ENGINEERING TRIPOS PART IB

Monday 6 June 2011 2 to 4

Paper 2

STRUCTURES

Answer not more than ***four*** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig. 5(a)

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Single-sided graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 The weightless pin-jointed structure shown in Fig. 1 has five members I to V. Each member has cross-sectional area *A*, Young's modulus *E*, and all behaviour is linear elastic. The unloaded structure is free of stress.

(a) Show that the structure has one redundancy and find a state of self-stress corresponding to a unit tension in bar II. [4]

(b) A vertical force *V* and a horizontal force *H* are applied as shown in Fig. 1. Specifying member II as the redundant bar:

(i) use the Force Method to determine the bar forces in the structure; [12]

(ii) find the components of displacement of the joint at which the loads are applied. [4]

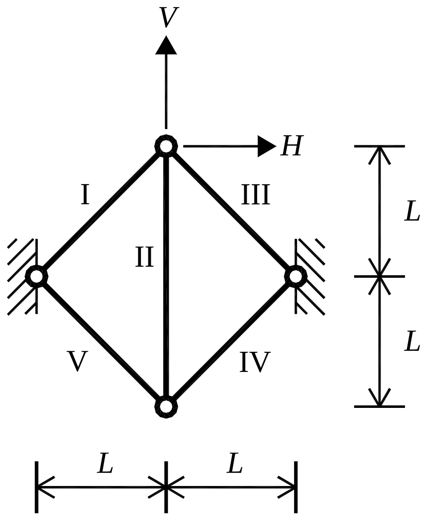


Fig. 1

2 (a) The simply supported beam in Fig. 2(a) is subjected to a distributed load of total magnitude *W* over half of its span. The beam has flexural rigidity *EI* and behaves linearly elastically. Show that the rotation at B is $7WL^2 / 192EI$. [4]

(b) The weightless frame shown in Fig. 2(b) is fixed to the ground at A and simply supported at C. A uniformly distributed load of total magnitude *W* is applied vertically over half of span BC. All members have flexural rigidity *EI*, are axially rigid and behave linearly elastically. The unloaded frame is unstressed.

(i) Determine the number of redundancies in the structure. [2]

(ii) Calculate the support reactions at the simply supported end C and the fixed end A. [8]

(iii) Determine the position and magnitude of the maximum moment in the frame and plot the bending moment diagram indicating important values. [6]

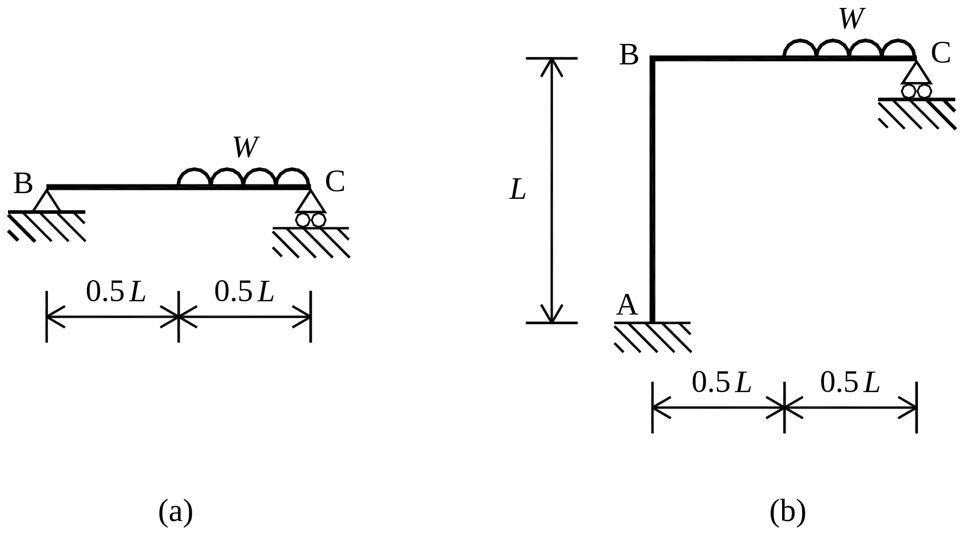


Fig. 2

3 (a) By considering that the constant shear flow q in an arbitrary thin walled cross-section is in equilibrium with the applied torque T, show that:

$$q = \frac{T}{2A_e}$$

where A_e is the area enclosed by the cross-section to the mid-thickness. [4]

