ENGINEERING TRIPOS PART IB

Tuesday 7th June 2011 2 to 4

Paper 4

THERMOFLUID MECHANICS

Answer not more than **four** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

Answer not more than **two** *questions from this section*

1 (a) An industrial gas turbine, shown in the upper part of Fig. 1, generates shaft power of 350 MW. The compressor inlet conditions are 1 bar, 20 °C and it operates with a pressure ratio of 20. The combustor exit temperature is 1500 °C and the turbine exit is at ambient pressure. The turbine and compressor have an isentropic efficiency of 85%.

(i) Sketch the *T-s* diagram of the gas turbine cycle. [2]

(ii) Calculate the compressor delivery temperature and show that the turbine exit temperature is 633 °C. [4]

(iii) Calculate the mass flow rate of air in the cycle, the heat flow rate into the combustor, and the efficiency of the cycle. [4]

Assume that the working fluid is air, with specific heat ratio γ = 1.40, and specific heat at constant pressure $c_p = 1.01$ kJ$\mathrm{kg^{-1}K^{-1}}$ You may also neglect the mass of the fuel, the combustor pressure drop and the changes in kinetic energy of the gas.

(b) The gas turbine from part (a) is used in a combined cycle power station, as shown in the lower part of Fig. 1. The superheated Rankine cycle has a maximum pressure of 30 bar, a condenser pressure of 0.04 bar and the steam turbine may be assumed isentropic. The Heat Recovery Steam Generator (HRSG) reduces the gas turbine exhaust temperature to 130 °C and raises the steam temperature to 600 °C. You may neglect the feed pump work (i.e. $h_6 \approx h_7$).

(i) Sketch the *T-s* diagram of the Rankine cycle. [2]

(ii) Find the mass flow rate of water and calculate the power output from the steam turbine. [6]

(c) Calculate the total power output and the efficiency of the combined cycle. [2]

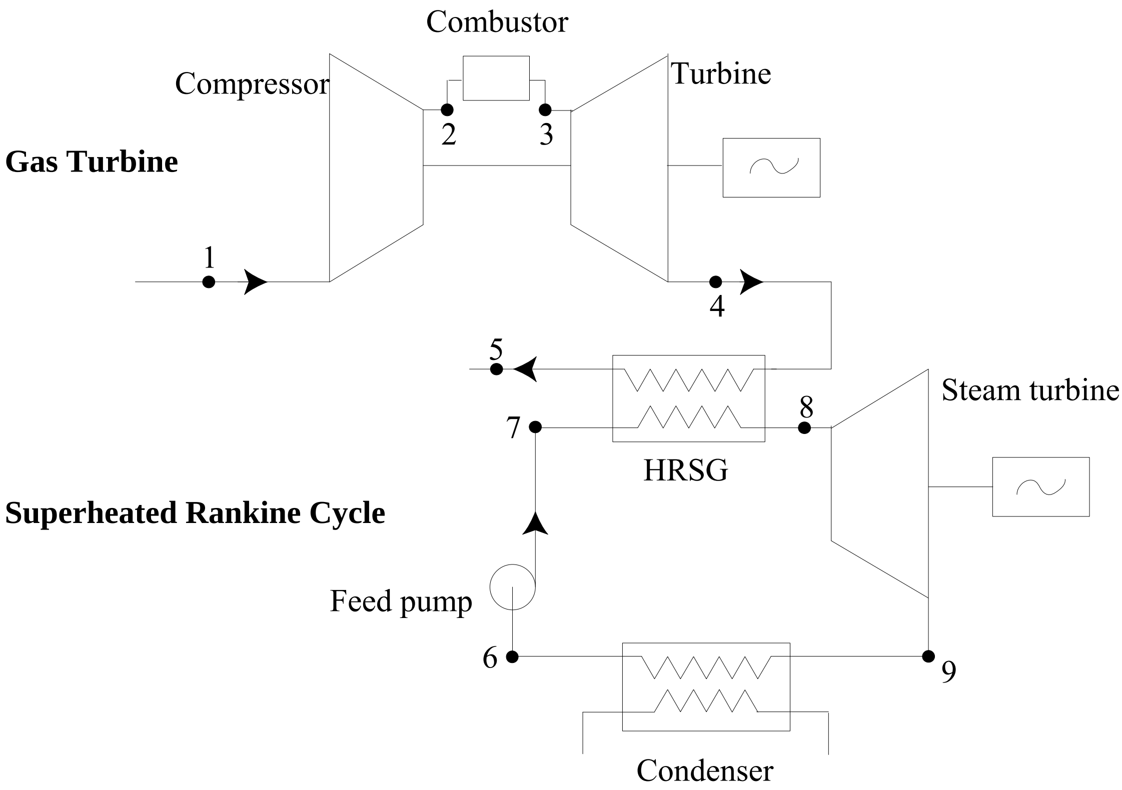
Combustor
Compressor
Turbine
2
3
Gas Turbine
1
4
5
Steam turbine
8
7
HRSG
Superheated Rankine Cycle
Feed pump
6
9
Condenser

