ENGINEERING TRIPOS PART IB

Thursday 9 June 2011 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig. 3.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

pcw05

SECTION A

*Answer not more than **two** questions from this section.*

1 A particular coffee machine heats water in an aluminium boiler with a heating coil built into the boiler wall as shown in Fig. 1.

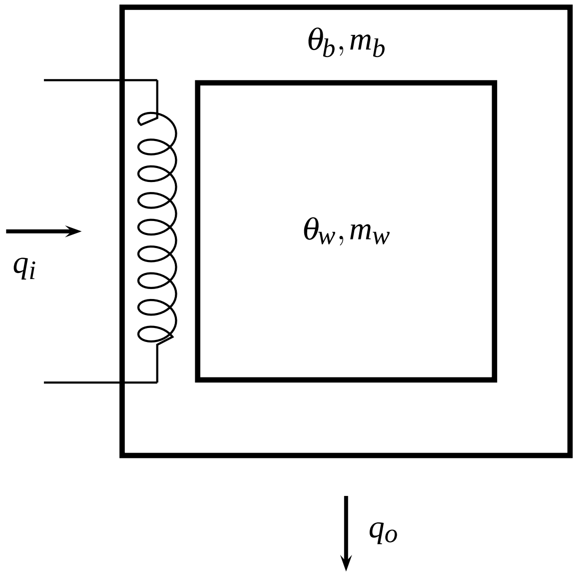


Fig. 1

Assume that, at any time, the boiler wall is at a uniform temperature $\theta_b(t)$ and that the water is at a uniform temperature $\theta_w(t)$. The mass of the boiler, m_b, and of the water, m_w, are both 0.5 kg. The rate of heat input is denoted q_i. Assume that the rate of heat loss, q_o, is constant at $q_o = 50$ W.

The dynamics are given by

$$\begin{aligned} m_b c_b \dot{\theta}_b &= q_i - q_o + h(\theta_w - \theta_b) \\ m_w c_w \dot{\theta}_w &= h(\theta_b - \theta_w) \end{aligned}$$

where the specific heat capacities c_b and c_w are 900 $\mathrm{J\,kg^{-1}\,K^{-1}}$ and 4200 $\mathrm{J\,kg^{-1}\,K^{-1}}$ respectively, and the heat transfer coefficient $h = 3\ \mathrm{J\,s^{-1}\,K^{-1}}$.

(a) A controller is added to control the water temperature by varying the rate of heat input, q_i, according to

$$q_i = k_1(\theta_d - \theta_w)$$

where θ_d is the desired temperature.

(i) Draw a block diagram for the feedback system showing the water and boiler wall temperatures as outputs of separate blocks, and find the closed loop transfer function from θ_d to θ_w. [7]

(ii) If k_1 is chosen such that the closed loop damping ratio is $1/\sqrt{2}$, what is the steady-state error in $^\circ$C? [5]

(b) The controller of part (a) is now replaced by a controller for the temperature of the boiler wall

$$q_i = k_2(\theta_d - \theta_b).$$

Find the new closed loop transfer function from θ_d to θ_w, and choose k_2 such that the steady state error is half of that in part (a)(ii). What are the locations of the poles of the closed loop transfer function for the chosen value of k_2? [5]

(c) Describe the response of the water temperature to a step change in demanded temperature for each design and comment on the advantages and disadvantages of each. [3]

2 (a) How can a *Bode diagram* for a linear system be constructed from experimental measurements? [3]

(b) The Bode diagram of an asymptotically stable linear system $G(s)$ is shown in Fig. 3. It is known that $G(s)$ is of the form:

$$G(s) = \frac{A(s+\alpha)}{s^n(s^2+\beta s+\gamma)} \ .$$

(i) Estimate values for the parameters A, α, β, γ and n. [6]

(ii) The system $G(s)$ is now used with closed-loop feedback as shown in Fig. 2. Estimate the phase margin and comment on closed-loop system stability when $K(s) = 1$. [2]

(c) The system of part (b) is changed such that

$$K(s) = \frac{s+2}{6(s+B)}$$

Find a value of B so that $|K(j\omega)G(j\omega)| \approx 1$ when $\omega = 4$. [3]

(d) For the value of $K(s)$ found in part (c), draw the bode diagram of $K(j\omega)G(j\omega)$ on the additional copy of Fig. 3. Estimate the phase margin of this revised system. Comment on the expected change in performance of the system by adding this compensator. [6]

