ENGINEERING TRIPOS PART IB

Monday 4 June 2012 2 to 4

Paper 2

STRUCTURES

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: none.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

fc05

SECTION A

1 (a) The two thin-walled cross-sections shown in Fig. 1(a) are a square and a cruciform of the same exterior dimensions. Show that the wall thickness of the cruciform must be $2\sqrt{2}$ times that of the box section for their stiffness in bending about the indicated axis to be equal. Assume that both cross-sections are made from the same material and that the wall thicknesses are uniform. [4]

(b) Two thin-walled tubes have uniform cross-sections of a square with the side-length L and an equilateral triangle with the base-length W and both have wall thickness t. Find the ratio of their torsion constants. [8]

(c) The cross-section of a tube is shown in Fig. 1(b). It comprises four equal thin-walled semi-circular segments of wall thickness t connected to each other on their ends at the corners of an imaginary square of side length L. The Young's and shear moduli of the tube are E and G, respectively. Calculate the torsional stiffness and the bending stiffness (using the Mechanics Data Book or otherwise) about the horizontal symmetry axis. [8]

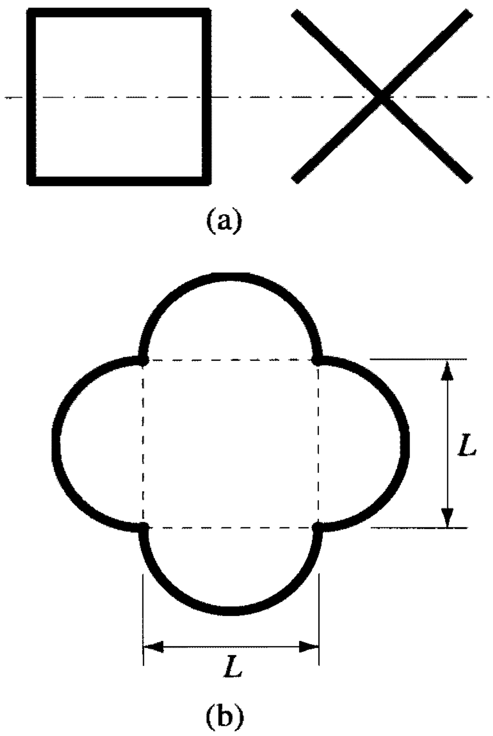


Fig. 1

2 (a) The upright cantilever in Fig. 2(a) has length L and is subjected to a transverse distributed load wx/L, where x is the axial distance from the tip. Using virtual work, or otherwise, show that the transverse deflection of the tip δ is given by

$$\delta = \frac{wL^4}{30EI}$$

where EI is the linear elastic bending stiffness. [6]

(b) The elastic frame in Fig. 2(b) consists of one beam and two built-in columns. The beam and columns have the same bending stiffness EI, pins connect the columns to the beam, and the beam is continuous along its length. The beam carries a uniform loading w_1. A linearly varying distributed load of maximum value w_2 is applied transversely to one of the columns.

(i) Find the vertical deflection of point A. [8]

(ii) Find the horizontal deflection of point A. [6]

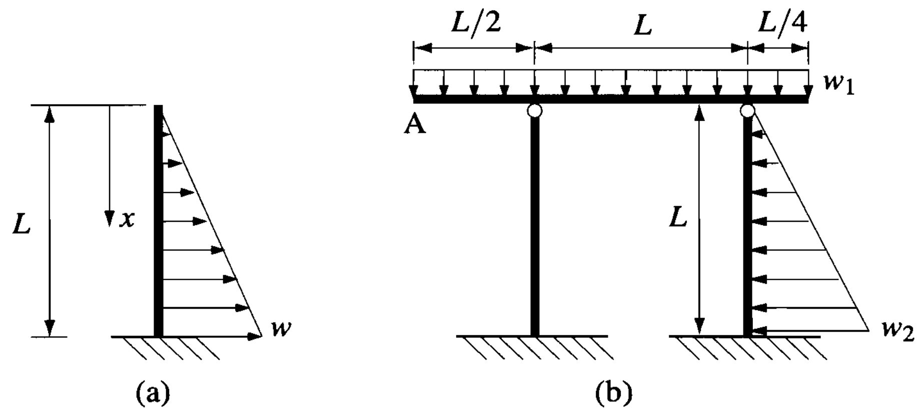


Fig. 2

3 A thin-walled, circular cylinder of radius $R = 1\,\mathrm{m}$ and uniform wall thickness $t = 3\,\mathrm{mm}$ has closed ends. It is subjected to an internal gauge pressure $p = 250\,\mathrm{kPa}$ and an axial torque $T = 600\,\mathrm{kNm}$.