Fig. 1

2 (a) Figure 2 shows a steady flow device within control volume CVA which delivers shaft power $\dot{W}_x$ while rejecting heat $\dot{Q}$ at temperature *T*. $\dot{W}_Q$ is the shaft power output achieved by a reversible heat engine receiving $\dot{Q}$ while rejecting $\dot{Q}_0$ to an environment at T_0. Using the 1st and 2nd laws of thermodynamics, prove that the specific *maximum available power* from a steady flow process operating in an environment at temperature T_0 is given by the change in the specific steady flow availability function, $b_1 - b_2$ where *b* is defined as

$$b = h - T_0 s$$ [6]

(b) For the questions below, the surroundings should be taken as being at 1 bar, 300 K. Assume that air behaves as a perfect gas, with $\gamma = 1.40$, $c_p = 1.005$ kJ $\mathrm{kg^{-1}\,K^{-1}}$ and $R = 0.2871$ kJ $\mathrm{kg^{-1}\,K^{-1}}$.

(i) A large reservoir supplies a stream of air at 8 bar, 700 K. Calculate the specific maximum available power of the stream. [2]

(ii) Calculate the fraction of the specific maximum available power that would be extracted if the stream were to be expanded through an isentropic turbine. [3]

(iii) Explain the difference between the answers to parts (i) and (ii) above, and how the difference might be recovered in theory. [1]

(c) The reservoir of part (b) (i), is to be used in a compressed air energy storage system. In this case, the reservoir is not insulated, and thus reaches thermal equilibrium with the surroundings.

(i) Calculate the fraction of the specific maximum available power of the hot stream of part (b) (i) which could be produced using an isentropic turbine expanding to ambient pressure. [2]

(ii) A turbine with an isentropic efficiency of 80% is used to expand the stream to ambient pressure. State the physical mechanisms that cause irreversibilities, and determine the fraction of the specific maximum available power that would be produced by the real turbine. [3]

(iii) The air in the reservoir has a specific humidity of 0.01 kg water per kg of air. Calculate the dew point of the air at exit from the real turbine of part (c) (ii). What problems would this dew point cause and what steps could be taken to avoid them? [3]

