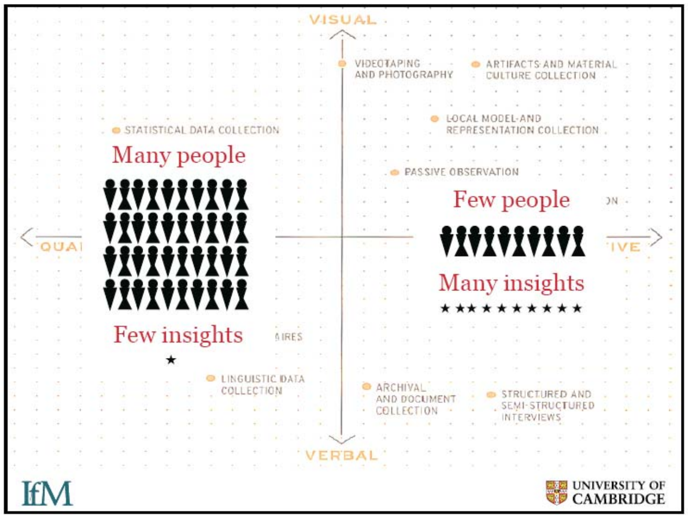
[12]

(i) and (ii) A good answer would consider all the elements of the ‘design mix’: Core, actual, augmented and meta product, as shown below:

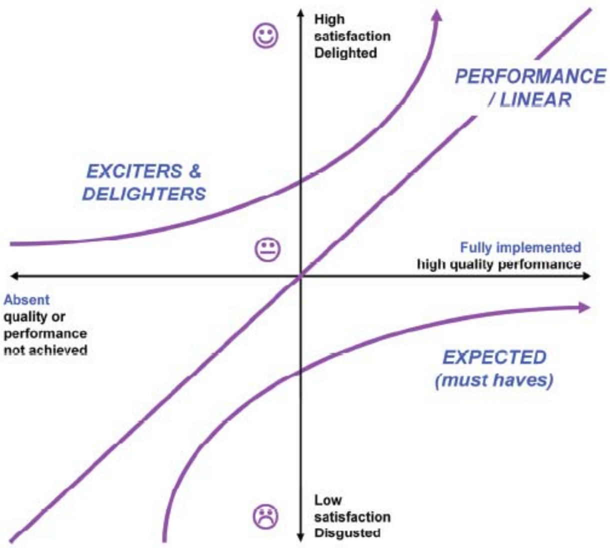


You should then go on to identify the market (i.e. the potential or actual users / customers for the product) and segment it by one of the following methods: product attributes (including price), product usage, demographics & geographic, psychographics.

Data on the chosen market segment could be acquired through: ‘Traditional’ methods: interviews; or ‘Empathic’ methods: observation. Data capture could also use a combination of qualitative and quantitative methods:



A Kano model would then be used to help position the product in terms of its key features:



Based on these insights, it should then be possible to define the product in terms of a simple 'elevator pitch':

- For (target customer)
- Who (statement of need or opportunity)
- The (product name) is a (product category)
- That (key benefit, compelling reason to buy)
- Unlike (primary competitive alternative)
- Our product (statement of primary differentiation)

(b) Describe the types of Intellectual Property (IP) that would be relevant for this product. For each type of IP, identify the issues that would need to be considered in protecting that IP. [8]

A good answer should consider all types of IP (registered and unregistered) and explain why they are, or are not, relevant:

Patents: It is very likely that you would want to apply for a patent for your product if you feel it is novel in its function, and could be imitated. Patents can be used to protect the technical, functional aspects of an invention, rather than the appearance (covered by design registration).

Registered design: If the appearance is an important part of the feature of the product (or you do not wish to / cannot apply or a patent), a registered design could be applied for which protects the appearance of all or part of a product. The appearance must be novel (i.e. not made public anywhere in the world) at the time of registration – and viewed as a 'fresh' design by an 'informed user' (rather than a design expert). Unlike patent applications, there is a 12 month period of grace after public disclosure when registration is still possible – perhaps providing an opportunity for test marketing before registration.

Trademark: A good way of differentiating the product would be through its name – establishing a clear brand identity, so that your product is instantly recognisable as distinctive, and, you hope, more valuable or attractive than the competition. This would be done through registering the name as a trademark.

Other, un-registered forms of IP that could be relevant include:

Copyright: This might apply to your advertising material, drawings, sketches, website, etc.

Confidentiality: You may choose just to keep some parts of your technology secret (in a 'black box') but this may be hard to do when you are getting other people to work with you on the development of the product.