ENGINEERING TRIPOS PART IB JUNE 2012

CHAIRMAN PROF. P DAVIDSON

TUESDAY 5TH JUNE 2012 2 TO 4

PAPER 4 SOLUTIONS – THERMOFLUID MECHANICS

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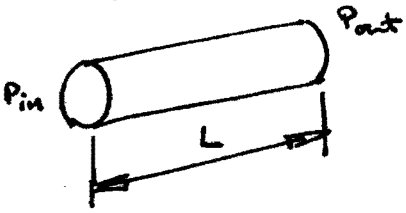
Created by Diane Unwin Friday, July 13, 2012

SECTION A

1) a) Consider a control volume around a section of tube of length L



For equilibrium $\tau_w (2\pi r_i) L = (P_{in} - P_{out}) \pi r_i^2$

$$\tau_w = \Delta P \frac{r_i}{2L}$$

Reynolds Analogy (Databook) $St = c_f/2$

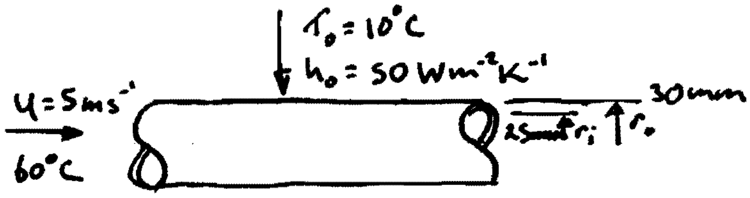
Stanton Number (Databook) $St = Nu/Re\,Pr = \frac{h}{\rho U c}$

Skin Friction Coefficient (Databook) $c_f = \tau_w / \tfrac{1}{2}\rho U^2$

so $c_f = \frac{r_i}{2L}\Delta P \Big/ \tfrac{1}{2}\rho U^2 = \frac{r_i \Delta P}{L \rho U^2}$

$$h = \rho U c\, St = \rho U c\, c_f/2$$

so $$h = \frac{\rho U c\, r_i \Delta P}{2 L \rho U^2} = \frac{\Delta P\, c\, r_i}{2 L U}$$

b) 

i) $$Re_D = \frac{\rho U D}{\mu} = \frac{1000 \times \cancel{5} \times (2 \times 25 \times 10^{-\cancel{3}})}{\cancel{5} \times \cancel{10^{-3}}}$$

$$= 50\,000$$

$$Nu_D = 0.023\, Re_D^{0.8}\, P_r^{0.4} \qquad 2300 < Re_D < 10^7$$

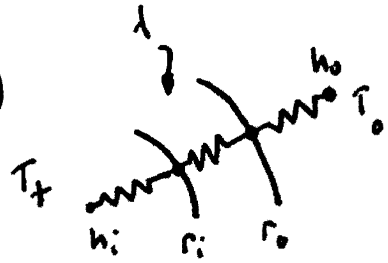
50 000 is within range

$$Nu_D = 0.023 \{50\,000\}^{0.8}\, 1^{0.4} = 182.1$$

$$Nu_D = \frac{hD}{\lambda} \quad \text{so} \quad h_i = \frac{\lambda\, Nu_D}{2 r_i}$$

$$= \frac{0.5 \times 182.1}{2 \times 25 \times 10^{-3}}$$

$$h_i = 1821 \ Wm^{-2}K^{-1}$$

ii) 

$$R_{TOTAL} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o} + \frac{\ln(r_o/r_i)}{2\pi\lambda}$$

$$= 1/1821 \times 2\pi \times 25 \times 10^{-3} + 1/50 \times 2\pi \times 30 \times 10^{-3} + \frac{\ln(30/25)}{2\pi\, 0.25}$$

$$= 0.0048 + 0.1061 + 0.1161$$

$$= 0.227\ KW^{-1}$$

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iii) $\dot{Q} = \dfrac{T_0 - T_f}{R_{TOTAL}} = \dfrac{60-10}{0.227} = 220.27 \text{ W/m}$

