

ENGINEERING TRIPOS PART IB JUNE 2012

CHAIRMAN PROF. P DAVIDSON

TUESDAY 5TH JUNE 2012 9 TO 11

PAPER 3 SOLUTIONS – MATERIALS

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SECTION A

1 (a) A typical microstructure of wood is shown in Fig. 1. When loaded along the grain (axially) the cell walls are loaded in tension/compression and give the same modulus as a solid with the same cross section. The modulus is given by the modulus of the solid material making the walls multiplied by the relative density of the wood (the wood density relative to the solid density). When loaded radially or tangentially, the cells deform by bending and the modulus scales as the relative density to the third power. [5]

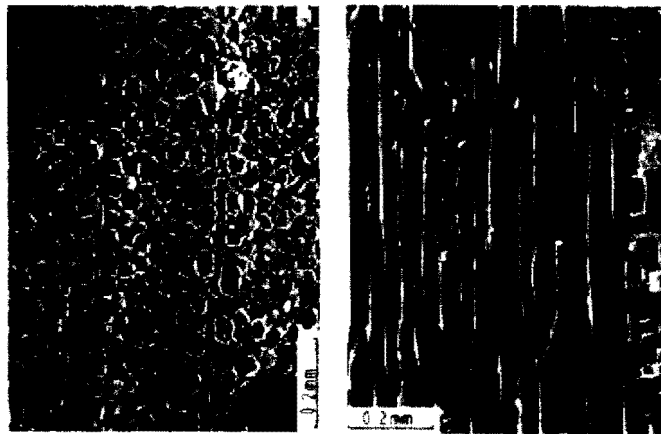


Fig. 1

(b) (i)
$$\bar{\rho} = \frac{\rho^*}{\rho_s} = \frac{4lt/2}{l^2} = 2\frac{t}{l} \quad [2]$$

(ii) By analogy with a unidirectional composite (see Materials data book) where $E_f = E_s$ and $E_m \simeq 0$:

$$E_x = V_f E_s = \frac{t}{l} E_s = \frac{1}{2} \bar{\rho} E_s$$

where E_s is the Young's modulus of the solid. [3]

- (c) (i) Fig. 2 shows the deformed shape of the unit cell for tension of the foam in the y direction. [3]

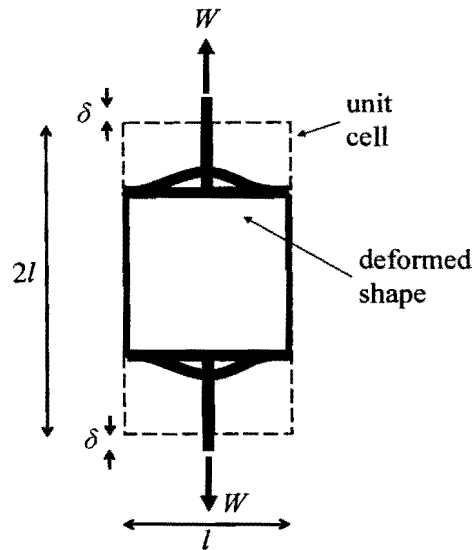


Fig. 2

- (ii) An expression for the Young's modulus of the foam in the y direction E_y in terms of $\bar{\rho}$ and E_s is obtained as follows:

The central deflection δ of a fixed-ended beam is given by

$$\delta = \frac{Wl^3}{192EI}$$

where $I = t^3/12$.

$$E_y = \frac{\bar{\sigma}}{\bar{\epsilon}} = \frac{W/l}{\delta/l} = \frac{W}{\delta} = \frac{192EI}{l^3} = \frac{192Et^3}{12l^3} = 16E_s \left(\frac{t}{l}\right)^3 = 2E_s \bar{\rho}^3$$

[7]

2 (a) Fig. 3 shows a free-body diagram of the forces acting on a vertical strip, of thickness δx , height $h(x)$ and unit depth at a typical horizontal co-ordinate x . [5]