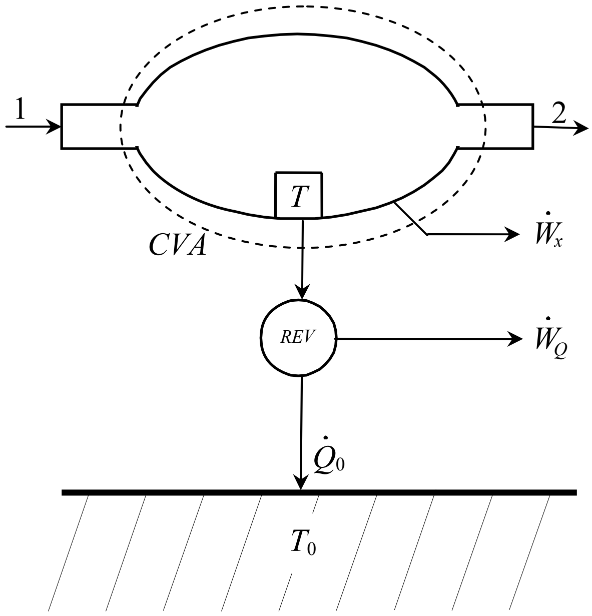

Fig. 2

3 (a) Show that the net radiation exchange Q_{12}, between black body 1, which has surface area A_1 and black body 2, is given by

$$Q_{12} = A_1 F_{12} \sigma \left(T_1^{\ 4} - T_2^{\ 4} \right)$$

where F_{12} is the shape factor, σ is the Stephan-Boltzmann constant, and T_1 , T_2 are the temperatures of surfaces 1 and 2 respectively. [4]

(b) A cylindrical light bulb is of an axisymmetric design. The filament, of diameter 0.2 mm, is maintained at 3000 K in an evacuated environment, and may be regarded as a black body. The bulb is sufficiently long that end effects may be ignored. A schematic of the cross section is shown in Fig. 3. The glass cylinder is effectively a black body (the radiation in the visible range which is transmitted is a very small proportion of the total), and the inner surface of the cylinder, which is of diameter 20 mm, is at 700 K. A fan is used to help remove heat from the outer bulb surface, which has a diameter of 24 mm. The thermal conductivity of the glass is 1.38 $\mathrm{Wm^{-1}K^{-1}}$.

(i) Show that the electrical power input per unit length to the filament is 2.88 $\mathrm{kWm^{-1}}$. [4]

(ii) Calculate the outer temperature of the glass cylinder, T_{outer}. [4]

(iii) If the environment surrounding the bulb may be regarded as a black body at 20 ºC, determine the fraction of the heat rejected to the environment which leaves by radiation. [4]

iv) The Nusselt number (Nu) for a cylinder in cross flow as a function of Reynolds number (Re) and Prandtl number (Pr) may be assumed to be given by the correlation

$$\mathrm{Nu}_d = 0.2\,\mathrm{Re}_d^{\ 0.6}\,\mathrm{Pr}^{0.33}$$

Use this correlation to find the required velocity of the cooling air, which is at a temperature of 20 ºC. The fluid properties in the near wall film are given below. [4]

Data: Air properties - Film:

Density, $\rho = 1.25\ kgm^{-3}$
Dynamic viscosity, $\mu = 2x10^{-5}\ kgm^{-1}s^{-1}$
Pr = 0.71
Thermal conductivity, $\lambda_{air} = 0.026\ Wm^{-1}K^{-1}$

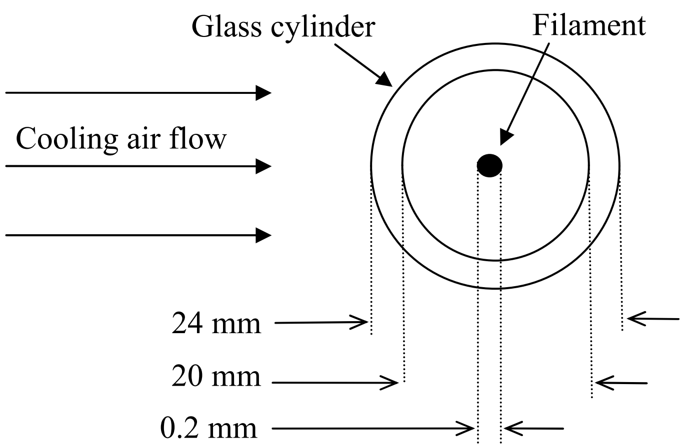

Fig. 3

SECTION B

Answer not more than **two** *questions from this section.*

4 (a) Show from first principles that for the steady laminar flow of a fluid between infinite parallel plates, subject to a streamwise pressure gradient,

$$\mu \frac{d^2u}{dy^2} = \frac{dp}{dz}$$

Here μ is the dynamic viscosity, u is the fluid velocity, p is the pressure and y and z are axes aligned at right angles to the parallel plates and in the flow direction respectively. [4]

