ENGINEERING TRIPOS PART IB

Monday 6 June 2011 9 to 11

Paper 1

MECHANICS: DYNAMICS OF RIGID BODIES

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

Answer not more than ***two*** *questions from this section*

1 (a) A solid cylinder of length h, radius a and mass m rolls on a smooth surface without slipping and with no rolling resistance. The axis of the cylinder has velocity v in the direction normal to the axis. The cylinder encounters a straight, upwards sloping ramp with a gradient of θ such that the cylinder remains in contact with the surface along its length and impact effects are negligible. How far along the slope will the cylinder travel before stopping? [6]

(b) A solid cone of length h, base radius a and mass m is connected at its tip to a rotating vertical shaft such that the base of the cone is in contact with a flat, horizontal surface and the axis of the cone remains horizontal (see Fig. 1). There is no slip at the contact point between the cone and the surface. The cone has angular velocity Ω about the z-axis.

(i) Using the moments of inertia of a disk, find the moments of inertia of the cone about the x and z axes. [8]

(ii) Compute the kinetic energy of the cone. [6]

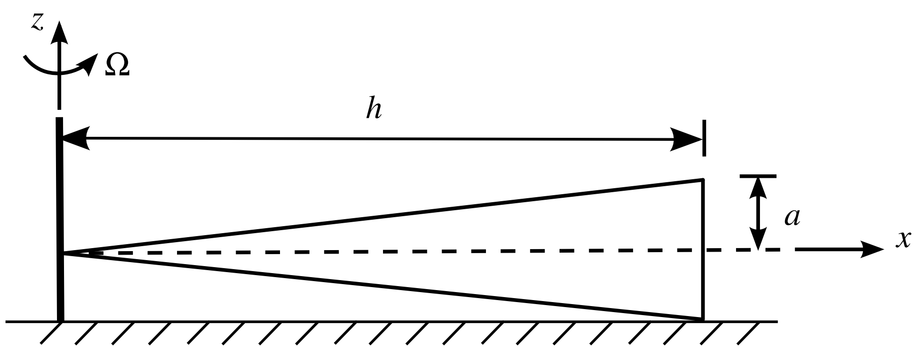


Fig. 1

2 Figure 2 shows a shaker mechanism composed of rigid links AB, BC and CD. The mechanism is driven by the hydraulic piston EB. The area of the piston is A_p and the piston is controlled by the volumetric flow rate Q of fluid into the piston. The extended length of the piston is given by $L' = L + r$.

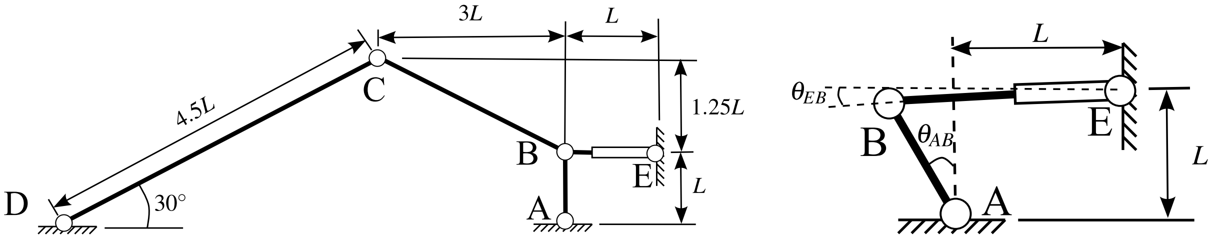
(a) If from time t_1 to t_2 the flow rate Q is constant, what is the change in length of the piston? [2]

(b) At the instant depicted in Fig. 2a, find the relationship between the piston extension speed $\dot{r}$ and the angular velocity of DC. [6]

(c) As the piston EB extends at a constant rate $\dot{r}$ it will rotate, with θ_{EB} being the rotation of EB from the horizontal. The bar AB will also rotate, with θ_{AB} being the rotation of AB from the vertical (see Fig. 2b).

(i) For the point B, find expressions for the horizontal component of the velocity and of the acceleration in terms of L, r, θ_{EB} and time derivatives of these quantities. [6]

(ii) Starting from an expression for the horizontal component of the velocity of B in terms of L, θ_{AB} and $\dot{\theta}_{AB}$, find an expression for the horizontal component of the velocity of B in terms of θ_{AB} and $\dot{r}$ only. Hint: Consider the geometry of the triangle ABE. [6]



(a) (b)

Not to scale.

