ENGINEERING TRIPOS PART IB JUNE 2012

CHAIRMAN PROF. P DAVIDSON

MONDAY 4TH JUNE 2012 9 TO 11

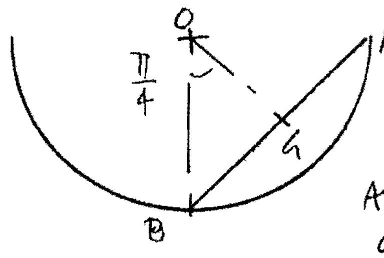
PAPER 1 SOLUTIONS – MECHANICS

AUTHORS :

PROF. J WILLIAMS

DR G CSANYI

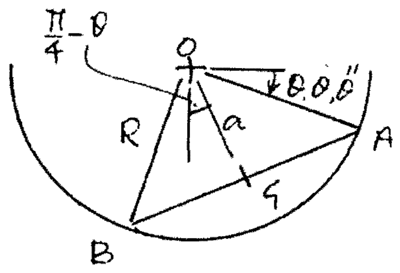
1.



initially

$AB = \sqrt{2}R$

$a = \frac{\sqrt{2}}{2}R$



time t

By energy

loss of p.e. = gain of k.e.

$$\left\{a\cos\left(\frac{\pi}{4}-\theta\right) - a\cos\frac{\pi}{4}\right\}mg = \frac{1}{2}m(a\dot\theta)^2 + \frac{1}{2}I_g\dot\theta^2$$

But $I_g = \frac{ma^2}{3} = \frac{mR^2}{6}$

$$\therefore \frac{\sqrt{2}}{\cancel{2}}\cancel{R}\left\{\cos\frac{\pi}{4}\cos\theta + \sin\frac{\pi}{4}\sin\theta - \cos\frac{\pi}{4}\right\}\cancel{m}g = \frac{1}{\cancel{2}}\cancel{m}\frac{R^{\cancel{2}}}{2}\dot\theta^2 + \frac{1}{\cancel{2}}\frac{\cancel{m}R^{\cancel{2}}}{6}\dot\theta^2$$

$$\cancel{\sqrt{2}}\left\{\frac{\cos\theta+\sin\theta}{\cancel{\sqrt{2}}} - \frac{1}{\cancel{\sqrt{2}}}\right\}g = \frac{2}{3}R\dot\theta^2$$

$\dot\theta^2 = \frac{3g}{2R}(\cos\theta+\sin\theta-1)$ $\qquad$ $\cos\theta+\sin\theta-1 = \frac{2R}{3g}\dot\theta^2$ — ①

So when $\theta = \pi/4$ $\qquad$ $\frac{2}{\sqrt{2}} - 1 = \frac{2R}{3g}\dot\theta^2$

i.e. $\dot\theta^2 = \frac{3g}{2R}(\sqrt{2}-1)$

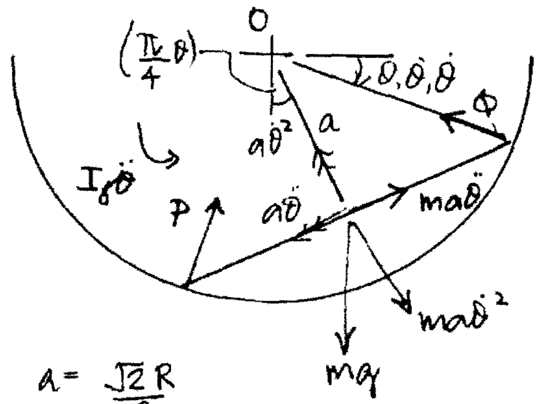
Diff ① wrt t

$$\frac{2R}{3g}\cdot 2\cancel{\dot\theta}\,\ddot\theta = -\sin\theta\,\cancel{\dot\theta} + \cos\theta\,\cancel{\dot\theta}$$

$$\ddot\theta = \frac{3g}{4R}(\cos\theta-\sin\theta)$$

when $\theta = \frac{\pi}{4}$ $\qquad$ $\ddot\theta = 0$

8.



$a = \frac{\sqrt{2}R}{2}$

moments about O

$mg\,a \sin(\frac{\pi}{4}-\theta) - ma^2\ddot\theta - I_g\ddot\theta = 0$

But $I_g = \frac{ma^2}{3} = \frac{mR^2}{6}$

$\therefore \left(m\frac{R^2}{2} + m\frac{R^2}{6}\right)\ddot\theta = mg\frac{\sqrt{2}}{2}R \sin(\frac{\pi}{4}-\theta)$

$\ddot\theta = \frac{3g}{2\sqrt{2}R}\sin(\pi/4-\theta)$ $\qquad \frac{2}{3}R\ddot\theta = \frac{g}{\sqrt{2}}\sin(\frac{\pi}{4}-\theta)$

$\therefore$ When $\theta = \pi/4 \quad \ddot\theta = 0$

putting $\ddot\theta = \dot\theta\frac{d\dot\theta}{d\theta}$

$\frac{2}{3}R\dot\theta\frac{d\dot\theta}{d\theta} = \frac{g}{\sqrt{2}}\sin(\pi/4-\theta)$

$\therefore \frac{2\sqrt{2}R}{3g}\dot\theta\, d\dot\theta = \sin(\pi/4-\theta)d\theta$