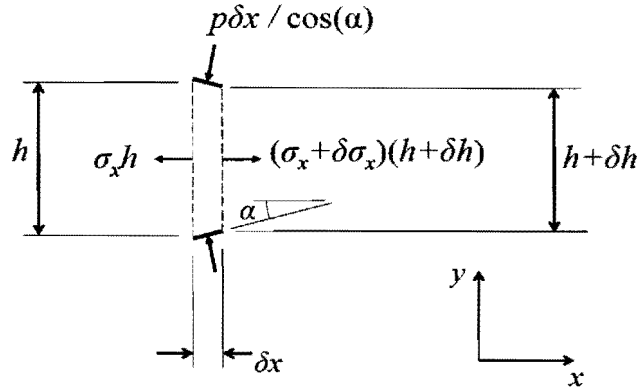


Fig. 3

(b) By considering the horizontal equilibrium of forces on the element sketched in Fig. 3:

$$(\sigma_x + \delta\sigma_x)(h + \delta h) - \sigma_x h - 2p\delta x \frac{\sin(\alpha)}{\cos(\alpha)} = 0$$

$$\delta\sigma_x h + \sigma_x \delta h - 2p\delta x \tan(\alpha) = 0$$

but

$$\delta h = -2\delta x \tan(\alpha)$$

thus

$$\delta\sigma_x h + \sigma_x \delta h + p\delta h = 0$$

and hence

$$\frac{d}{dh}(h\sigma_x) + p = 0$$

[5]

(c) The material is assumed to be a rigid ideally-plastic solid of yield strength Y . For a small die angle α it can be assumed that the die contact pressure p is a principal stress. The Tresca yield criterion therefore gives $\sigma_x + p = Y$ and thus

$$\frac{d}{dh}(h\sigma_x) + Y - \sigma_x = 0$$

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(cont.)

and

$$\frac{d\sigma_x}{dh}h + Y = 0$$

An expression for the drawing stress σ_{draw} as a function of the reduction ratio is obtained:

$$\int_0^{\sigma_{draw}} d\sigma_x = -Y \int_{h_{in}}^{h_{out}} \frac{1}{h} dh$$

$$\frac{\sigma_{draw}}{Y} = \ln \frac{h_{in}}{h_{out}}$$

[6]

(d) The maximum possible reduction ratio h_{in}/h_{out} that can be obtained through frictionless dies occurs when $\sigma_{draw} = Y$ and therefore when

$$\ln \frac{h_{in}}{h_{out}} = 1$$

i.e. when

$$\frac{h_{in}}{h_{out}} = e = 2.718$$

[4]

3 (a) *Hardenability* is the ability of a steel to form martensite on quenching - the lower the cooling rate needed to achieve a martensitic structure, the higher the hardenability. The implication for steel selection is that a larger component can be quenched to form martensite right to its centre, if the hardenability is high. This is a pre-requisite for subsequent tempering to give a strong, tough microstructure throughout the component.

A hardenable steel has a low critical cooling rate (CCR) for avoidance of diffusion controlled transformations occurring, and this forms martensite easily at the higher cooling rates encountered in welding. Forming martensite in steel welds is highly undesirable due to the likelihood of brittle fracture. Hence *weldability* requires a high CCR - the opposite of *hardenability*.

Alloying pushes the nose of the C-curves to longer times. The redistribution of the alloying elements takes time, delaying the phase transformation. Alloy steels therefore have a higher hardenability and a lower weldability than plain carbon steels.

[4]

(b) BS503M40 1% nickel steel (Fig 7.1, page 34 of the Materials Data Book) is a low hardenability material since a very rapid quench is needed to form martensite.

AC_3 is the point at which austenite becomes unstable and ferrite begins to form. AC_1 is the eutectoid temperature below which it is also possible to form iron carbide (either within pearlite or bainite).

M_S is the martensite start temperature. Undercooling below M_S causes the start of the displacive shear transformation to martensite.

In general the rate at which a phase transformation proceeds depends on the undercooling and the diffusion rate. The further from the transition temperature the greater the driving force, but the lower the temperature the slower the diffusion. There is a maximum rate at some temperature below the transition temperature which is reflected in the 'nose' of the TTT diagram. Both nucleation and growth depend on the same two temperature-dependent effects, and these combine in to a single transformation rate curve with a peak at a critical undercooling - hence the shape of the C-curves when plotted against time to a given fraction transformed.

[6]

(c) AC_1 and AC_3 can be determined from the Iron-Carbon equilibrium diagram (Fig 6.3, page 30 of the Materials Data Book). Since AC_3 occurs at 800°C and ferrite is forming first Fig 6.3 in the data book shows that there is about 0.3 wt.% carbon in BS503M40 steel.