Generation $\dfrac{\dot{Q}}{\text{Volume}} = \dfrac{220.27}{1 \times \pi \times (25\times10^{-3})^2} = \underline{\underline{1.12 \times 10^5 \text{ W/m}^3}}$

c)

i) $Gr = 1\times10^6$
$\lambda_0 = 0.05 \text{ Wm}^{-1}\text{K}^{-1}$
$Pr = 1$

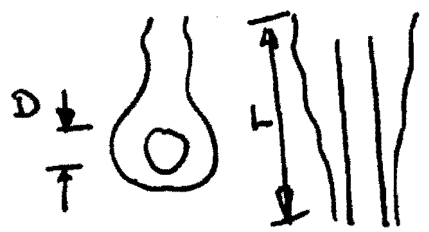
$Nu = 0.5\,(Gr\,Pr)^{0.25}$ (Given)
$= 0.5(1\times10^6 \times 1)^{0.25}$
$= 15.81$

so $h_{natural} = \dfrac{Nu\,\lambda}{D} = \dfrac{15.81 \times 0.05}{2\times30\times10^{-3}} = \underline{\underline{13.18 \text{ Wm}^{-1}\text{K}^{-1}}}$

~20% of previous value (50 Wm^{-1}K^{-1})

ii) From Databook:

$Gr \propto L^3$



Boundary layer growth driven by D in horizontal position. In vertical arrangement, boundary layer growth driven by L.

L >> D so Gr will be much higher.

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NOTE: $h = \dfrac{Nu\,\lambda}{L} = \dfrac{C(GrPr)^n\lambda}{L}$. Both C and n will change for new arrangement so h may go down.

2) a) Frist Law for a steady flow : $\dot{q} - \dot{w}_x = \Delta h$

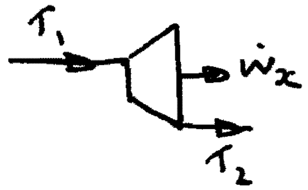
Second Law for a steady flow : $\Delta s = \int_T \frac{d\dot{q}}{T} + \Delta s_{irrev}$

Steady flow availability : $b = h - T_0 s$ (given)

i) $$b_2 - b_1 = h_2 - h_1 - T_0(s_2 - s_1)$$
$$= \dot{q} - \dot{w}_x - T_0 \left\{ \int \frac{d\dot{q}}{T} + \Delta s_{irrev} \right\}$$
$$= \underset{\text{Shaftwork}}{-\dot{w}_x} + \underbrace{\int \left(1 - \frac{T_0}{T}\right) d\dot{q}}_{\text{Heat transfer}} - \underset{\text{Losses due to}}{T_0 \Delta s_{irrev}}$$

ii) Shaftwork — Heat transfer — Losses due to:
- Viscous losses
- Mixing losses
- $\dot{q}$ across finite ΔT

b) 

i) 

$$T_{2s} = T_1 \left\{ \frac{P_2}{P_1} \right\}^{\frac{\gamma-1}{\gamma}} = 1800 \left\{ \frac{1}{18} \right\}^{\frac{1.4-1}{1.4}}$$
$$= 788.2K$$

Turbine efficiency $\eta = 0.85$ so $T_2 = T_1 - \eta(T_1 - T_{2s})$
$$= 1800 - 0.85(1800 - 788.2)$$
$$= 939.95 = 940K$$

i) continued

Specific work $-\dot{w}_x = c_p(T_2 - T_1)$

$$= 1.005(940 - 1800)$$

$$\dot{w}_x = +864.4 \text{ kJ/kg}$$

ii) $b_2 - b_1 = (h_2 - h_1) - T_0(s_2 - s_1)$

$$= c_p(T_2 - T_1) - T_0 \Delta s$$

From Databook $\Delta s = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$

$$= 1.005 \ln\left(\frac{940}{1800}\right) - 0.287 \ln\left(\frac{1}{18}\right)$$

$$= 0.1766 \text{ kJ/kg/K}$$

so $\Delta b = -864.4 - 300 \times 0.1766$

$$= -917.6 \text{ kJ/kg}$$

the lost potential is due to irreversibilities in the turbine. Not due to heat transfer, as the question states that the turbine is adiabatic.