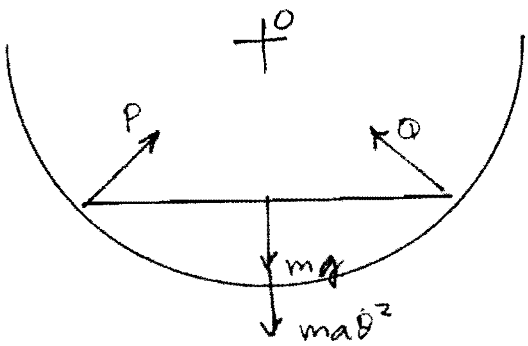
hence $\frac{\sqrt{2}R}{3g}\dot\theta^2 = +\cos(\pi/4-\theta) + A$

When $\theta = 0, \dot\theta = 0 \quad \therefore A = \cos\pi/4$

$\dot\theta^2 = \frac{3g}{\sqrt{2}R}\left\{\cos(\pi/4-\theta) - \cos\pi/4\right\}$

$\therefore$ when $\theta = \pi/4 \quad \dot\theta^2 = \frac{3g}{\sqrt{2}R}\left(1 - \frac{\sqrt{2}}{2}\right)$

$\dot\theta^2 = \frac{3g}{2R}(\sqrt{2}-1)$



Summary P=Q

$$2P\cos\frac{\pi}{4} = mg + ma\dot{\theta}^2$$

$$\cancel{2}P\,\frac{\sqrt{2}}{\cancel{2}} = mg + m\frac{R}{\sqrt{2}}\cdot\frac{3g}{2R}(\sqrt{2}-1)$$

$$\therefore\ P.\sqrt{2} = mg + \frac{3}{2}mg\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\therefore\ P = \frac{5\sqrt{2}-3}{4}\,mg$$

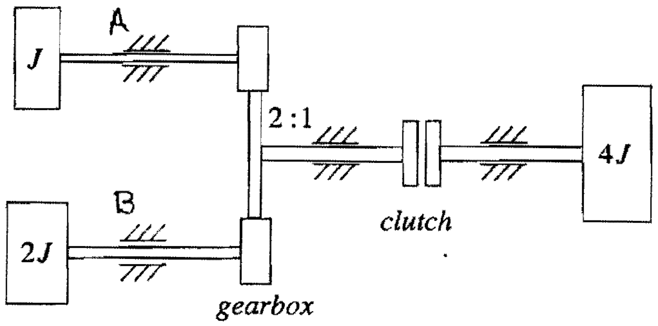
Examiner's Comments - Section A

Q1) The moment of inertia was computed correctly by almost all, including those who chose to compute it about O rather than G. The biggest stumbling block for part c) was the correct orientation of the centripetal and angular accelerations. An alarming number thought that angular acceleration is radial and centripetal acceleration is tangential. Few realised that the reactions must in any case be normal, marks were awarded to those who did, as they were to those who stated that the two reactions are equal in magnitude due to symmetry.

Q2) Over half the solutions assumed angular momentum conservation along the axis of the clutch, and so quickly got the wrong answer and, with no chance for a consistency check, proceeded to substitute the results into b) and c). Those who did this correctly were marked generously.

Q3) Part a) of this was an easy piece of geometry, and most who attempted it got it right. In part b) most of the confusion came from identifying which of the two angles to use when determining the potential and kinetic energy terms (for those who did it that way), or by forgetting either the mass or the acceleration terms for those who took moments to write down the equation of motion. Part c) was attempted by comparatively few, and again was very poorly done, most forgot some terms in the force balance,: very few figured out that they needed to work at the limit of the oscillation by setting the angular velocity to zero.

2.



Let ω_f be final speed of 4J flywheel. Smaller flywheels rotate at $2\omega_f$.

(a)

It is tempting to suppose, as many candidates did, that moment of momentum is conserved along the axis of the larger flywheel and so to say that

$$4J\Omega = 4J.\omega_f + J.2\omega_f + 2J.2\omega_f$$

So that $\omega_f = \frac{4}{10}\Omega$. But this is not correct.

Moment of mtm. is conserved along an axis about which there is no externally applied torque. The reactions at the bearings A and B provide such a moment.

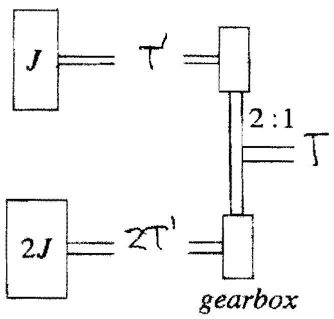
To get the correct solution it is necessary <u>either</u> to consider the changes in moment of mtm of each part of the system as below <u>or</u> to replace the two smaller flywheels by a single equivalent inertia which is co-axial with the 4J flywheel.

<u>Method 1</u> for largest flywheel $\int_0^\tau T\,dt = 4J(\Omega - \omega_f)$ —①

τ = time of slip

where T is torque at any time transmitted by clutch.

