ENGINEERING TRIPOS PART IB

Tuesday 5 June 2012 9 to 11

Paper 3

MATERIALS

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachment: Figure for question 4 (1 page), to be submitted with the solution.

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

dds4

SECTION A

1 (a) Discuss, with reference to sketches, how the microstructure of wood affects the elastic properties along and transverse to the grain. [5]

(b) Figure 1 shows a two dimensional foam that is analagous to the structure of wood.

(i) Find the relative density $\overline{\rho} = \frac{\rho^*}{\rho_s}$ in terms of t and l, where ρ^* is the mean density of the foam and ρ_s is the density of the solid cell walls. [2]

(ii) Show that the effective Young's modulus of the foam in the x direction E_x is

$$E_x = \frac{1}{2}\overline{\rho}E_s$$

where E_s is the Young's modulus of the solid. [3]

(c) Consider the unit cell shown within the dashed rectangle in Fig. 1.

(i) Sketch the deformed shape of the unit cell for tension of the foam in the y direction. [3]

(ii) Find an expression for the Young's modulus of the foam in the y direction E_y in terms of $\overline{\rho}$ and E_s. Note that the central deflection δ of a fixed-ended beam, as shown in Fig. 2, is given by

$$\delta = \frac{Wl^3}{192EI}$$

[7]

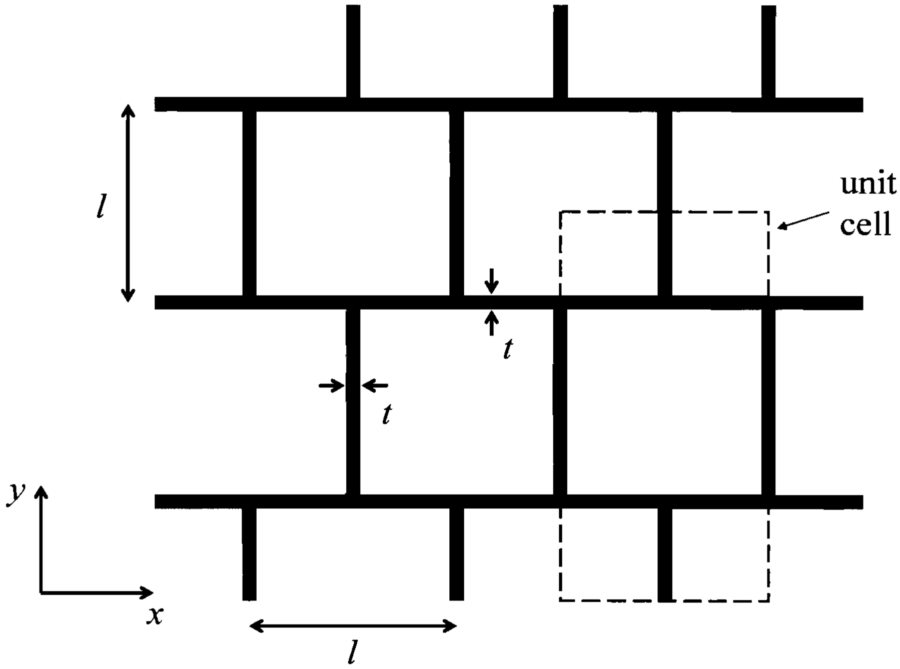


Fig. 1

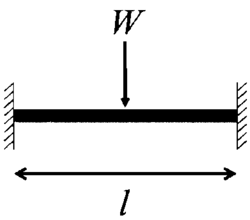


Fig. 2

2 Figure 3 shows a two-dimensional sheet drawing process. The material is stress-free at entry and is pulled through two converging dies by a drawing stress σ_{draw}. The dies are frictionless and subtend an angle of 2α. The sheet thickness is reduced from h_{in} to h_{out}.