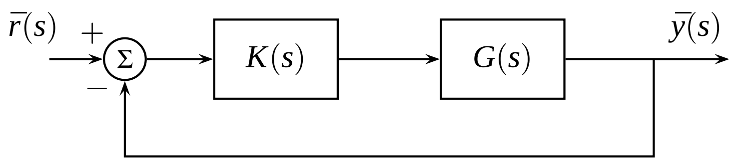


Fig. 2

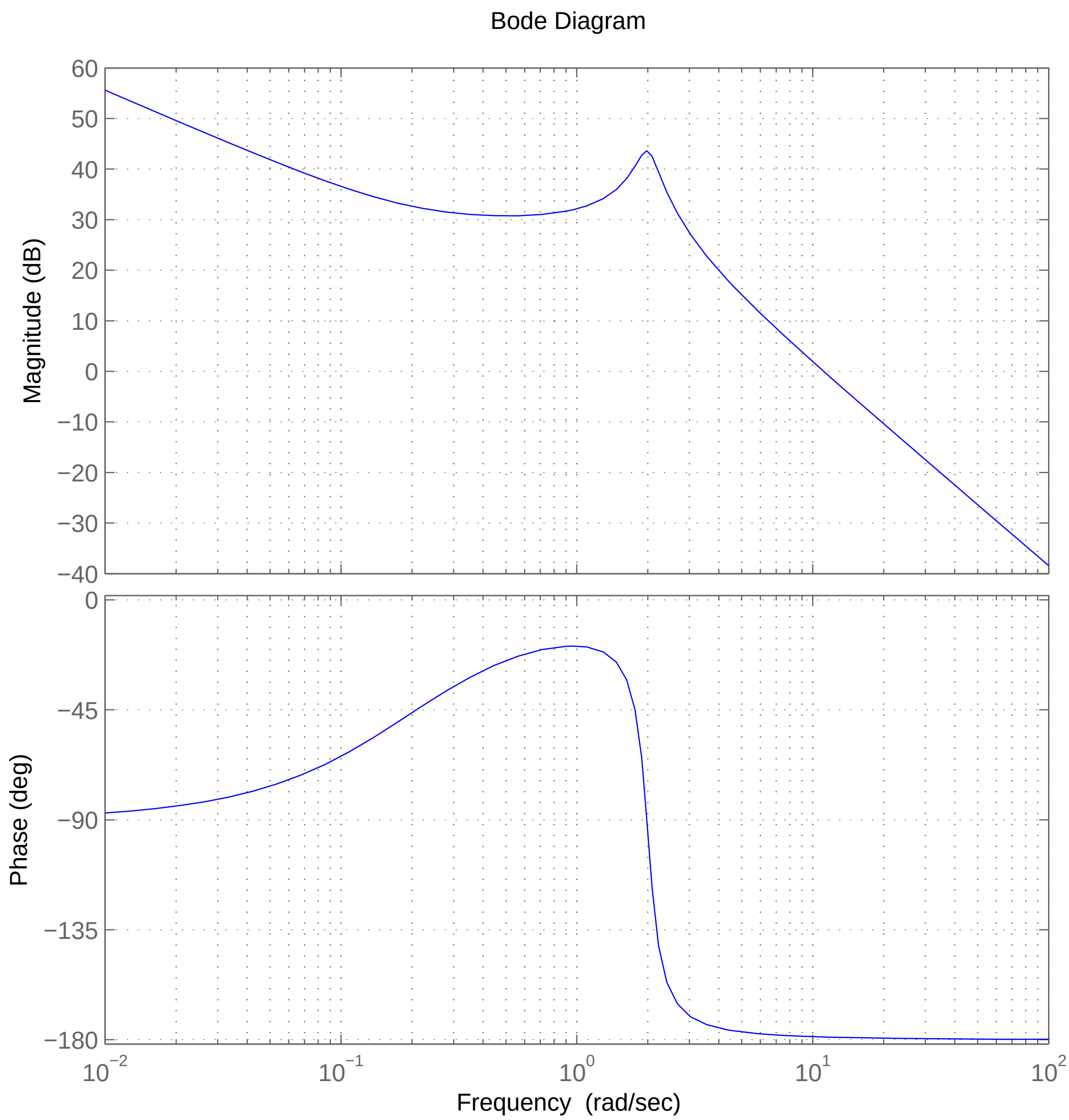


Fig. 3

Note: an additional copy of Fig. 3 is attached at the end of this paper. This should be annotated with your constructions and handed in with your answer to this question.