Since accns. of two smaller flywheels must be same then if torque on smaller is T' then that applied to larger is $2T'$



To relate T' to T, consider g'box

g'box torques

By v work $T.1 = T'2 + 2T'.2$

compatible velocities

$\therefore$ $\underline{T' = T/6}$

Now for smallest flywheel $\int_0^{\tau} T'\,dt = J.2\omega_f$

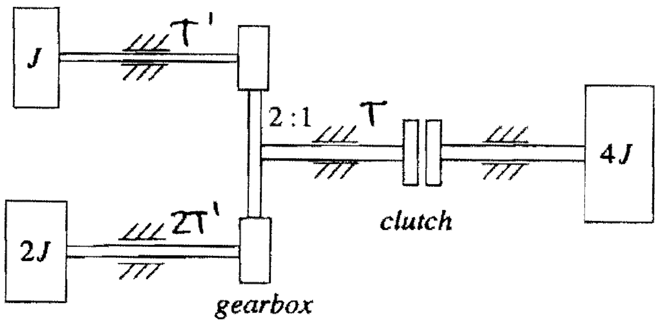
i.e. $\frac{1}{6}\int_0^{\tau} T\,dt = J.2\omega_f$ — ②

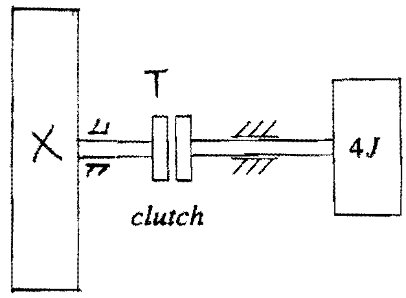
Hence from ① & ② $4\cancel{J}(\Omega-\omega_f) = 12\cancel{J}\omega_f$

$\therefore\ \underline{\omega_f = \frac{\Omega}{4}}$

Method 2

'equivalent system'





There must be an equivalent rotating system consisting of a single flywheel of unknown inertia, say X, fixed to a shaft which is colinear with the input shaft and which is dynamically indistinguishable from the specified system.

Now for this equivalent system, moment of mtm is conserved along axis ∵ there are no off-set bearing reactions.

$\therefore\ \underline{4J\Omega = (4J+X)\omega_f}$ — ①

If torque at any stage transmitted by clutch is T, then

$\underline{T = X\dot{\omega}}$ — ②

But as before by considering equilibrium of g'box

$T' = T/6$

Hence for smallest flywheel, $T' = J.2\dot{\omega}$

i.e. $T = 6.J.2\dot{\omega} = 12J\dot{\omega}$ — ③

Hence from ② & ③ $T = X\dot{\omega} = 12J\dot{\omega}$, i.e. $X = 12J$

then from ① $4J\Omega = (4J+12J)\omega_f$ $\underline{\omega_f = \frac{\Omega}{4}}$

(b) original ke $= \frac{1}{2} \cdot 4J \cdot \Omega^2 = 2J\Omega^2$

final ke $= \frac{1}{2} 4J\, \omega_f^2 + \frac{1}{2} J(2\omega_f)^2 + \frac{1}{2} \cdot 2J(2\omega_f)^2$

$= (2+2+4) J \cdot \left(\frac{\Omega}{4}\right)^2 = \frac{J\Omega^2}{2}$

$\therefore \frac{\text{ke lost}}{\text{original}} = \frac{\frac{3}{2} J\Omega^2}{2J\Omega^2} = \underline{\frac{3}{4}}$

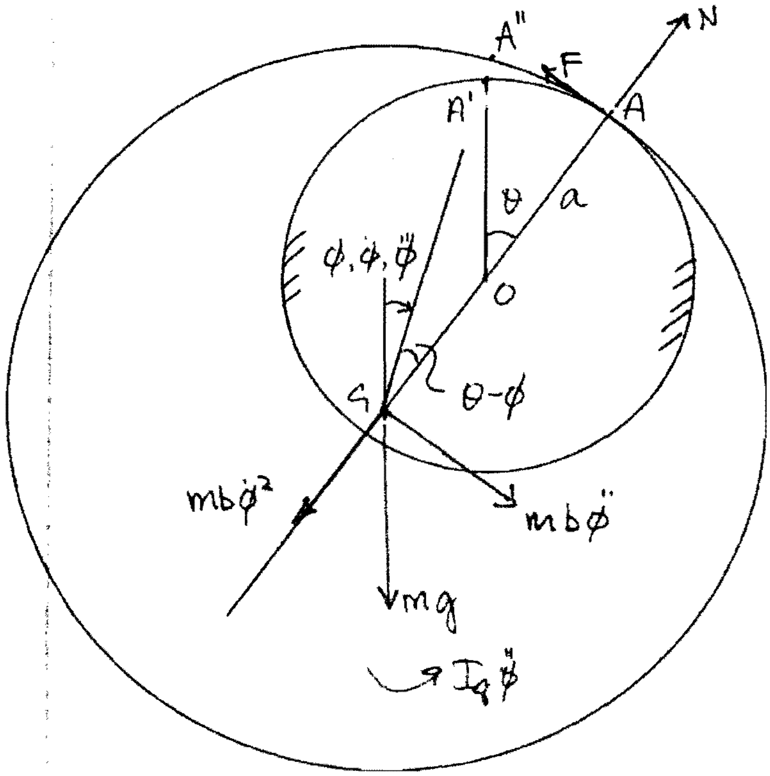
(c) If T constant then for largest flywheel

$$\int_0^\tau T\, dt = 4J(\Omega - \omega_f)$$

becomes $T \cdot \tau = 4J \frac{3}{4} \Omega$

i.e. $\tau = \underline{\frac{3J\Omega}{T}}$

3



Hoop rotates ϕ such that arc AA' = arc AA''