(b) A device for measuring fluid viscosity is shown in Fig. 4. It consists of a vertical circular tube of diameter D, and a "free piston" of mass m, and height h. The radial gap is ε where $\varepsilon << D$ and $\varepsilon << h$ (i.e. the expression in part (a) may be used). Fluid, which completely fills the tube, is injected from below at a rate Q such that the piston is maintained at a fixed height. You may assume that all velocities are small, and that variations in hydrostatic pressure are negligible.

(i) Derive an expression for the velocity of the fluid in the annular gap, and then by considering the forces that act on the piston, find a relationship for the pressure difference $p_1 - p_2$ as a function of m, g, D and ε. [8]

(ii) Derive an expression for the flow rate of the fluid in terms of D, ε, $p_1 - p_2$, h and μ. [6]

(iii) What is the rate at which energy is dissipated due to fluid friction? [2]

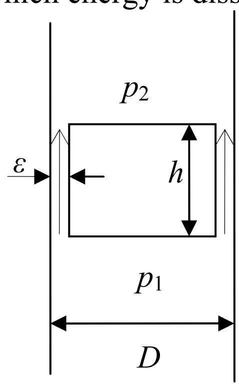

Fig. 4

5. Figure 5 shows a simple "ejector pump" in which a flow forced through a thin-walled inner tube, diameter *d*, is used to induce a flow into the smooth entry outer body, which in the mixing section has a diameter D. In the inner tube the velocity is v, in the annulus at the axial location 1, the velocity is u, and at the exit location, 2, a sufficient distance from location 1 to enable the flows to fully mix, the velocity is w. The flow is incompressible, has density ρ, and wall friction is negligible. Far from the ejector pump the pressure is p_a. In the thin-walled tube, the annulus at location 1, and at the exit plane 2, the velocities are uniform.

(a) Show that the velocity w is given by

$$w = kv + (1-k)u$$

where $k = \frac{d^2}{D^2}$. [4]

(b) Explain why the pressure at location 1, p_1, is equal to $p_a - \frac{\rho u^2}{2}$, and hence show, via a momentum balance between locations 1 and 2, that u, v and w, are also related by

$$2w^2 - 2kv^2 = (1-2k)u^2$$ [7]

(c) If $k = \frac{1}{2}$, $v = 10\ \mathrm{ms^{-1}}$ and $\rho = 1.2\ \mathrm{kgm^{-3}}$ find the values of u, w, and $p_a - p_1$. Proceed by direct substitution of these numerical values into the equations above. [4]

(d) The whole ejector pump is supported by the thin-walled inner tube, with the outer body being connected to the tube in a way that does not affect the flows. Assuming location 0 is far upstream of the ejector pump, what is the axial force in the tube wall at this location, using the same parameters as in part (c), and $D = 0.2\ \mathrm{m}$? [5]