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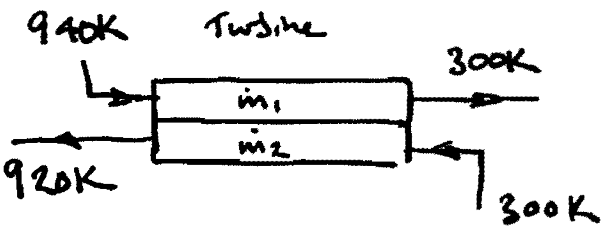
c) i)



$$\dot m_1 \cancel{c_p}(940-300) = \dot m_2 \cancel{c_p}(920-300)$$

$$\frac{\dot m_1}{\dot m_2} = \frac{920-300}{940-300} = \underline{\underline{0.97}}$$

ii)

$$\Delta b = h_2 - h_1 - T_0(s_2 - s_1)$$
$$= c_p(T_2 - T_1) - T_0 \Delta s$$
$$= c_p(T_2 - T_1) - T_0\left\{ c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln \cancelto{0}{\left(\frac{P_2}{P_1}\right)} \right\}$$
$$= c_p\left[(T_2 - T_1) - T_0 \ln\left(\frac{T_2}{T_1}\right)\right]$$

Turbine exit $\quad \Delta b = 1.005\left[(300-940) - 300\ln\left(\frac{300}{940}\right)\right]$

$= \underline{\underline{-298.9 \text{ kJ/kg (turbine exit flow)}}}$

Process $\quad \Delta b = 1.005\left[(920-300) - 300\ln\left(\frac{920}{300}\right)\right]$

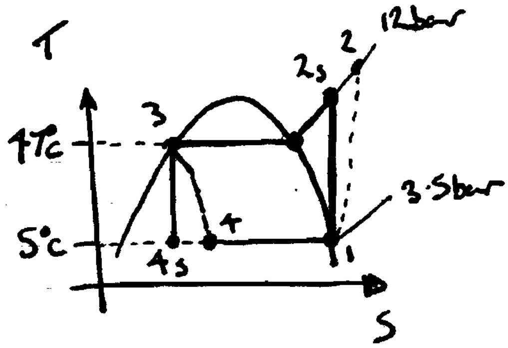
$= 285.2$ kJ/kg (process flow)

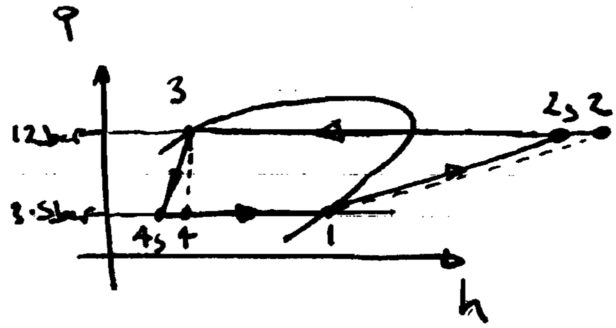
so

$= \underline{\underline{294.4 \text{ kJ/kg}}}$ of turbine exit flow

ii) The lost power potential is due to irreversibilities generated in the wall of the heat exchanger due to heat transfer across a finite temperature difference.
They are <u>not</u> due to viscous sources as $\underline{\Delta P = 0}$

3) a) i)





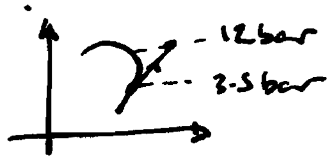
ii) condenser exit (3), 12bar sat. liquid 47°C
compressor inlet (1), 3.5bar sat. vapour 5°C

iii) $COP_R = \dfrac{\dot{q}_{in}}{\sum \dot{w}_x} = \dfrac{h_1 - h_4}{(h_2 - h_1) - (h_3 - h_4)}$

(1) Sat. vapour, 3.5bar, 5°C $h_1 = h_g = 401.5$ kJ/kg
$s_1 = s_g = 1.7245$ kJ/kg/K

(2s) $\Delta s_{1 \to 2s} = 0$ $s_{2s} = s_1 = 1.7245$ kJ/kg/K

Use the chart:



Follow a constant s line to get
$h_{2s} \approx 427$ kJ/kg

(2) $\eta = \dfrac{h_{2s} - h_1}{h_2 - h_1}$ so $h_2 = h_1 + \dfrac{h_{2s} - h_1}{\eta}$
$= 401.5 + \dfrac{427 - 401.5}{0.7}$
≈ 437.9 kJ/kg

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(3) Sat. liquid 12bar, 47°C $h_3 = 265 \text{ kJ/kg}$