$\therefore OA\theta = GA(\theta-\phi)$

i.e. $a\theta = b(\theta-\phi)$

$\therefore \underline{b\phi = (b-a)\theta}$

$I_g = mb^2$

Taking moments about A

$$mgb\sin\theta + mb^2\ddot\phi + I_g\ddot\phi = 0$$

If θ small $\sin\theta \simeq \theta = \dfrac{b}{b-a}\phi$

$$\therefore \cancel{m}g\frac{b^{\cancel{2}}\phi}{b-a} + \cancel{m}b^{\cancel{2}}\ddot\phi + \cancel{m}b^{\cancel{2}}\ddot\phi = 0$$

$$\underline{\ddot\phi = -\frac{g}{2(b-a)}\phi}$$

$$\underline{\omega_n^2 = \frac{g}{2(b-a)}}$$

$\therefore$ If $b = \tfrac{3}{2}a$ $\quad \underline{\omega_n^2 = \dfrac{g}{a}}$

Resolving
$$\begin{cases} F = mb\ddot\phi + mg\sin\theta \\ N = mb\dot\phi^2 + mg\cos\theta \end{cases}$$

Now if $b = \tfrac{3}{2}a$ $\quad \theta = 3\phi$

and if $\quad \phi = \Phi \cos \omega_n t \quad$ in extreme position

$$\ddot{\phi} = -\omega_n^2 \Phi = -\frac{g}{a}\Phi \quad \text{and} \quad \dot{\phi} = 0$$

$$\therefore \quad \frac{F}{N} = \frac{-b/a\,\Phi + \sin 3\Phi}{\cos 3\Phi}$$

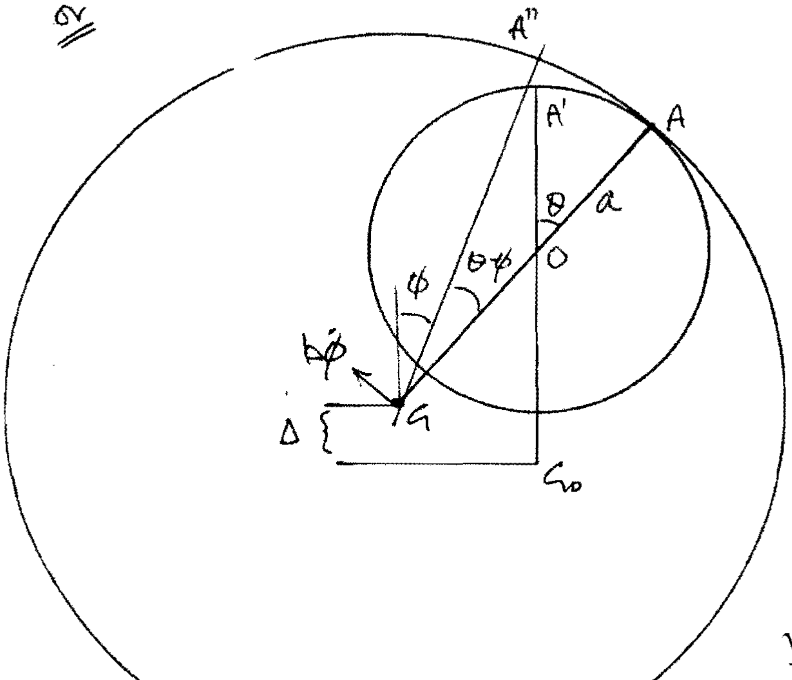
If Φ small $\quad \dfrac{F}{N} = \dfrac{-\frac{3}{2}\Phi + 3\Phi}{1}$

i.e. COF for no slip $\geq \frac{3}{2}\Phi$

i.e. if COF $= 0.2 \quad \Phi_{max} < \dfrac{2 \times 0.2}{3}$

$\Rightarrow 7.64^\circ$

or



$OA.\theta = GA(\theta - \phi)$

i.e. $a\theta = b(\theta - \phi)$

$$\therefore \quad \phi = \frac{b-a}{b}\theta$$

$V(\theta) = mg\Delta$

But $\Delta = (b-a)(1-\cos\theta)$

$$\frac{V(\theta)}{mg} = (b-a)(1-\cos\theta)$$

$$\frac{V'(\theta)}{mg} = (b-a)\sin\theta$$

$V'(\theta) = 0$ when $\theta = 0$

$$\frac{V''(\theta)}{mg} = (b-a)\cos\theta$$

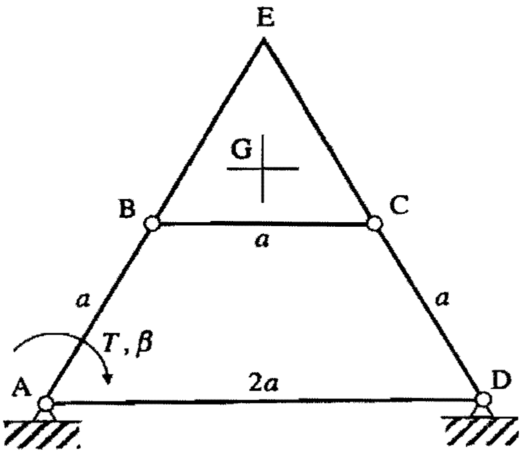
But K.e. $= \frac{1}{2}m(b\dot{\phi})^2 + \frac{1}{2}I_g\dot{\phi}^2 = \frac{1}{2}\,2mb^2\dot{\phi}^2 \quad \dfrac{m\cancel{b^2}(b-a)^2\dot{\theta}^2}{\cancel{b^2}}$

$$\omega_n^2 = \frac{V''(\theta)_0}{I(\theta)_0} = \frac{(b-a)mg}{2(b-a)^2 m} = \frac{g}{2(b-a)}$$