[4]

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(cont.

(d) In each case start by austenitizing the steel by holding at 845C for at least 30 minutes.

(i) 100% martensite may be obtained by cooling the sample to room temperature in 2 seconds (or less).

(ii) 90% bainite, 10% martensite may be obtained by cooling to 400°C in 1 second, holding at this temperature for 40 seconds, and then quenching to room temperature.

(iii) 40% ferrite, 10% pearlite, 50% martensite may be obtained by cooling to 650°C in 2 seconds, holding at this temperature for 10 minutes, and then quenching to room temperature.

[6]

SECTION B

4 (a) (i) Figure 4 shows the graphical construction required for solving this question. At 94°C, the composition C_l of the liquid phase is about 15 wt.% and the composition C_v of the vapour is about 28 wt.%.

(ii) From Figure 4, we see that a system of average composition C_v needs to be taken to 92°C for its liquid phase to have the composition C_l . The corresponding vapour phase is now at a composition 38 wt.%.

(iii) The maximal weight fraction that can be reached with this process is 95.6 wt.% Ethanol. As this point is approached, the compositions of the vapour and liquid phases converge toward the same value. Very little can be gained by extracting the vapour anymore. At exactly 95.6 wt.% Ethanol, vapour and liquid phases cannot coexist any more and extracting the vapour would be pointless.

(iv) Starting from a mixture of water-ethanol with less than 95.6 wt.% Ethanol, it is possible to purify water by collecting the liquid phase instead of the gas phase. Starting from a composition between 0 and 95.6 wt.% Ethanol, repeating this process indefinitely would lead in principle to pure water. If one starts with more than 95.6 wt.% Ethanol, one could increase water content by collecting the vapour phase, but only down to 95.6 wt.% Ethanol.

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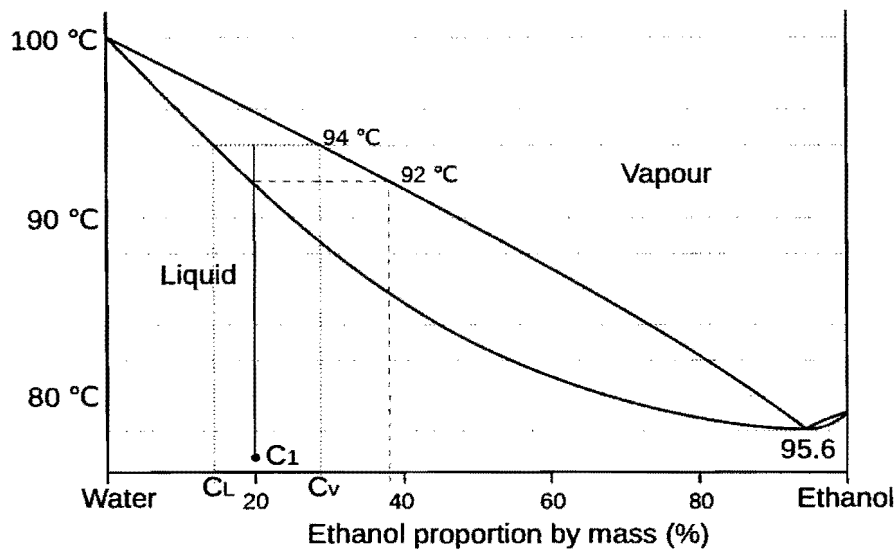


Fig. 4

- (b) (i) See Figure 5A for the location of the temperatures. T_4 and T_5 can not be found precisely; we only know that they are below T_3 and above T_2 respectively. To place accurately the temperatures T_1 and T_6 , we need to identify from the free energy graphs the composition of the liquid and solid phases. By drawing a tangent line to both free energy curves (see Figure 5B), we find for T_1 that the liquid composition is 50 wt.% and the solid composition is about 75 wt.%. T_1 can then be found by looking for the temperature producing these two phases. The same reasoning can be applied to find T_6 .

(ii) A mixture containing 60% of A and 40% of B would be pure solid at T_4 and T_3 , and pure liquid at T_1 , T_2 and T_5 . At T_6 , there is a mixture of liquid and solid. The composition of the liquid phase is 20 wt.%B, the composition of the solid is 50 wt.%B, and the proportion of the two is $1/3$ liquid and $2/3$ solid.