$s_3 = 1.2199 \text{ kJ/kg/K}$

(4s) $\Delta s_{3-4s} = 0$ so $s_{4s} = s_3 = 1.2199 \text{ kJ/kg/K}$

find dryness fraction $x = \left.\dfrac{s_{4s} - s_f}{s_g - s_f}\right|_{3.5\text{ bar}}$

$= \dfrac{1.2199 - 1.0244}{1.7245 - 1.0244}$

$= 0.279$

so $h_{4s} = h_f + x(h_g - h_f)\Big|_{3.5\text{bar}}$

$= 206.8 + 0.279(401.5 - 206.8)$

$\approx 261.2 \text{ kJ/kg}$

(4) $\eta = \dfrac{h_3 - h_4}{h_3 - h_{4s}}$ so $h_4 = h_3 - \eta(h_3 - h_{4s})$

$= 265 - 0.7(265 - 261.2)$

$\approx 262.3 \text{ kJ/kg}$

so $COP_R = \dfrac{401.5 - 262.3}{(437.9 - 401.5) - (265 - 262.3)}$

≈ 4.1

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iv) The Carnot equivalent should operate between the highest and lowest temperatures in the cycle

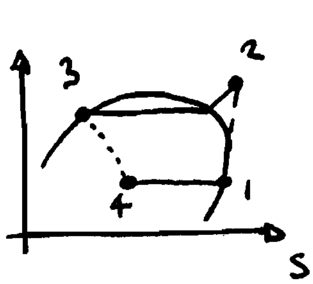
From the chart, the compressor outlet temperature is when:

$h_2 \sim 438$ kJ/kg, 12 bar so $T \approx 66°C$

NOTE: This is quite approximate using the chart. Interpolation using the table gives ~19°C superheat

$$\text{so} \quad COP_{R,\,CARNOT} = \frac{(273.15+5)}{66 - 5} = 4.6$$

b)



$\dot{q} - \dot{w}_x = \Delta h$

so $\Delta h = 0$ across throttle

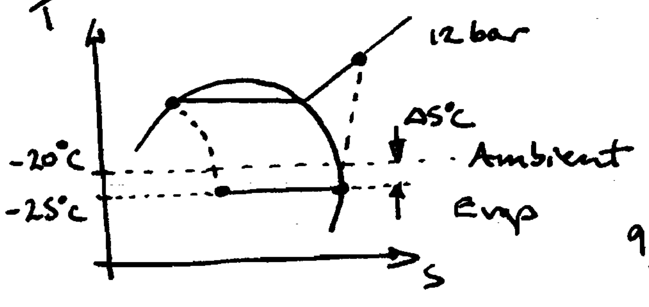
Now $h_3 = h_4$, $\dot{w}_{34} = 0$

$$\text{so} \quad COP_R = \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_3}{h_2 - h_1} = \frac{401.5 - 265}{437.9 - 401.5}$$

$$= 3.75$$

so change ~10%

c) We need a 5°C difference for the evaporator



So evaporator must be -25°C sat maximum, so

$P_{evap} = 1.06$ bar

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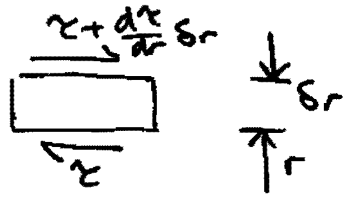
GP

Q4) a) Tube open to atmosphere ⇒ zero streamwise pressure gradient



$$2\pi r \tau = 2\pi (r + \delta r)\left(\frac{d\tau}{dr}\delta r + \tau\right)$$

$$\cancel{r\tau} = r\frac{d\tau}{dr}\delta r + \cancel{r\tau} + \frac{d\tau}{dr}\cancel{\delta r\,\delta r} + \tau\,\delta r$$

$$0 = r\frac{d\tau}{dr} + \tau$$

$$= \frac{d}{dr}(r\tau)$$

b) Newtonian fluid: $\tau = \mu \frac{du}{dr}$

$$\therefore \quad 0 = \frac{d}{dr}\left(r\mu\frac{du}{dr}\right)$$

$$r\mu\frac{du}{dr} = A$$

$$u = \frac{A}{\mu}\ln r + B$$

$r = R_1 \quad u = 0 \qquad 0 = \frac{A}{\mu}\ln R_1 + B \qquad ①$

$r = R_2 \quad u = V \qquad V = \frac{A}{\mu}\ln R_2 + B \qquad ②$

P1 cont.