(a) Calculate the shear stress and the components of stress in the axial and hoop-wise directions. Assume that the through-thickness component of stress is zero. [5]

(b) Sketch the three-dimensional set of Mohr's circles for the state of stress found in (a) and determine the size of the largest shear stress and its direction by indicating an appropriate sequence of rotations of the original coordinate axes. [8]

(c) If the original axial stress, hoop-wise stress and shear stress are $\sigma/2$, σ and τ, respectively, show that yield according to the von Mises criterion is satisfied when

$$3\left(\frac{\tau}{Y}\right)^2 + \frac{3}{4}\left(\frac{\sigma}{Y}\right)^2 = 1$$

where Y is the uniaxial yield stress. [7]

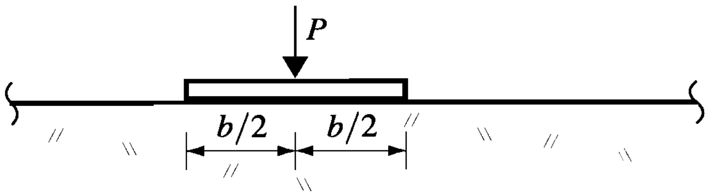
SECTION B

4 Figure 3(a) shows the cross-section of a long foundation slab used to support a building. The slab is of width b and is designed to carry a line load P (force per unit length into the page). The ground may be considered as a rigid-perfectly plastic material with a yield stress in shear k.

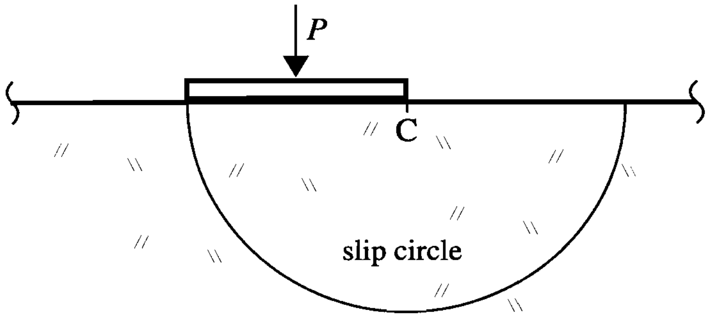
(a) Initially consider the slab to be rigid. By using the slip-circle mechanism shown in Fig. 3(b), where rotation takes place about point C, show that an upper-bound estimate of the line load at collapse is given by $P = 2\pi kb$. [6]

(b) There is concern that the foundation slab may also fail. Consider that the slab is a rigid-perfectly plastic plate with plastic moment per unit length m. Using the two-slip-circle mechanism shown in Fig. 3(c), where a plastic hinge forms along the slab at B, find another upper-bound estimate of the line load at collapse. [8]

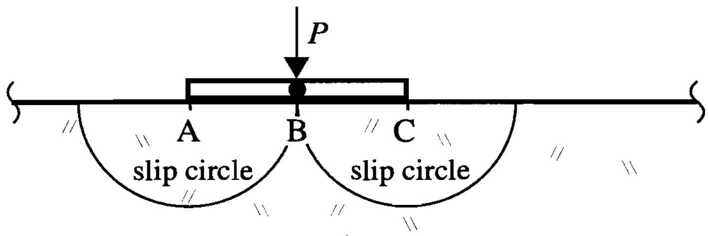
(c) If the slab has thickness t and is made of rigid-perfectly plastic material with a yield stress in shear ck, determine the required thickness of the slab to ensure that slab failure does not occur in the failure mode shown in Fig. 3(c). [6]



(a)



(b)



(c)

Fig. 3

5 Figure 4 shows a portal frame with a fully plastic moment M_p, carrying a downward load $2W$ distributed uniformly across the beam, and a horizontal point load W acting at the mid-height of the shorter column. The frame is rigidly connected to the foundations.

(a) Sketch *four* reasonable collapse mechanisms where plastic hinges occur at some subset of the six points marked A–F. For each mechanism, show clearly where plastic hinges have formed. [6]

(b) Find an upper-bound estimate of the collapse load for each of the mechanisms sketched in (a), making the assumption that any hinge at D occurs in the centre of the beam. [7]

(c) By considering that the hinge at D no longer has to occur in the centre of the beam, find an optimal upper-bound estimate of the collapse load that improves (even marginally) upon those found in (b). [7]

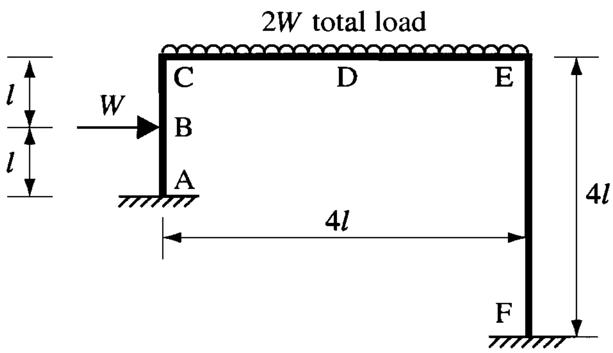


Fig. 4

6 Figure 5 shows a universal beam UB356×127×39 which is continuous over two 8 m spans. The structure is designed to carry two 100 kN point loads, one at the centre of each span. The beam is made of structural steel with a yield stress of 355 MPa.

(a) Using the elastic method, estimate the value of the load factor λ when first yield will occur in the structure. [8]

(b) Using the plastic method, find an upper bound for the load factor λ in the structure. [6]

(c) Using the answers from (a) and/or (b), or otherwise, find a lower bound for the load factor λ in the structure. [6]

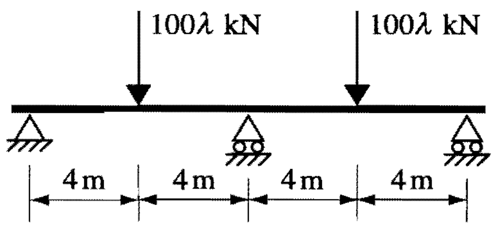


Fig. 5

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