- 5 (a) (i) The response of a spring to a ramp in strain is a ramp in stress. So 1d corresponds to β . A dashpot produces a force proportional to the strain rate. So 1b corresponds to α . δ contains a spring and dashpot in parallel, so

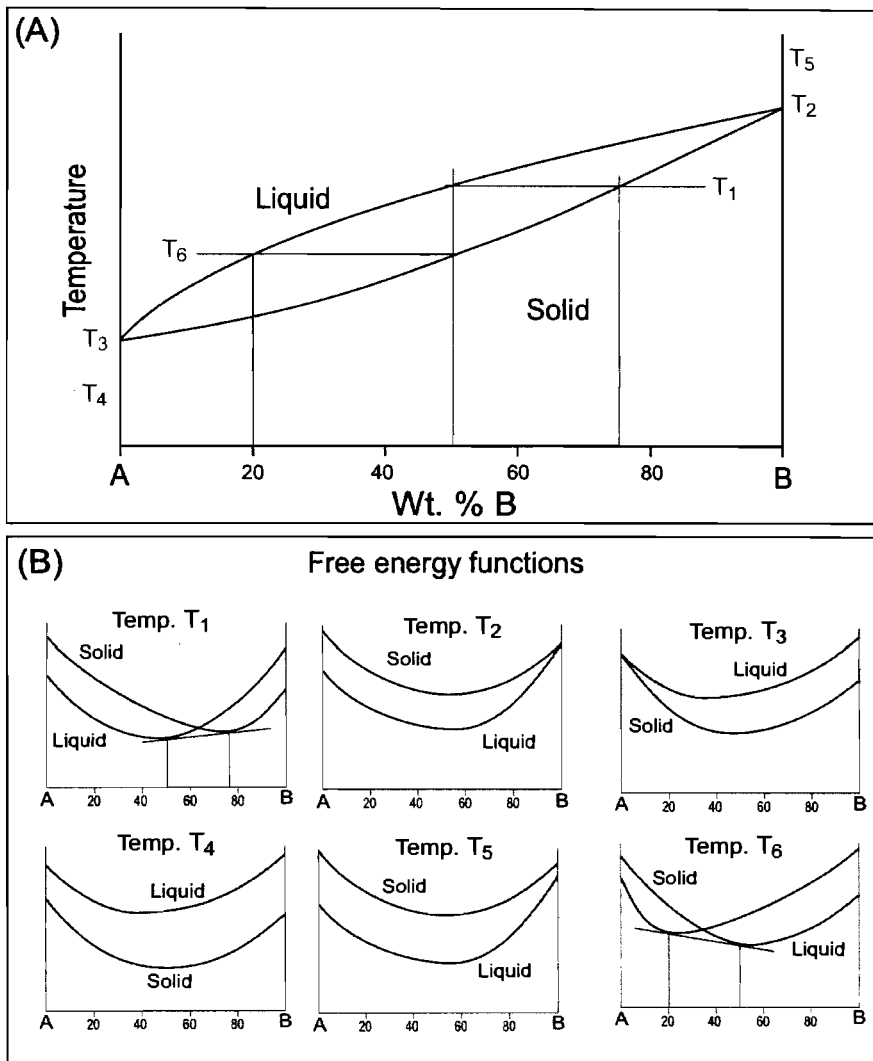


Fig. 5

the stresses in each components should simply add up for a given strain; 1a is the only one corresponding to a sum of 1d and 1b. γ must converge to zero stress at long time scales for a constant strain, with a certain relaxation time; 1c fits indeed with this description.

(ii) The response of a spring to a ramp in stress is a ramp in strain. So 2c corresponds to β . For a dash, the strain is the integral of the stress, so 2a corresponds to α . γ contains a spring and a dashpot in series, so the strains

of both components should add up for a given stress; 2d is the only graph corresponding to a sum of 2a and 2c. δ must converge toward a finite strain for a constant stress, with a certain relaxation time. 2b fits with this requirement.

(b) (i) For a linear elastic solid, $\tau = G\gamma$. Hence, $\dot{\gamma}_r = \dot{\tau}/G$. For a Newtonian fluid, $\dot{\tau} = \mu\dot{\gamma}$, Hence, $\dot{\gamma}_o = \tau/\mu$.