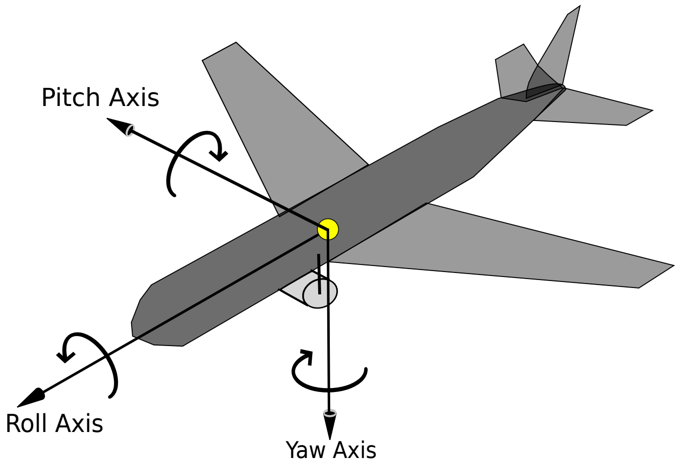
Fig. 2

3 An aircraft travelling at speed v carries a spinning bomb supported below its fuselage, as shown in Fig. 3. The bomb is modelled as a solid cylinder of mass m and radius r. It spins at ω rpm (backspin) on a shaft that is supported at each end by the bearings A and B. The bearings are a distance L apart. The shaft is parallel to the axis of the cylinder, but offset by a distance e (see Fig. 3b).

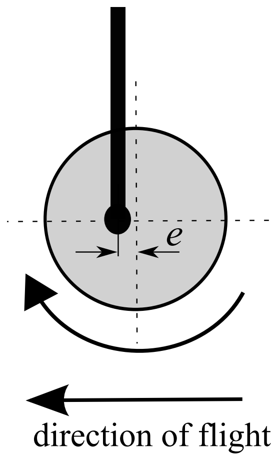
(a) Compute the dynamic forces applied to the bearings due to the offset e. [4]

(b) As the aircraft pitches upwards with a turning radius of R, what additional reactions will this manoeuvre induce in each bearing? [4]

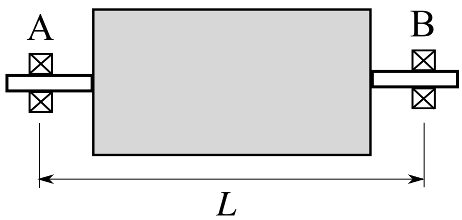
(c) When making a particular turn, the aircraft rolls at $\dot{\theta}_1$ rad s^{-1} and yaws at $\dot{\theta}_2$ rad s^{-1} (both positive according to the axes in Fig. 3a). For the case $e = 0$, find the reactions in each bearing due to the spinning of the cylinder during this manoeuvre. [12]



(a) aircraft axes



(b) side view



(c) front view

Fig. 3

SECTION B

*Answer not more than **two** questions from this section*

4 A rubber ball of mass m and radius r is rotating with angular velocity ω, and is dropped from a height h onto the ground, as shown on Fig. 4a, and bounces. The collision is perfectly elastic and air resistance can be neglected.

(a) Calculate the vertical velocity of the ball immediately after the first collision. [2]

(b) Assuming that there is no slip, determine the angular velocity of the ball after the first collision, and hence the horizontal velocity that it acquires. [8]

(c) The ball bounces and travels a distance L horizontally before it next hits the ground, as shown in Fig. 4b. Determine L in terms of m, r, ω and h. [4]

(d) Suppose that the coefficient of friction between the ball and the ground is μ. By considering the ratio of horizontal to vertical impulses, find the maximum value of ω for which there will be no slip. Use this to determine the maximum possible distance travelled, L_{max}, between the first and second collisions. [6]

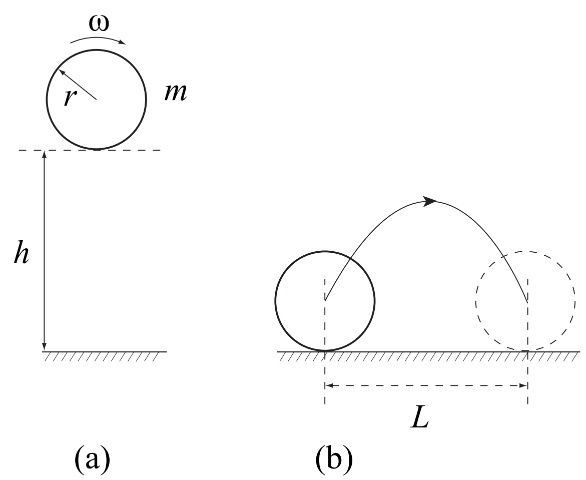


Fig. 4

5 A thin pencil of length l and mass m is placed vertically on its tip on a rough table and released so that it falls in some direction, as shown in Fig. 5.