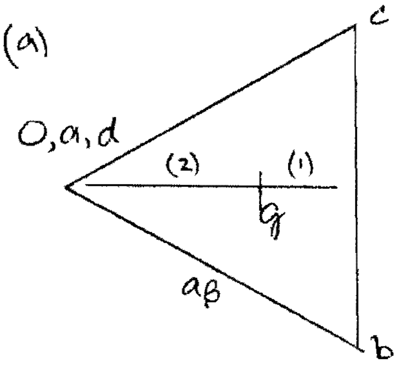
Data Book

$\therefore$ if $b = 3/2\,a \quad \omega_n^2 = g/a$

4.



(a)



accn diagram
(no centripetal terms)

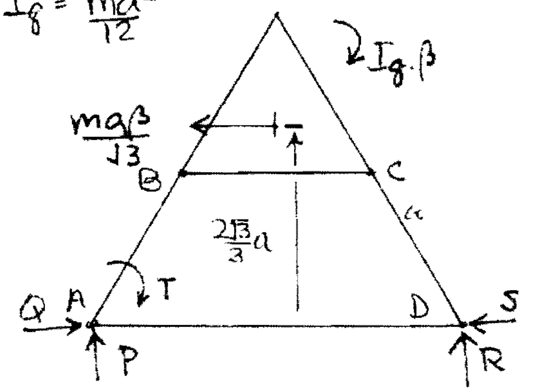
$$\omega_{BC} = \frac{bc}{BC} = \frac{a\beta}{a} = \beta$$

hence, using accn image to find g, accn of G = og

$$= \frac{2}{3}\frac{\sqrt{3}}{2}\,\overrightarrow{a\beta} = \frac{a\beta}{\sqrt{3}}$$

$I_g = \frac{ma^2}{12}$

(b)



now consider D'Alembert for mechanism

by virtual work, since velocity (or displacement) diagram same form as accn diagram

$$T\cancel{\omega} - \frac{ma\beta}{\sqrt{3}}\frac{a\cancel{\omega}}{\sqrt{3}} - \frac{ma^2}{12}\beta\cancel{\omega} = 0$$

$$\therefore T = \frac{5ma^2\beta}{12}$$

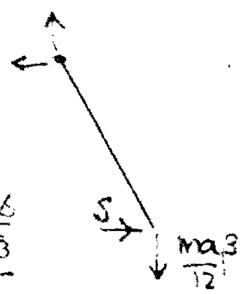
(c) Find R by taking moments about A

$$T - \frac{ma\beta}{\sqrt{3}}\frac{2\sqrt{3}a}{3} + \frac{ma^2}{12}\beta - R.2a = 0$$

ie $$\frac{5}{12}ma^2\beta - \frac{2}{3}ma^2\beta + \frac{ma^2}{12}\beta = 2Ra$$

$$R = -\frac{ma\beta}{12}$$

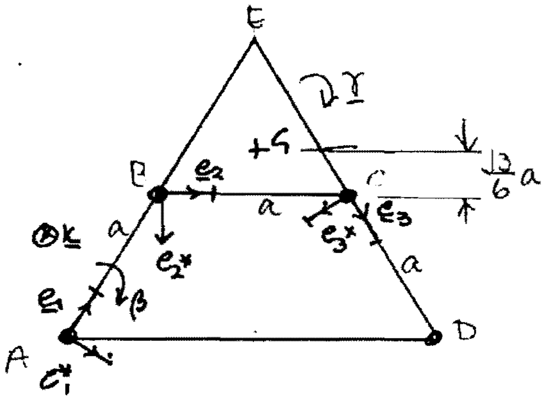
To find S take moments about C for bar CD



$$\sqrt{R^2+S^2} = \frac{ma\beta}{6\sqrt{3}}$$

$$S\frac{\sqrt{3}}{2}a = R.\frac{a}{2} \qquad \therefore S = \frac{ma\beta}{12\sqrt{3}}$$

4 (a) by vectors



Define $e_1, e_1^*, e_2, e_2^*, e_3, e_3^*$ as indicated $\boldsymbol{\beta} = \beta k$ (all vels zero)

and $k \times e_1 = e_1^*$ etc.

$$a_B = \overset{0}{a_A} + \boldsymbol{\beta} \times AB$$

$$= a\beta k \times e_1 = a\beta e_1^*$$

$$a_G = a_B + \gamma k \times \left(\frac{a}{2} e_2 - \frac{\sqrt{3}}{6} a e_2^*\right)$$

Suppose angular accln. of BCE is γ

$$a_G = a\beta e_1^* + \frac{a\gamma}{2} e_2^* + \frac{\sqrt{3}}{6} a\gamma e_2$$