(a) Draw a free-body diagram to show the forces acting on a vertical strip, of thickness δx, height $h(x)$ and unit depth at a typical horizontal co-ordinate x. [5]

(b) By considering the horizontal equilibrium of forces on the element sketched in part (a), show that

$$\frac{\mathrm{d}}{\mathrm{d}h}(h\sigma_x) + p = 0$$

[5]

(c) The material may be assumed to be a rigid ideally-plastic solid of yield strength Y. For a small die angle α it can be assumed that the die contact pressure p is a principal stress. Use the Tresca yield criterion to obtain an expression for the drawing stress σ_{draw} as a function of the reduction ratio h_{in}/h_{out}. [6]

(d) What is the maximum possible reduction ratio h_{in}/h_{out} that can be obtained through frictionless dies? [4]

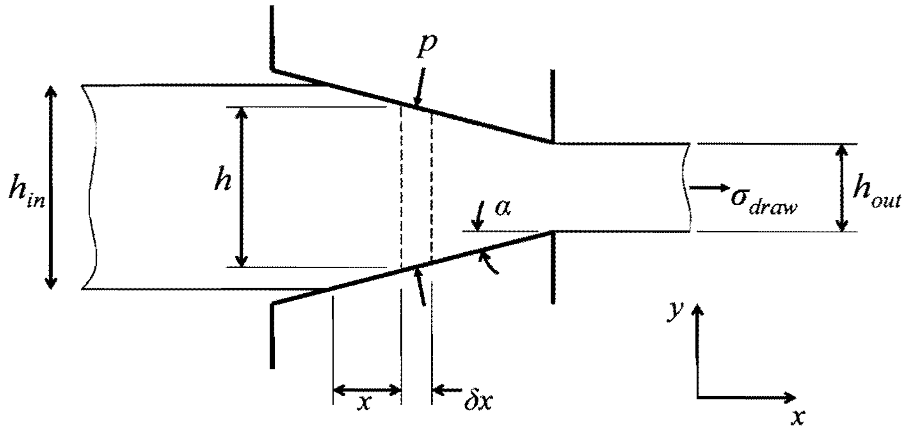


Fig. 3

3 (a) Explain what is meant by *hardenability* and *weldability* in the context of carbon and low alloy steels. Why are these terms considered to be opposites? [4]

(b) Consider the isothermal transformation diagram for BS503M40 1% nickel steel (Fig. 7.1, page 34 of the Materials Data Book). Is the hardenability of this steel low or high? What do AC_3, AC_1 and M_S represent? Explain the shape of the contours of constant fraction transformed in terms of the thermodynamics and kinetics of phase transformations. [6]

(c) Refer to the Iron-Carbon equilibrium diagram (Fig. 6.3, page 30 of the Materials Data Book). Which lines on Fig. 7.1 can be determined from Fig. 6.3? Hence estimate the percentage of carbon in BS503M40 steel (the presence of Ni may be neglected for this purpose). [4]

(d) Briefly explain appropriate heat treatments that may be used to obtain the following microstructures in BS503M40 steel in the shortest possible time:

(i) 100% martensite

(ii) 90% bainite, 10% martensite

(iii) 40% ferrite, 10% pearlite, 50% martensite [6]

THIS PAGE DELIBERATELY LEFT BLANK

SECTION B

4 (a) Figure 4 shows the phase diagram for a water-ethanol mixture that describes the equilibrium between the liquid and vapour phases. This diagram will be used to explore the process of distillation.

(i) Starting from a mixture at the composition C_1, what would be the equilibrium compositions C_L and C_V of the liquid and vapour phases at 94°C? [3]

(ii) The vapour is then rapidly extracted and placed in a secondary container. At what temperature should this extracted mixture be held so that the liquid phase is at the composition C_1? What would be the composition of the corresponding vapour phase? [3]

(iii) What would be the highest weight fraction of ethanol that can be obtained by repeating this cycle of temperature control and vapour extraction? Explain your answer. [3]

(iv) Can such a process be used to purify water from a mixture of water-ethanol? If yes, what is the highest purity that can be reached? Explain your answers. [3]