(a) Calculate the moment of inertia of the pencil about its end point. [2]

(b) Determine the angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$ when the pencil has rotated through an angle θ. [6]

(c) Give expressions for the normal force N and the frictional force F acting at the contact point. [6]

(d) Show that no matter how high the coefficient of friction is, the pencil cannot rotate beyond an angle given by $\cos\theta = 1/3$ without slipping. [3]

(e) Determine the angle at which the frictional force reverses direction, and make a rough sketch of the ratio F/N as a function of θ. [3]

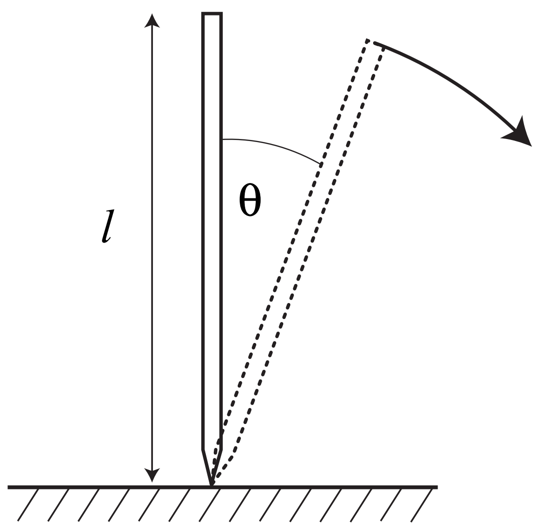


Fig. 5

6 A car wheel, shown in Fig. 6, is out of balance. The wheel rims are located at a radius R and separated by a distance d. A test has shown that the out-of-balance is equivalent to $m = 30$ grams located on the wheel at a radius $r = 7R/6$ and at an angle of $\theta = 45°$, on a plane that is at a distance $z = d/6$ from the centerline of the wheel, as shown. Static and dynamic balance is achieved by adding two balance masses m_1 and m_2, one to each rim of the wheel.

(a) Find the required masses and angular positions of the two balancing weights. [8]

(b) You discover at the garage that only 6 gram and 12 gram balancing masses are available. Based on your answer to (a), devise a scheme to achieve static and dynamic balance using four masses, a pair of 6 gram weights on one rim, and a pair of 12 gram weights on the other rim. [8]

(c) The 12 gram weights from part (b) fall off. Determine the new out-of-balance and the plane on which it acts. [4]

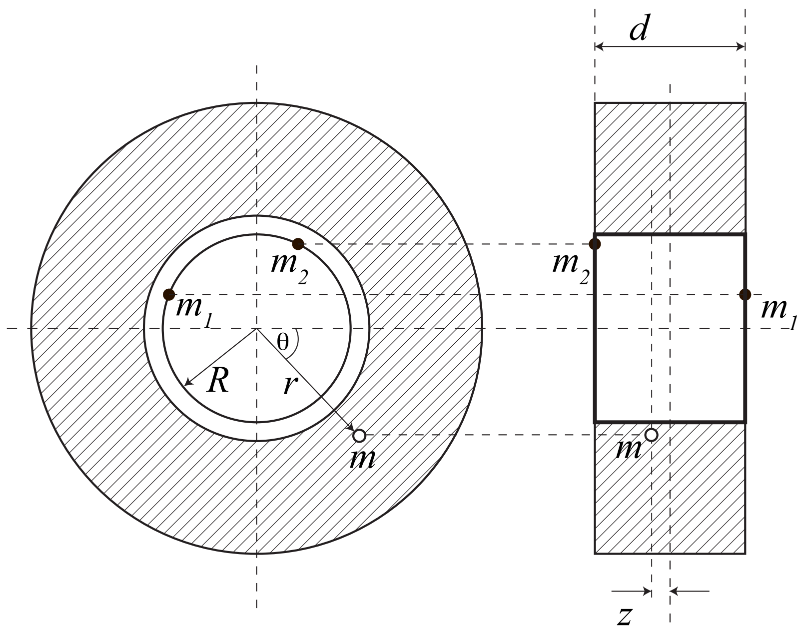


Fig. 6

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