But $a_C = a_B + \gamma k \times a e_2$

$$= a\beta e_1^* + a\gamma e_2^*$$

But a_C must be $\perp$r to DC, ie. $a_C \cdot e_3 = 0$

$$\therefore \quad a\beta e_1^* \cdot e_3 + a\gamma e_2^* \cdot e_3 = 0$$

But $e_1^* \cdot e_3 = \cos 30° = \frac{\sqrt{3}}{2}$ and $e_2^* \cdot e_3 = \frac{\sqrt{3}}{2}$

$$\therefore \quad a\beta + a\gamma = 0$$

$$\gamma = -\beta$$

$$\therefore \quad a_G = a\beta e_1^* - \frac{a\beta}{2} e_2^* - \frac{\sqrt{3}}{6} a\beta e_2$$

Now component of a_G $\perp$r to BC is $a_G \cdot e_2^*$

$$\text{ie.} = a\beta e_1^* \cdot e_2^* - \frac{a\beta}{2} e_2^* \cdot e_2^* - \frac{\sqrt{3}}{6} a\beta \cancel{e_2 \cdot e_2^*}$$

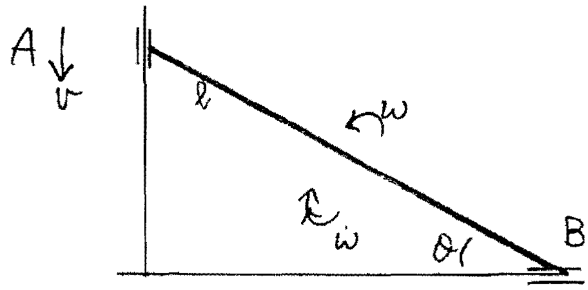
$$= a\beta \cdot \frac{1}{2} - \frac{a\beta}{2} = 0$$

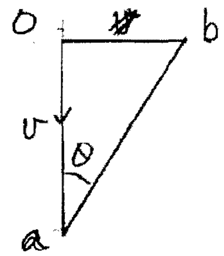
ie. a_G must be $\parallel$el to BC and of

magnitude $a_G \cdot e_2 = a\beta e_1^* \cdot e_2 - \frac{a\beta}{2} \cancel{e_2^* \cdot e_2} - \frac{\sqrt{3}}{6} a\beta e_2 \cdot e_2$

$$= a\beta \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} a\beta = \frac{a\beta}{\sqrt{3}} \text{ in direction } e_2$$

5.



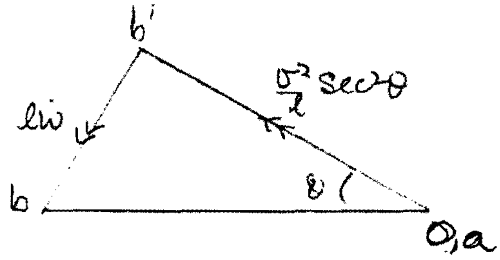


velocity diagram.

$$\overset{\curvearrowleft}{\omega} = \frac{ab}{AB} = \frac{v}{l\cos\theta} = \frac{v}{l}\sec\theta$$

accelerations

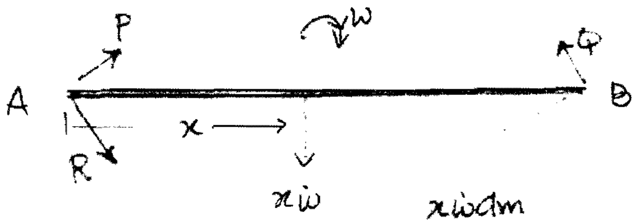
B → A at $l\omega^2 = \dfrac{v^2\sec^2\theta}{l}$

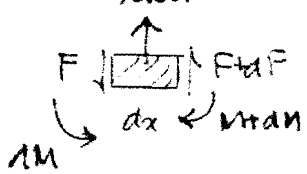


$$\tan\theta = \frac{l\dot\omega}{\frac{v^2}{l}\sec^2\theta}$$

$$\therefore \overset{\curvearrowright}{\dot\omega} = \frac{v^2}{l^2}\sec^2\theta\tan\theta$$

acch diagram





Consider inertial loading on rod.

$$dF = -x\dot\omega\,dm$$

i.e. $\dfrac{dF}{dx} = -x\dot\omega\dfrac{m}{l}$

$$\therefore F = -\frac{x^2\dot\omega m}{2l} + A \qquad -①$$

But $\dfrac{dM}{dx} = F$

$$\therefore M = -\frac{x^3\dot\omega m}{6l} + Ax + B$$

But BM = 0 at $x = 0$ & $x = l$

$\therefore$ B = 0 and $0 = A - \frac{l^2 m \dot{w}}{6l}$

$\therefore$ $A = \frac{l m \dot{w}}{6}$

$\therefore$ $M = -\frac{x^3 \dot{w} m}{6l} + \frac{l m \dot{w} x}{6}$

BM max when S.F. F = 0 i.e. $x = \frac{2lA}{m\dot{w}}$ from ①

i.e. $x^2 = \frac{2l}{m\dot{w}} \cdot \frac{l m \dot{w}}{6}$ $\therefore$ $x = \frac{l}{\sqrt{3}}$