3 (a) What is the Nyquist stability criterion as it applies to the feedback system shown in Fig. 4 where K is a constant gain and $G(s)$ is an asymptotically stable linear system? [4]

(b) For the feedback system shown in Fig. 4

$$G(s) = \frac{e^{-s\tau}}{s+1}.$$

(i) Find the minimum value of K such that $|KG(j\omega)| > 1$ for all $\omega < 2$. [4]

(ii) For the value of K found in (b)(i), sketch the Nyquist diagrams for $\tau = 0$ and $\tau = 0.5$. What is the maximum value of τ, τ_{crit}, for which the system remains stable? [6]

(iii) For the case $\tau = \tau_{\text{crit}}$ and K set such that the gain margin is equal to 2, find the corresponding phase margin. For what frequencies is the magnitude of the steady-state gain from r to y equal to unity? [6]

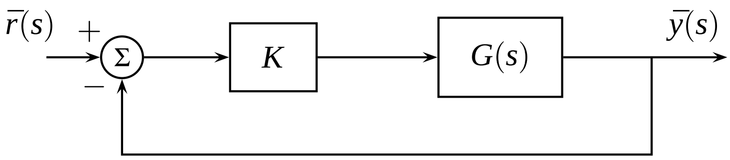


Fig. 4

SECTION B

*Answer not more than **two** questions from this section.*

4 (a) Show that the following functions form a valid Fourier transform pair:

$$x(t) = \frac{\sin \Omega t}{\pi t}, \quad X(\omega) = \begin{cases} 1, & |\omega| < \Omega \\ 0, & \text{Otherwise} \end{cases}$$

where Ω is a known constant. [5]

(b) Hence show that the following convolution result applies to two functions of the form of $x(t)$ in part (a):

$$\frac{\sin \Omega_1 t}{\pi t} * \frac{\sin \Omega_2 t}{\pi t} = \frac{\sin \Omega_3 t}{\pi t}$$

where Ω_3 should be determined and expressed as a function of the constants Ω_1 and Ω_2. [5]

(c) A filter with frequency response $H(\omega)$ is defined as follows:

$$H(\omega) = \begin{cases} -j, & \omega > 0 \\ 0, & \omega = 0 \\ j, & \omega < 0 \end{cases}$$

(note that this frequency response is *purely imaginary* except at $\omega = 0$).

Suppose that a signal as in part (a), $x(t) = \frac{\sin \Omega t}{\pi t}$, is input to this filter. Show that the output of the filter will be [6]

$$y(t) = \frac{1 - \cos(\Omega t)}{\pi t}$$

(d) The signal $y(t)$ from part (c) is now reapplied as the input to an identical filter $H(\omega)$. Determine the output of this second filter. Comment on the result of applying the filter $H(\omega)$ twice to a general input signal. [4]

5 (a) The length-N Discrete Fourier Transform (DFT) is defined as follows, for a digitally sampled signal x_n:

$$X_m = \sum_{n=0}^{N-1} x_n \exp(-jnm2\pi/N)$$

Explain how the Discrete Fourier Transform is derived, starting with the Fourier transform of a continuous-time signal that has been sampled with a train of delta functions having sampling period T. To which frequency does the component m correspond in the above DFT? [5]

(b) Show that, for real-valued data x_n,

$$X_{N-p} = X_p^*$$

where $*$ denotes the complex conjugate operation. [2]

(c) In a physical experiment it is desired to detect the presence of sinusoidal signals by first digitising and then applying a DFT of length $N = 4$ samples.

(i) If the maximum frequency of the sinusoids is 10Hz, suggest, with justification, an appropriate sampling frequency for the digitisation, taking account of the properties of a practical anti-aliasing filter. [2]

(ii) Determine the required resolution, in bits, for the A/D converter (ADC), if the average signal-to-noise ratio at its output must be at least 30dB, assuming that the sinusoids always cover the entire range of the ADC. [4]

(iii) The digitised signals are now of the form

$$x_n = \sin(\omega nT),$$

with $T = 0.01$s. Determine the $N = 4$ DFT for the two cases $\omega = 50\pi$ and $\omega = 25\pi$, and comment on these results. [7]

6 (a) Describe the principal methods for multiple channel access in wireless communications, outlining the differences between them. [5]

(b) A binary data stream of rate $R = 100$kbit/s is to be transmitted in a wireless channel at a carrier frequency of f_c using ASK. Rectangular pulses are used which fill the entire bit period and the binary data is transmitted as $0 \rightarrow -5$V and $1 \rightarrow +5$V.