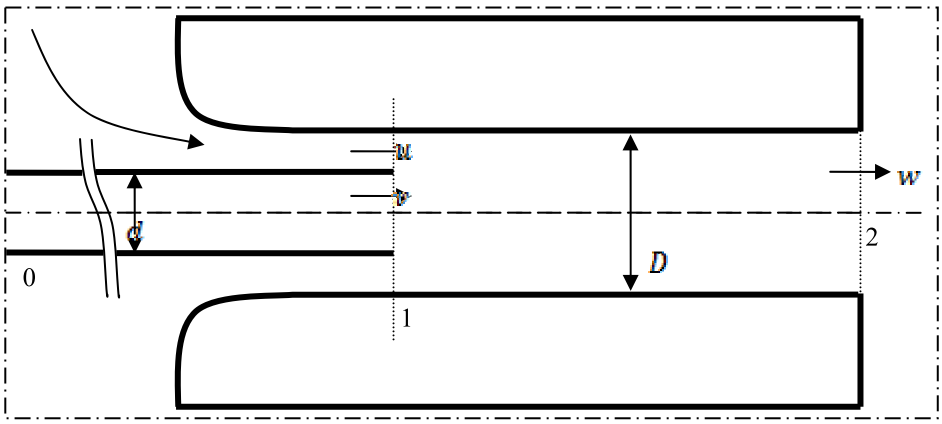

Fig. 5

6 (a) The stagnation pressure drop Δp_0 between two stations a few pipe diameters upstream and downstream of an orifice in a pipe carrying a flow rate $Q_{orifice}$ of an incompressible fluid of density ρ can be expressed as

$$\Delta p_0 = \frac{1}{2}\rho\left(\frac{Q_{orifice}}{A}\right)^2\left(\frac{1-\alpha}{\alpha}\right)^2$$

Here A is the pipe area before and after the orifice, and αA is the effective orifice area. State the physical principles that would be required in order to deduce this expression. [4]

(b) Figure 6 shows a plan view of an irrigation system. The whole pipe network may be regarded as lying in a horizontal plane. The reservoir, which is open to the atmosphere, contains water of density ρ. The pipe from the reservoir to the pump is short, and there are negligible losses at the pipe's entry. The increase in stagnation pressure across the pump is $\Delta p_{0,\,\mathrm{pump}}$. All pipes have the same internal diameter, d with friction coefficient c_f, and all pipes downstream of the pump (i.e. 6 sections in all) are of the same length L. You may assume that there is no change in stagnation pressure through the pipe junctions and the bend.

The valves (V_i) can be regarded as adjustable orifice plates, as in part (a), where the opening is represented by α, with $0 \le \alpha \le 1$. These valves are mounted in the three "delivery" pipes, which have open ends at the points E_i, also of inner diameter d, where the water is ejected into open space. The flow from the reservoir is Q, and the valves are arranged so that a) the flow in each delivery pipe is the same (i.e. $Q/3$), and b) so that the pumping power is minimised. For your solutions, you may find the substitutions $k = 4c_f L/d$ and $n = 8\rho/\pi^2 d^4$ convenient.

(i) Explain why $\alpha_3 = 1$ and then by considering the change in stagnation pressure of the flow between the reservoir and the exit at E_3, show that the stagnation pressure rise across the pump can be expressed as

$$\Delta p_{0,\,\mathrm{pump}} = nQ^2(1+15k)/9$$

where k and n are defined above. [7]

(ii) Using the result from (b) (i), and by considering in turn the flows which exit at E_1 and E_2, obtain expressions for α_1 and for α_2 as a function of k. [7]

(iii) With the valve openings unchanged, what is the increase in the flow in the delivery pipes if $\Delta p_{0,\,\text{pump}}$ increases by 50%? [2]

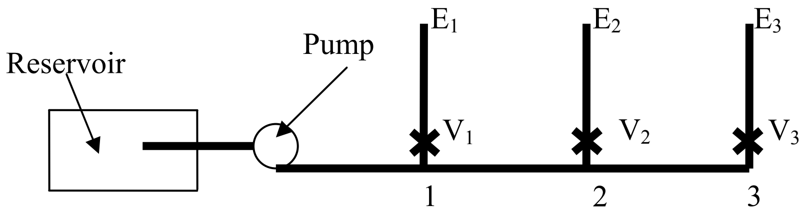

Fig. 6

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