and $M = -\frac{l^3 m \dot{w}}{3\sqrt{3}.6l} + \frac{m \dot{w} l^2}{6\sqrt{3}}$

$= \frac{2}{3} \frac{m \dot{w} l^2}{6\sqrt{3}}$

$= \frac{2}{3} \frac{m l^2}{6\sqrt{3}} \frac{v^2}{l^2} \sec^2\theta \tan\theta$

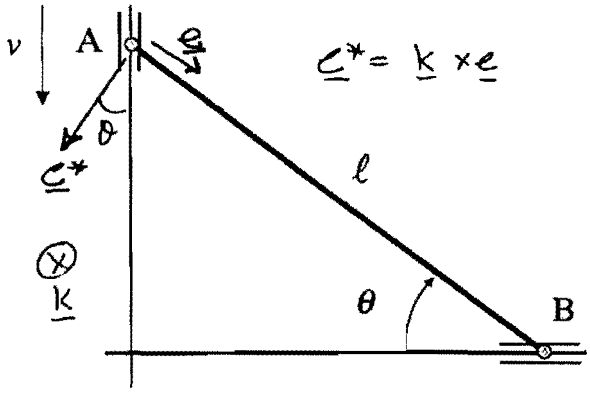
i.e. $|M| = \frac{m v^2 \sec^2\theta \tan\theta}{9\sqrt{3}}$

Notice that there must be a force, say R, applied at A to maintain this point moving at constant velocity v.

Resolving forces shows that $R = \phi = -\frac{ml}{3} \dot{w} \sec\theta$

and that $P = -\frac{ml}{2} \dot{w} \csc\theta$

5(a) By vectors



$$\underline{v}_B = \underline{v}_A + \underline{\omega} \times \underline{AB}$$

$$= v\sin\theta\, \underline{e} + v\cos\theta\, \underline{e}^* + \omega\, \underline{k} \times l\underline{e}$$

$$= v\sin\theta\, \underline{e} + (v\cos\theta + l\omega)\, \underline{e}^*$$

But $\underline{v}_B \cdot (\underline{e}\sin\theta + \underline{e}^*\cos\theta) = 0$

$$\therefore \quad v\sin^2\theta + \cos\theta(v\cos\theta + l\omega) = 0$$

ie $\quad v + l\omega\cos\theta = 0$

$$\omega = -\frac{v}{l}\sec\theta$$

Similarly $\quad \underline{a}_B = \underline{a}_A - l\omega^2\underline{e} + \dot{\omega}\underline{k} \times l\underline{e}$

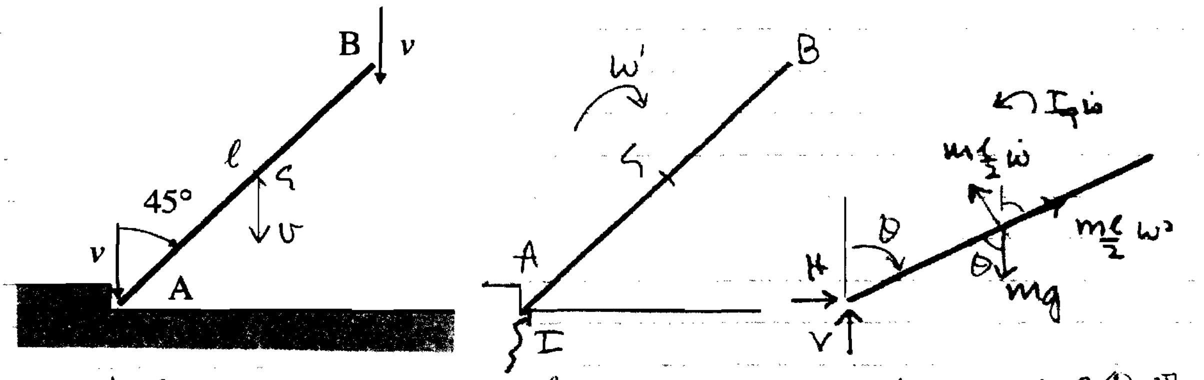
$\underline{a}_A = 0 \quad = 0 - l\omega^2\underline{e} + l\dot{\omega}\,\underline{e}^*$

But $\underline{a}_B \cdot (\underline{e}\sin\theta + \underline{e}^*\cos\theta) = 0$

$$\therefore \quad -l\omega^2\sin\theta + l\dot{\omega}\cos\theta = 0$$

$$\therefore \quad \dot{\omega} = \frac{l\omega^2}{l}\frac{\sin\theta}{\cos\theta} = \omega^2\tan\theta$$

or $\quad \dot{\omega} = \frac{v^2}{l}\sec^2\theta\tan\theta$



before impact after impact Subsequently $\frac{\pi}{4} < \theta < \pi/2$

(a) Since impulse applied at A and is of short duration reasonable to write that moment of momentum is conserved about A.

$$\text{Thus} \quad mv \frac{l}{2\sqrt{2}} = m\frac{l}{2}\omega' \frac{l}{2} + I_g \omega'$$

$$\text{i.e.}\ ml\frac{v}{2\sqrt{2}} = ml^2\left(\frac{1}{4}+\frac{1}{12}\right)\omega'$$

$$\therefore \quad \omega' = \underline{\frac{3v}{2\sqrt{2}\,l}}$$

(b) Initial ke $= \frac{1}{2}mv^2$.