Determine and sketch the power spectrum of the transmitted signal, clearly identifying the positions of any nulls, central lobes and side-lobes. [6]

(c) FDMA is used to multiplex K users in a channel of bandwidth B. The channel noise is additive, white and Gaussian. Write down an expression for the channel capacity per user, assuming that the total transmitted power is P and that the noise power spectral density in N_0. Sketch the capacity per user as a function of B. [4]

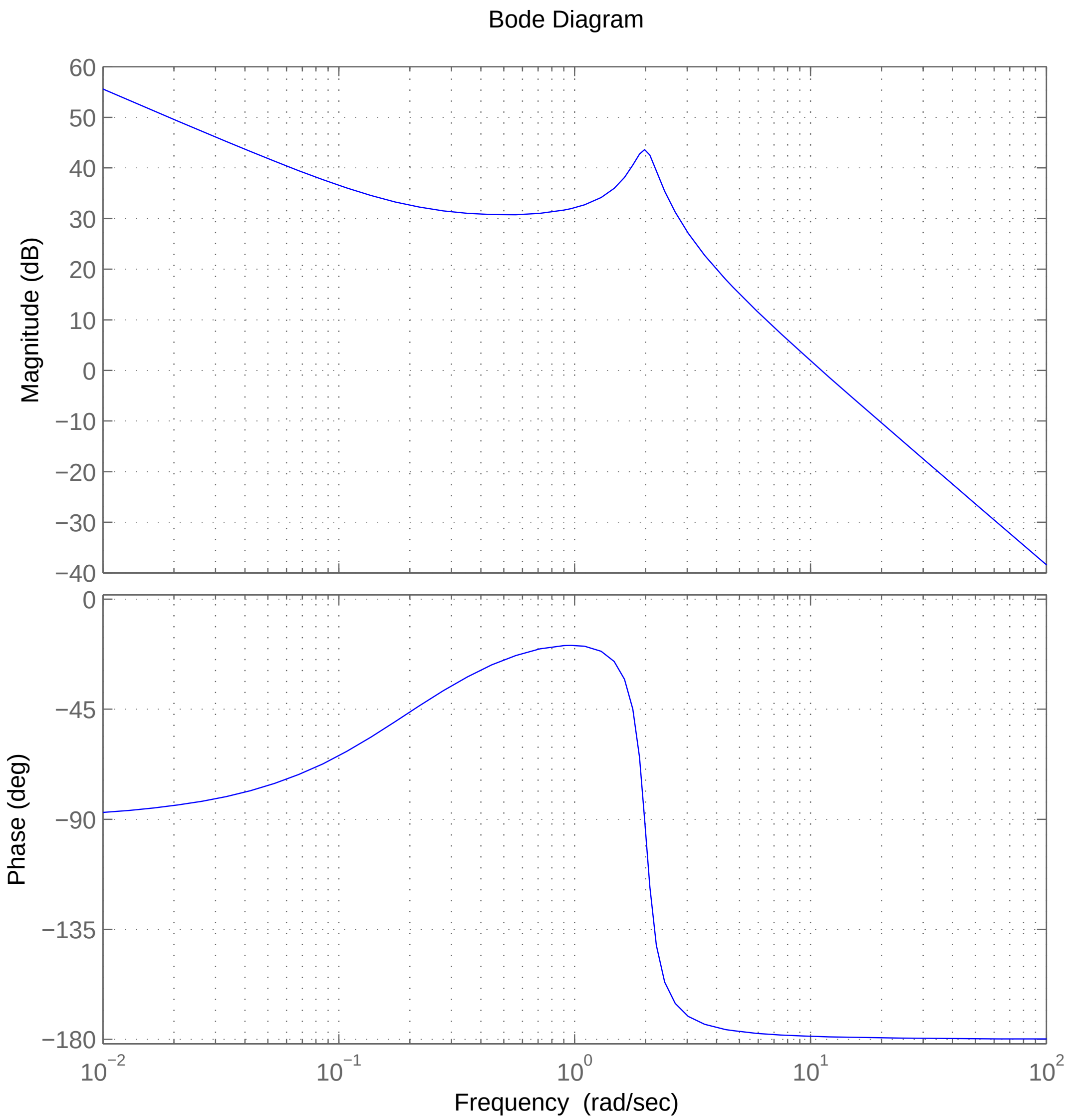
(d) Suppose FDMA is used to transmit a number of ASK users which are modulated as in part (b), in a channel that extends from 100MHz to 200MHz.

(i) Determine the maximum number of users that may be accommodated in this channel, assuming that the spectrum of the ASK signal does not cause interference beyond the first side-lobe of each individual user's transmitted spectrum. [3]

(ii) Repeat the calculation if a guard band is introduced between users. The guard band is equal to the spacing between the nulls in each user's spectrum. [2]

END OF PAPER

Candidate Number: xs



Copy of Fig. 3. This should be annotated with your constructions and handed in with your answer to question 2.