ENGINEERING TRIPOS PART IB

Monday 4 June 2012 9 to 11

Paper 1

MECHANICS

Answer not more than **four** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

jaw05

SECTION A

Answer not more than **two** *questions from this section.*

1 A uniform rod of length $\sqrt{2}R$ and mass m slides in a smooth, hemi-cylindrical trough of radius R whose axis at O is horizontal. The rod is released from rest in its initial position when it makes an angle of 45° with the vertical as shown in Fig. 1. In the subsequent motion it remains in the same vertical plane.

(a) Show that when the rod has rotated through angle θ from its initial position, its loss of gravitational potential energy is given by the expression

$$\frac{R}{\sqrt{2}}\left\{\cos\left(\frac{\pi}{4}-\theta\right)-\cos\left(\frac{\pi}{4}\right)\right\}mg \ .$$ [6]

(b) Obtain expressions for the angular velocity of the rod $\dot{\theta}$ and its angular acceleration $\ddot{\theta}$ in terms of the angle θ. Confirm that when $\theta = 45°$,

$$\dot{\theta}^2 = \frac{3\left(\sqrt{2}-1\right)}{2}\frac{g}{R} \ .$$ [6]

(c) Find expressions for the reactions at the ends A and B of the rod when it is first horizontal. [8]

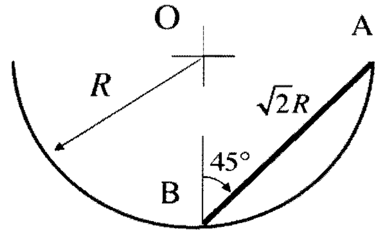

Fig. 1

2 Figure 2 shows three flywheels, one with polar moment of inertia J, one with $2J$ and one with $4J$, connected by a clutch and gearbox. The gearbox has a velocity ratio of 2:1. The shafts and gears are supported in frictionless bearings and can be considered to have negligible inertia. The two smaller flywheels are initially at rest and the clutch is disconnected. The clutch is engaged at a moment when the largest flywheel is rotating at speed Ω.

(a) Determine the angular velocities of the flywheels when all slipping in the clutch has ceased. [10]

(b) What proportion of the initial kinetic energy of the largest flywheel has been dissipated during this period of slip? [5]

(c) During the period of slip, the clutch delivers a constant torque T. Find the time before slip ceases in terms of J, Ω and T. [5]

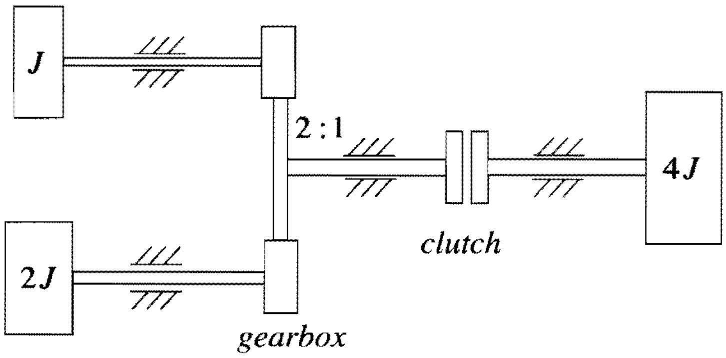

Fig. 2

3 A uniform hoop of radius b centre G hangs from a circular peg of radius a. The hoop is displaced slightly so that its point of contact with the peg moves from A to A′ defined by the small angle θ, as illustrated in Fig. 3. There is no slip between the hoop and the peg.

(a) Show that the angle of rotation of the hoop ϕ is related to the angle θ by the expression

$$\phi = \frac{b-a}{b}\theta \ .$$ [5]

(b) The hoop is now released. Find the angular frequency with which it will oscillate in terms of the specified quantities. [8]

(c) The coefficient of friction between the hoop and the peg is 0.2. For the case $b = 3a/2$ estimate the maximum amplitude of the angle through which the hoop can swing before some slip might be anticipated. [7]

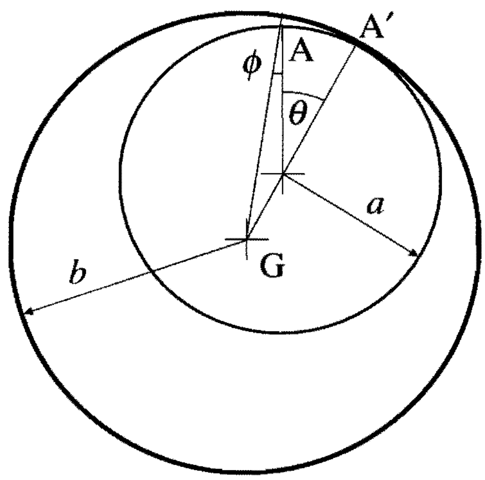

Fig. 3

SECTION B

Answer not more than **two** *questions from this section.*

4 A four bar mechanism of the form shown in Fig. 4 operates in the horizontal plane. The lengths of the links AB and CD are a and the length of the fixed base link AD is $2a$. The coupler is a uniform equilateral triangular lamina of mass m and side length a. When the angle BAD is 60°, a torque T is applied at A so that the link AB starts to move from rest with angular acceleration β. The inertia of links AB and CD can be considered small.