(b) A cylindrical pressure vessel of 500 mm outside diameter and 5 mm wall thickness forms a 3 m long cantilever as shown in Fig. 3(a). The pressure vessel is subjected to an internal pressure of 1.5 MPa and an eccentric point load of 20 kN at its free end as shown in Fig. 3(b). Points A and B are of particular interest and are both located close to the root of the cantilever at the mid-thickness of the pressure vessel wall. Point A and point B are positioned at the top and the mid-depth of the pressure vessel respectively.

(i) Assuming that the support has no influence on the hoop stresses locally, determine the total longitudinal and hoop stresses at both A and B. [5]

(ii) Calculate the total shear stresses at both A and B. [5]

(iii) The pressure vessel is made of steel with a uniaxial yield stress of 275 MPa. Both the internal pressure and the point load are to be increased by a factor λ. Considering only points A and B, use the von Mises' Yield Criterion to determine the value of λ at which full yielding would occur. [6]

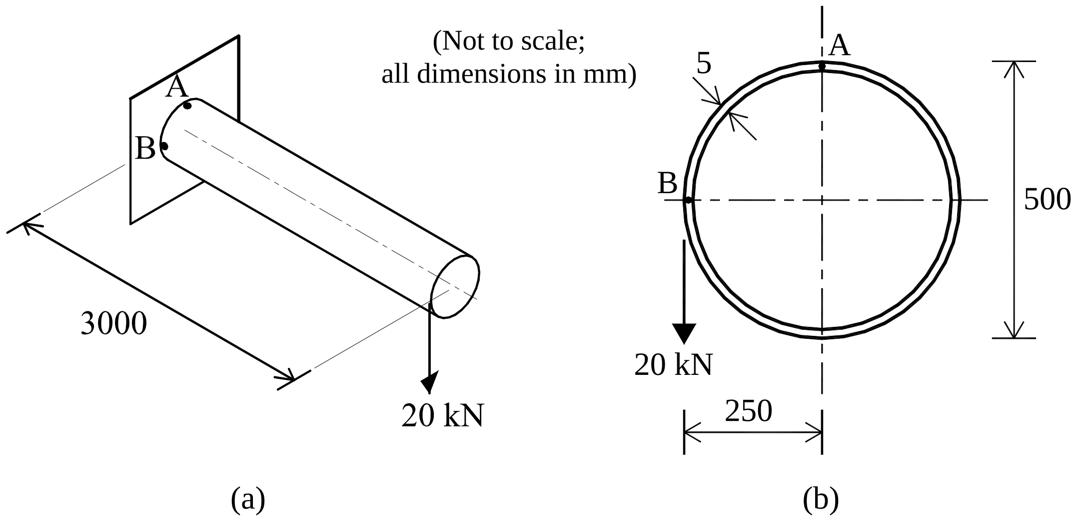


Fig. 3

SECTION B

4 The portal frame shown in Fig. 4 has a uniform fully plastic moment M_p . The frame is subjected to a horizontal uniformly distributed load of total magnitude *H*, and a vertical point load *V* applied at mid-span.