(ii) The total thickness L is $L = 2d + e$. the relative displacement between the external surfaces x is the sum of the shear distance in each part: $x = d\gamma_r + e\gamma_o + d\gamma_r$. Hence the result.

(iii)

$$\dot{\gamma} = \frac{2d\dot{\tau}/G + e\tau/\mu}{2d + e}$$

(iv) The shear stress is uniform in the system, and the shear displacements are additive. We are therefore dealing with a system of spring and dashpot in series. For such a system, we have:

$$\sigma = E\varepsilon_s = \eta\dot{\varepsilon}_d$$

$$\varepsilon = \varepsilon_s + \varepsilon_d$$

where ε_s and ε_d are respectively the strain in the spring and dashpot components. The differential equation for the model γ is therefore:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}$$

By analogy with the expression for $\dot{\gamma}$ above, we find $E = G\frac{2d+e}{2d}$ and $\eta = \mu\frac{2d+e}{e}$.

6 (a) This technique involves heating the steel in a carbon rich atmosphere. Carbon diffuses into the surface of the component, which is then quenched.

The high surface concentration of carbon has two effects on the gear wheel: (1) It increases the hardenability of the steel (redistributing the carbon atoms delays the austenite to pearlite / bainite phase transformation). (2) It increases the hardness of the transformed material (via enhanced solid solution or precipitation hardening).

dds4

(cont.)

(b) The coefficient of diffusion has the following dependence with temperature: $D = D_0 \exp(-Q/RT)$. We can use the data provided to calculate D_0 and Q .

$$\ln D(1223K) - \ln D(1173K) = -\frac{Q}{R} \left(\frac{1}{1223} - \frac{1}{1173} \right)$$

Hence, $Q = 1.25 \cdot 10^5$ J. D_0 is then found from the following expression at one of the known temperatures.

$$D_0 = \frac{D(T)}{\exp(-Q/RT)} = 9.16 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$$

At 1000 °C, we find $D(1223 \text{ K}) = 6.71 \cdot 10^{-11} \text{ m}^2 \text{ s}^{-1}$.

(c) (i) For such a surface treatment, the gear material is accurately modelled as a half-infinite steel space. The boundary conditions are: (i) $C(0, t) = C_s$ for $t > 0$, and $C(\infty, t) = C_0$ at all time. These are consistent with the limits of the proposed function. (ii) The initial state is $C(x, 0) = C_0$ for x strictly positive. This is also satisfied.

(ii) We need here to show that:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

By calculating the relevant partial derivatives, we find:

$$\frac{\partial C}{\partial x} = -(C_s - C_0) \frac{1}{2\sqrt{Dt}} \frac{2}{\sqrt{\pi}} \exp\left(-\frac{x^2}{4Dt}\right)$$

$$\frac{\partial^2 C}{\partial x^2} = -(C_s - C_0) \frac{-2x}{4Dt} \frac{1}{2\sqrt{Dt}} \frac{2}{\sqrt{\pi}} \exp\left(-\frac{x^2}{4Dt}\right)$$

$$\frac{\partial C}{\partial t} = -(C_s - C_0) \left(-\frac{1}{2} \cdot t^{-\frac{3}{2}} \frac{x}{2\sqrt{D}}\right) \frac{2}{\sqrt{\pi}} \exp\left(-\frac{x^2}{4Dt}\right)$$

By substituting these terms in the diffusion equation, we find that the left and right hand sides are identical, hence the diffusion equation is satisfied.

(d) In a carbon rich atmosphere, the carbon concentration at the steel surface corresponds to the saturation concentration at the carburisation temperature. The relevant phase diagram is in the Materials databook, and the saturation concentration is about $C_s = 1.6 \text{ wt.}\%$.

dds4

(TURN OVER for continuation of Question 6

(e) The carbon content required for a Vickers Hardness $H_v=800$ is about 0.6 wt.%. We need to find the time t such that $C(1\text{mm}, t) = 0.6$ wt.%.

$$C(1\text{mm}, t) = 0.6 = 1.4(1 - \text{erf}(Y)) + 0.2 \rightarrow \text{erf}(Y) = 0.71 \rightarrow Y = 0.75$$

$$\frac{x}{2\sqrt{Dt}} = 0.75 \rightarrow t = \left(\frac{10^{-3}}{1.5\sqrt{6.71 \cdot 10^{-11}}} \right)^2 = 6620\text{s} \approx 2 \text{ hours}$$

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