(a) Show that at this instant the acceleration of G, the centroid of BCE, is parallel to AD and find its magnitude in terms of a and β. [8]

(b) If friction in the joints can be neglected, obtain an expression for T in terms of a, m and β. [7]

(c) Find also the magnitude of the reaction at the pivot D. [5]

Note that the polar moment of inertia about its centre of a lamina of mass m with the form of an equilateral triangle of side a is $ma^2/12$.

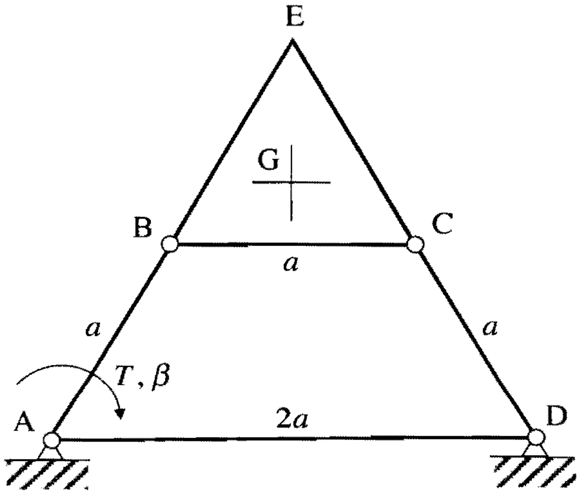

Fig.4

5 A straight uniform, rigid rod of length ℓ and mass m has its two ends constrained by frictionless pivots which can slide, without friction, in two straight guides as illustrated in Fig. 5. The guides are fixed and at right angles to one another. The pivot at end A moves at constant speed v.

(a) (i) Evaluate the angular velocity of the rod. [4]

(ii) Show that the magnitude of the angular acceleration of the rod is given by the expression

$$\frac{v^2}{\ell^2}\sec^2\theta\tan\theta$$

where θ is the angle indicated. [6]

(b) Hence, or otherwise, find the location of the maximum bending moment in the rod and determine its magnitude in terms of m, v and θ. [10]

Gravity can be neglected.

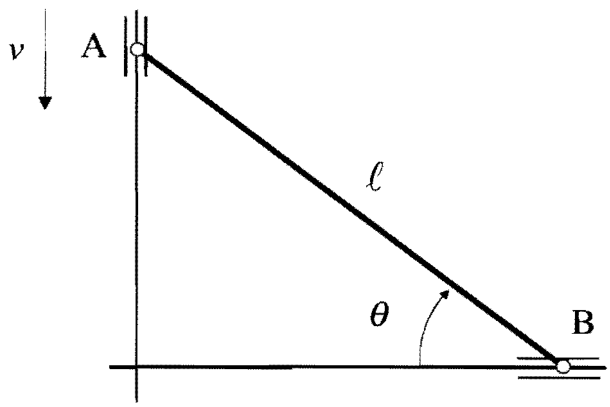

Fig. 5

6 A uniform rod AB of length ℓ is dropped at an angle of $45°$ to the vertical so that both ends have vertical velocity v when end A hits the ground, as illustrated in Fig. 6. End A just makes contact with a fixed step so that in the subsequent motion the rod pivots about its contact point.

(a) Show that immediately after the initial impact, the angular velocity ω of the rod is given by

$$\omega = \frac{3v}{2\sqrt{2}\ell} .$$ [6]

(b) Determine the proportion of the initial kinetic energy of the rod that is lost in the impact at A. [6]

(c) In a particular case $v^2 = 0.5\ell g$. Determine the inclination of the rod to the vertical when contact with the step is lost. [8]

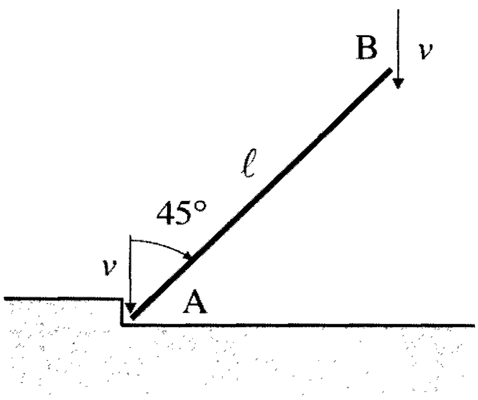

Fig. 6

END OF PAPER