$$\text{ke after impact} = \frac{1}{2}m\left(\frac{\ell}{2}\omega'\right)^2 + \frac{1}{2}I_g(\omega')^2$$

$$\therefore \text{loss of ke} = \frac{1}{2}mv^2 - \frac{1}{2}\frac{ml^2}{3}\frac{9v^2}{8l^2} = \frac{5}{16}mv^2$$

$$\frac{\Delta E}{E} = \frac{5/16}{1/2} = \underline{5/8}$$

(c) <u>During subsequent</u> motion energy conserved

$$\text{loss of pe} = mg\frac{\ell}{2}(\cos \pi/4 - \cos\theta) \qquad \pi/4 < \theta < \pi/2$$

$$\text{gain of ke} = \frac{1}{2}\frac{1}{3}ml^2\omega^2 - \frac{1}{2}\frac{1}{3}ml^2\omega'^2$$

$$\text{i.e.} \quad \frac{ml^2}{6}\omega^2 - \frac{3}{16}mv^2$$

$$\therefore \quad mg\frac{l}{2}\left(\cos\pi/4 - \cos\theta\right) = \frac{ml^2}{6}\omega^2 - \frac{3}{16}mv^2$$

$$\frac{l^2\omega^2}{6} = \frac{gl}{2}\left(\cos\frac{\pi}{4} - \cos\theta\right) + \frac{3}{16}mv^2$$

Taking moments about A

$$mg\frac{l}{2}\sin\theta = \frac{ml^2}{3}.\dot\omega$$

$$\therefore \quad \dot\omega = \frac{3g}{2l}\sin\theta$$

When rod loses contact with step $H \to 0$ (not V!)

$\therefore$ resolving horizontally

$$\cancel{m}\frac{\cancel{l}}{\cancel{2}}\dot\omega\cos\theta = \cancel{m}\frac{\cancel{l}}{\cancel{2}}\omega^2\sin\theta$$

So substituting for $\dot\omega$ and ω^2

$$\frac{3g}{2l}\cancel{\sin\theta}\cos\theta = \left\{\frac{3g}{l}\left(\cos\pi/4 - \cos\theta\right) + \frac{9}{8}\frac{v^2}{l^2}\right\}\cancel{\sin\theta}$$

Thus if $v^2 = \frac{lg}{2}$

$$\frac{3}{2}\cos\theta = 3\left(\cos\pi/4 - \cos\theta\right) + \frac{9}{16}$$

$$\therefore \quad \frac{9}{2}\cos\theta = \frac{3\sqrt{2}}{2} + \frac{9}{16}$$

$$\text{ie.} \quad \cos\theta = \frac{\sqrt{2}}{3} + \frac{1}{8}$$

$$\text{ie.} \quad \theta = 53.4^\circ$$

[V can be evaluated by resolving

$$V = mg - \frac{ml}{2}\dot\omega\sin\theta - \frac{ml}{2}\omega^2\cos\theta$$

Substitute for $\dot\omega$ and ω given, at this instant,

$$V = mg - \frac{3mg}{4}\left(\sin^2\theta + \cos^2\theta\right) = \frac{mg}{4}$$]

Examiner's comments – Section B

Q4) The solution of this sort of problem is made much simpler by a decent diagram drawn with a ruler. There were very few of these and far too many sloppily sketched free-hand efforts that bore no relation to the equilateral triangle specified in the problem. Most of the more successful attempts used velocity and acceleration diagrams but the idea of the velocity or acceleration image is not understood: if when you walk from B to C on the mechanism and you have to turn left to get to G then as you go from b to c on either the velocity of the acceleration diagrams you will also have to turn left to locate g. Despite the problem specifying that the mechanism was operating in the horizontal plane a significant number of candidates included gravity.

Many of the minority that attempted an analytical treatment got lost in a morass of $\underline{e}$ and $\underline{e}$*s and abandoned ship. The idea of taking a dot product between a vector and a unit vector to find the magnitude of the component of the vector in the direction of the unit vector is apparently unknown – or forgotten.

Q5) Some candidates, having demonstrated that $\omega = \frac{v}{\ell}\sec\theta$, were unable to successfully differentiate again with respect to time to derive $\dot{\omega} = \frac{v}{\ell}\sec\theta\tan\theta \times \dot{\theta}$. Having drawn a perfectly fine (and simple) acceleration diagram, a lot of candidates reverted to vectors to find the acceleration of the rod's centre of mass (say) G instead of simply locating its image g half-way between a and b on the diagram they had just drawn: doing this immediately demonstrates that the acceleration of G is parallel to the guide B.

Not unusual in part (b) was the statement that BM is a max when SF is zero and that this would be half way along the rod. Tempting to think this true, perhaps if θ is 45°, but not in fact the case because there must be a force at A in the direction of v to maintain the constant speed of this end of the rod. No candidate appreciated this and included such a force in their FBD though it's perfectly possible to get a correct solution for the BM with this omission by integrating twice and setting the BM zero at each end of the rod.

Q6) Momentum and energy. Generally well done – only arithmetic slips marring parts (a) and (b). In part (c) loss of contact with the step occurs when the *horizontal* component of the reaction falls to zero. At this instant the vertical component is still positive and equal to $mg/4$. A good number of complete solutions.

JKW
June 12