(a) Sketch all the feasible collapse mechanisms. [5]

(b) Plot an interaction diagram showing values of *H* and *V* for which collapse would occur for each mechanism. [7]

(c) Determine the value of the collapse load when $H = 4V$. [3]

(d) The designer has increased the fully plastic moment of the horizontal member in the portal frame to $2M_p$. Show how your interaction diagram in (b) would be modified and recalculate the collapse load when $H = 4V$. [5]

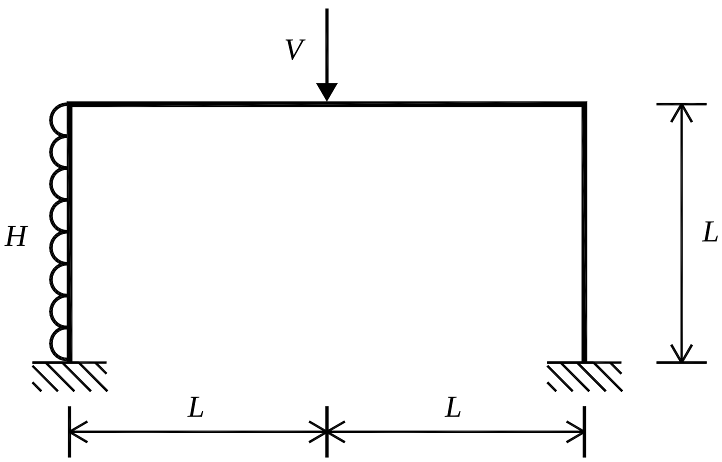


Fig. 4

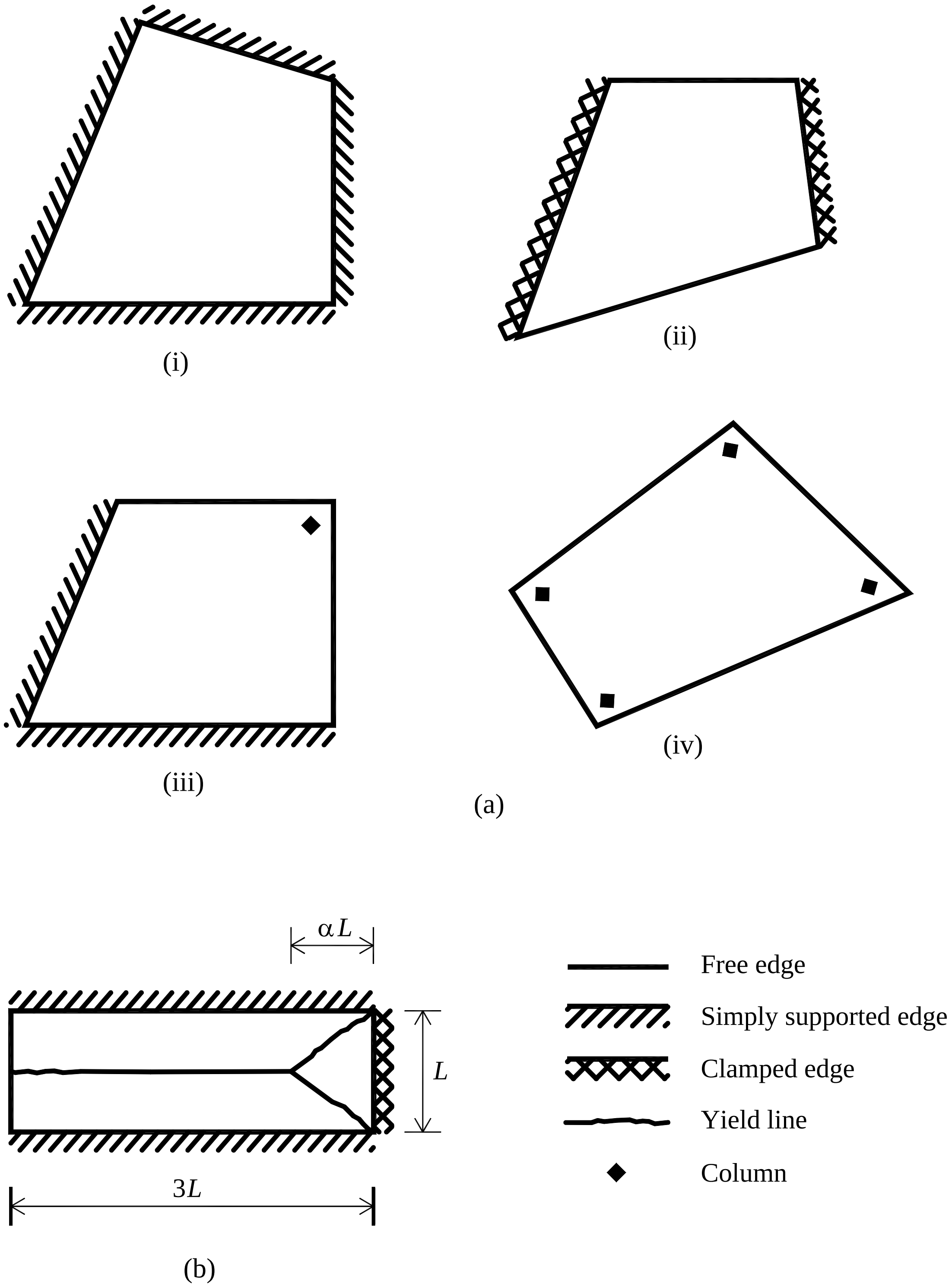
5 (a) Give a set of guidelines to ensure that a yield line mechanism satisfies compatibility constraints. [3]