② − ① $$V = \frac{A}{\mu} \ln \frac{R_2}{R_1} \Rightarrow A = \frac{\mu V}{\ln(R_2/R_1)}$$

sub in ② $$V = \frac{V \ln R_2}{\ln R_2/R_1} + B$$

$$B = V\left(\frac{\ln R_2/R_1 - \ln R_2}{\ln R_2/R_1}\right) = -V \frac{\ln R_1}{\ln(R_2/R_1)}$$

$$\Rightarrow \frac{u}{V} = \frac{\ln r}{\ln(R_2/R_1)} - \frac{\ln R_1}{\ln(R_2/R_1)}$$

c) $$F_L = 2\pi R_2 \cdot \mu \frac{du}{dr}\bigg|_{r=R_2}$$

$$\mu \frac{du}{dr}\bigg|_{r=R_2} = \frac{A}{R_2} = \frac{\mu V}{R_2 \ln(R_2/R_1)}$$

$$F_L = \frac{2\pi \mu V}{\ln(R_2/R_1)}$$

d) Non-dim F_L is $\dfrac{F_L}{\frac{1}{2}\rho V^2 . R_2}$

$$\frac{F_L}{\frac{1}{2}\rho V^2 R_2} = f\left(\frac{\rho V R_2}{\mu}\right)$$

Reynolds number

To keep Reynolds number need $V_{mod} = 10\, V_{orig}$ (same ρ, same μ)

SP 1

d) With Re matched:

cont.

$$\left.\frac{F_L}{\frac{1}{2}\rho V^2 R_2}\right|_{mod} = \left.\frac{F_L}{\frac{1}{2}\rho V^2 R_2}\right|_{orig}$$

$$\frac{F_{L\,mod}}{F_{L\,orig}} = \frac{V^2_{mod}}{V^2_{orig}} \cdot \frac{R_{2\,mod}}{R_{2\,orig}}$$

$$= 10^2 \cdot \frac{1}{10} = 10$$

GP 2

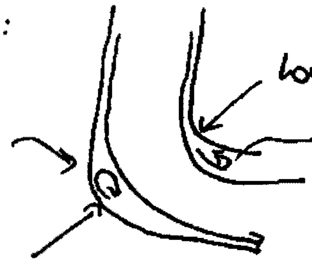
Q5 a) i) Inlet loss:

flow separation at inlet ⇒ dissipation

ii) Pipe loss due to viscous dissipation in boundary layers

iii) Bend loss:

separation at bend inlet on outside of corner due to adverse p-gradient



separation at bend exit on inside of corner due to adverse p-gradient

iv) Nozzle is highly accelerating so loss coefficient likely to be small (attached boundary layers).

b)
$$p_{0pump,2} - \overbrace{\frac{1}{2}\rho V_3^2 \cdot 4c_f\frac{L_3}{D}}^{\text{pipe loss}} - \overbrace{\frac{1}{2}\rho V_3^2 \cdot \frac{D^4}{d^4} K_N}^{\text{nozzle loss}}$$

$$= p_a + \frac{1}{2}\rho V_3^2 \frac{D^4}{d^4}$$

$$p_{0pump,2} = p_a + \frac{1}{2}\rho V_3^2\left[4c_f\frac{L_3}{D} + \frac{D^4}{d^4}\left(1 + K_N\right)\right]$$

GP 2

c) Pipe 1:

$$p_{0pump,1} = p_a + \rho g z_1 - \frac{1}{2}\rho V_1^2 \left(K_B + 4 c_f \frac{L_1}{D}\right)$$