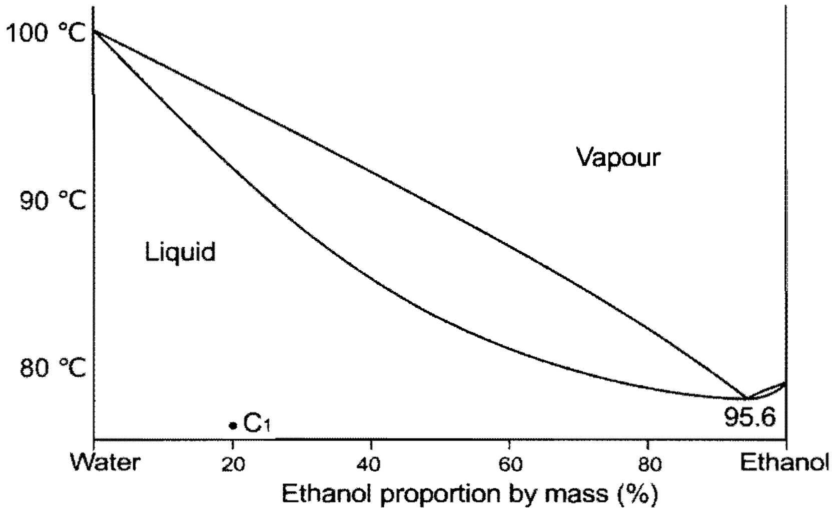


Fig. 4

(b) Figure 5(a) shows the phase diagram of a mixture of two materials X and Y, and Fig. 5(b) a collection of graphs of the free energy functions of the solid and liquid phases at six different temperatures.

(i) A copy of Fig. 5 is provided as an attachment to this paper. On this copy, mark as precisely as you can the temperatures T_1 to T_6. [4]

(ii) A particular mixture contains 60 wt.% of X and 40 wt.% of Y. What would be the composition and the proportion of the phases present at each of the temperatures T_1 to T_6? [4]

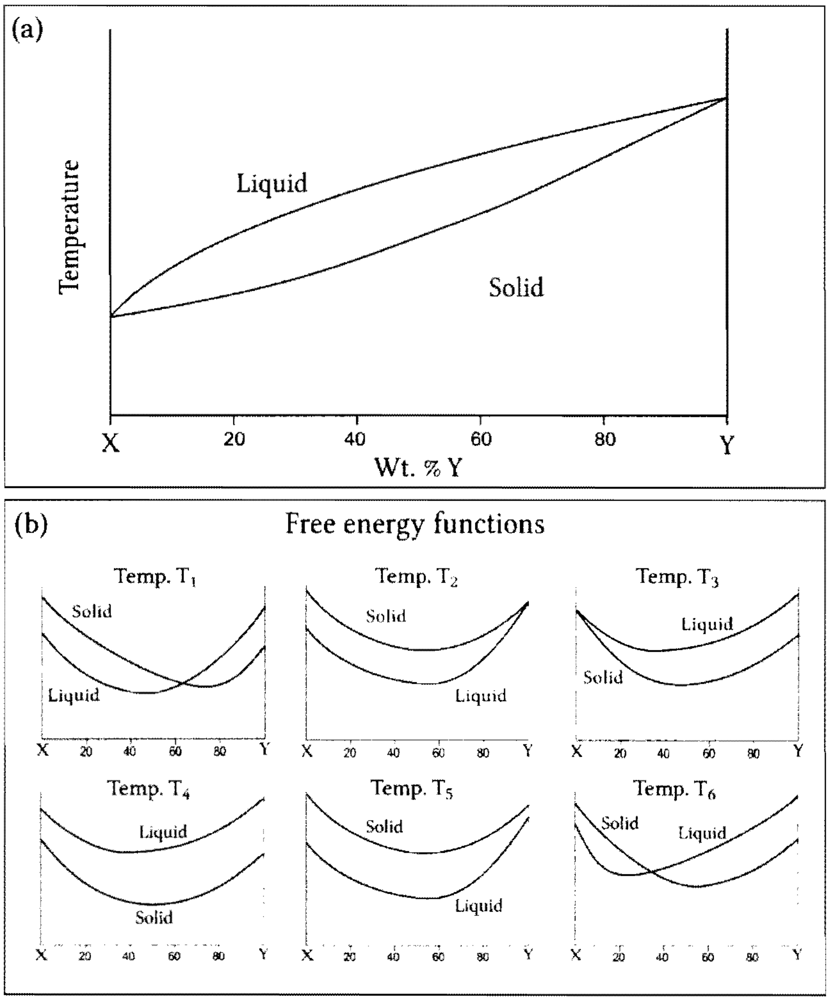


Fig. 5

5 (a) Figure 6(A) shows diagrams of four different viscoelastic models, referred to as α, β, γ and δ.