(b) The four slabs marked (i) to (iv) in Fig. 5(a) are subjected to uniformly distributed loads and have the same plastic moment capacity about any axis in both hogging and sagging bending. Sketch a likely collapse mechanism for each slab and indicate the axes of rotation and any variable parameters to be used in a yield line analysis. *A separate copy of these figures is provided at the end of the paper; your answers should be drawn on this sheet and handed back with your answer book.* [7]

(c) The slab shown in Fig. 5(b) is subjected to a uniformly distributed load of intensity w, and has a plastic moment capacity in both hogging and sagging of m. The yield lines intersect at a distance αL from the clamped edge, as shown:

(i) determine the collapse load in terms of m and α ; [7]

(ii) numerically evaluate m and calculate the lowest upper bound of the collapse in terms of α. [3]



(i)
(ii)
(iii)
(iv)
(a)
αL
L
3L
(b)
Free edge
Simply supported edge
Clamped edge
Yield line
Column

Fig. 5

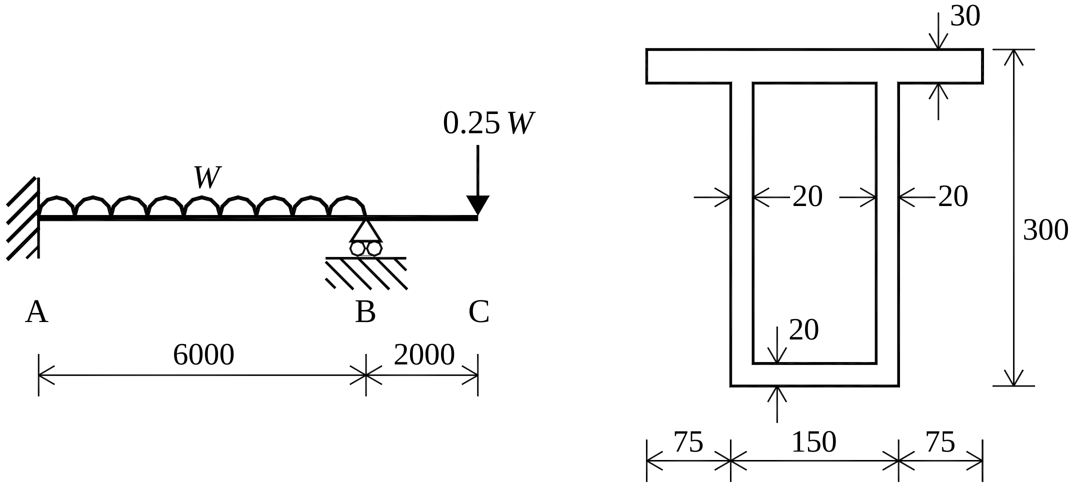
6 The structure shown in Fig. 6(a) carries a load of total magnitude W uniformly distributed along span AB and a point load of $0.25W$ at C. The structure is made from grade 43 steel (σ_y = 275 MPa) and has the cross-section shown in Fig. 6(b). Self-weight may be ignored.