Pipe 2:

$$p_{0pump,2} = p_a + \rho g z_2 - \frac{1}{2}\rho V_2^2 \left(K_B + 4 c_f \frac{L_2}{D}\right)$$

equate:

$$\rho g z_1 - \frac{1}{2}\rho V_1^2 \left(K_B + 4 c_f \frac{L_1}{D}\right) = \rho g z_2 - \frac{1}{2}\rho V_2^2 \left(K_B + 4 c_f \frac{L_2}{D}\right)$$

$$z_1 = z_2 \Rightarrow V_1^2 \left(K_B + 4 c_f \frac{L_1}{D}\right) = V_2^2 \left(K_B + 4 c_f \frac{L_2}{D}\right)$$

$$\frac{V_1}{V_2} = \sqrt{\frac{K_B + 4 c_f \frac{L_2}{D}}{K_B + 4 c_f \frac{L_1}{D}}} = \alpha$$

d) continuity: $V_3 = V_1 + V_2$

$$= V_1 (1 + \alpha)$$

$$\therefore V_1 = \frac{V_3}{(1+\alpha)}$$

$W_x = Q . \Delta p_{0,pump}$

(~~the~~ if reversible)

$$p_{0pump,1} = p_a + \rho g z_1 - \frac{\frac{1}{2}\rho V_3^2}{(1+\alpha)^2} \left(K_B + 4 c_f \frac{L_1}{D}\right)$$

$$\Delta p_{o\,pump} = p_{o\,pump_2} - p_{o\,pump_1}$$

$$= \frac{1}{2}\rho V_3^2\left[4c_f\frac{L_3}{D} + \frac{D^4}{d^4}(1+K_N) + \frac{K_B + 4c_f\frac{L_1}{D}}{(1+\alpha)^2}\right] - \rho g z_1$$

$$W_x = \frac{\pi D^2}{4}\cdot V_3\cdot \Delta p_{o\,pump}.$$

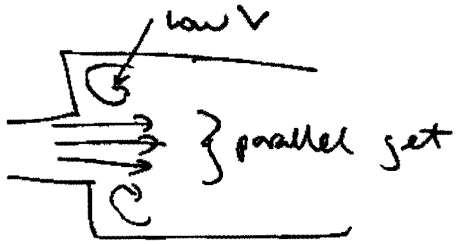
GP 3

Q6 a) $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_1^2 \left(\frac{D}{d}\right)^4$

$$\frac{1}{2}\rho V_1^2\left(\left(\frac{D}{d}\right)^4 - 1\right) = p_1 - p_2$$

$$V_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left[\left(\frac{D}{d}\right)^4 - 1\right]}}$$

b) pressure is uniform across jet because streamlines are straight and parallel (no curvature). Outside the jet, V is low so no significant pressure gradient



Assume shear stress on wall between 2 and 3 is negligible: SFME:

$$\left(p_2 + \rho V . V_2\right) . \frac{\pi D^2}{4} = \left(p_3 + \rho V^2\right) . \frac{\pi D^2}{4}$$

$$p_2 + \rho V^2 . \frac{D^2}{d^2} = p_3 + \rho V^2$$

$$p_3 - p_2 = \rho V^2 \left(\frac{D^2}{d^2} - 1\right)$$

GP 3 c)

$$(p_{03} - p_{02}) = \left(p_3 + \tfrac{1}{2}\rho V^2\right) - \left(p_2 + \tfrac{1}{2}\rho V^2 \frac{D^4}{d^4}\right)$$

$$= (p_3 - p_2) + \tfrac{1}{2}\rho V^2\left[1 - \frac{D^4}{d^4}\right]$$

$$= \rho V^2\left(\frac{D^2}{d^2} - 1\right) + \tfrac{1}{2}\rho V^2\left[1 - \frac{D^4}{d^4}\right]$$

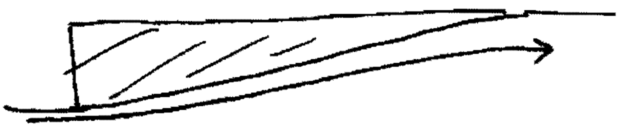
$$= \tfrac{1}{2}\rho V^2\left[\frac{2D^2}{d^2} - 1 - \frac{D^4}{d^4}\right]$$

$$= -\tfrac{1}{2}\rho V^2\left(\frac{D^2}{d^2} - 1\right)^2$$

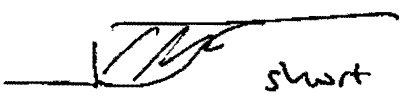
i.e always -ve.

p_0 reduces due to viscous dissipation as jet at plane 2 mixes to uniform flow at plane 3.

d)



Long duct = gradual deceleration ∴ reduced chance of boundary layer separation. But long length increases boundary layer thickness due to large surface area.



short duct, strong deceleration, boundary layer separation likely but less loss than no insert.