(a) Show that the plastic moment capacity of the section is 568 kNm. [6]

(b) Perform a *lower bound analysis* to determine the maximum safe value of W and sketch the overall bending moment diagram. [6]

(c) Perform an *upper bound analysis* to determine the collapse load W. [6]

(d) Comment on the relationship between your lower and upper bound estimates. [2]

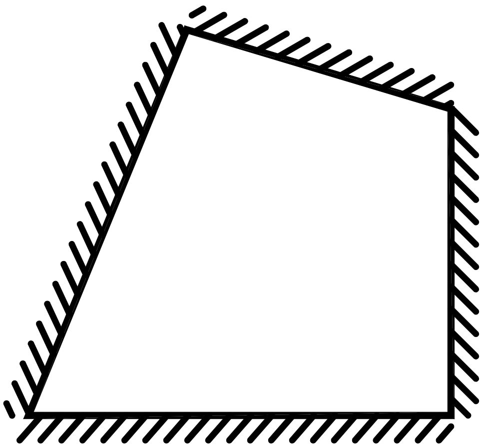


(a) (b)

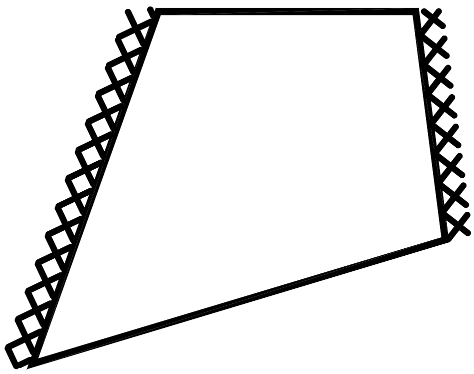
(Not to scale; all dimensions in mm)
Fig. 6

ENGINEERING TRIPOS PART IB **CANDIDATE NUMBER: ________**
STRUCTURES

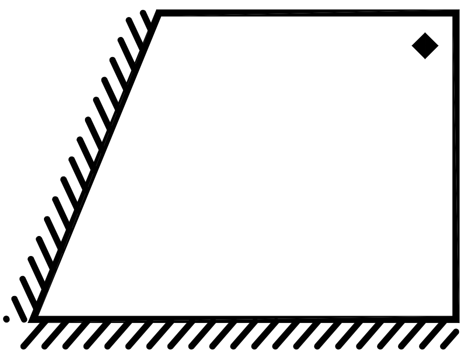
Additional copy of Fig. 5(a) that can be annotated and should be handed in with your solutions. Additional copies are available from the Invigilator.



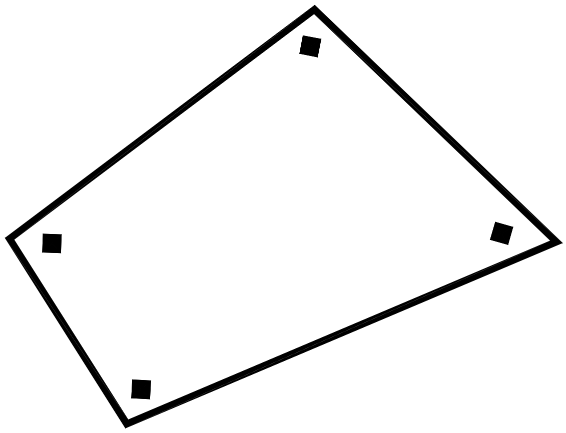
(i)



(ii)



(iii)



(iv)

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