(i) Figure 6(B) shows a linear ramp in strain and, below, the corresponding evolution of stress in each of these models, presented in a random order. Match each of the stress-time graphs 1a to 1d to its related viscoelastic model α, β, γ or δ. Briefly justify your answer. [5]

(ii) Figure 6(C) shows a linear ramp in stress and, below, the evolution of the strain in each of these models, presented in a random order. Match each of the strain-time graphs 2a to 2d to its related viscoelastic model. Briefly justify your answer. [5]

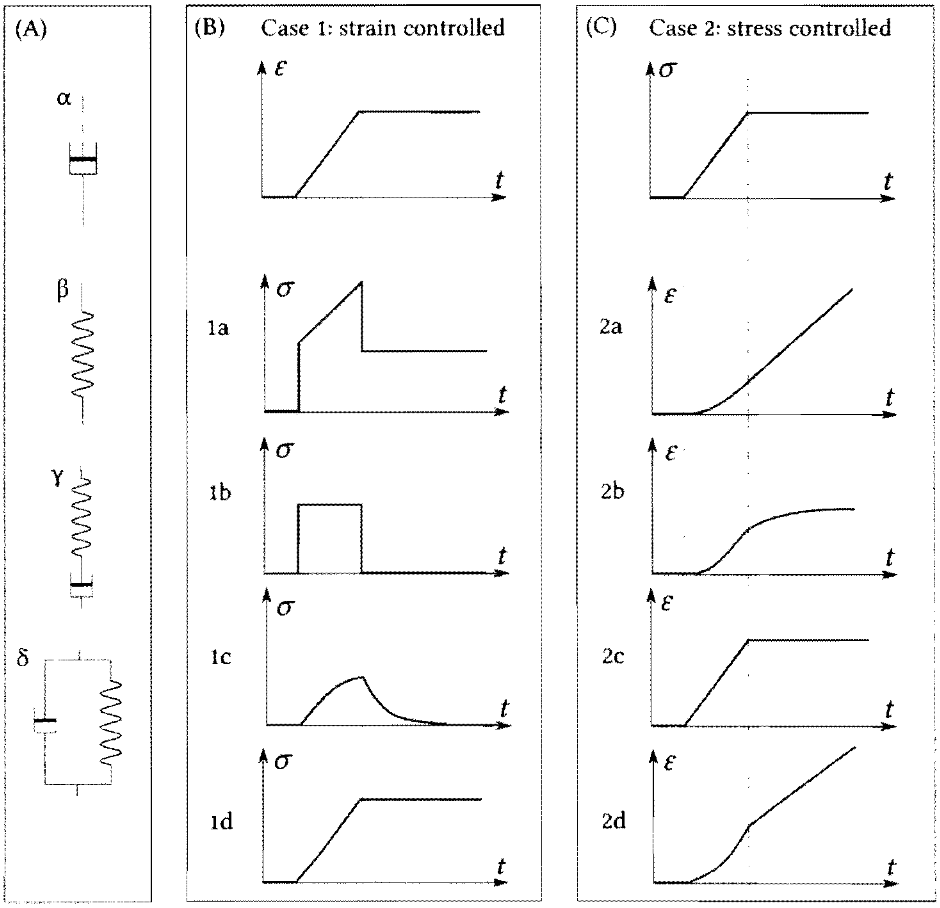


Fig. 6

(b) Two flat blocks of rubber, each of thickness d are separated by a layer of oil of thickness e (see Fig. 7). A uniform shear stress $\tau(t)$ is applied to the system parallel to the layers. The shear modulus of rubber, assumed to follow a linear elastic behaviour, is G. The oil is Newtonian and has a viscosity μ.

(i) Write expressions for the shear rates $\dot{\gamma}_r$ and $\dot{\gamma}_o$ in the rubber and oil respectively as a function of τ, $\dot{\tau}$, G and μ. [2]

(ii) The overall shear deformation γ is x/L (see Fig. 7). Show that it can be written as:

$$\gamma = \frac{2d\gamma_r + e\gamma_o}{2d+e}$$

[2]

(iii) Using parts (b)(i) and (b)(ii), obtain an expression for $\dot{\gamma}$ as a function of τ and $\dot{\tau}$. [3]

(iv) The system behaviour corresponds to one of the models shown in Fig. 6(A). Explain which one is relevant and where appropriate give expressions for the corresponding spring and dashpot constants as a function of the model parameters defined above. [3]

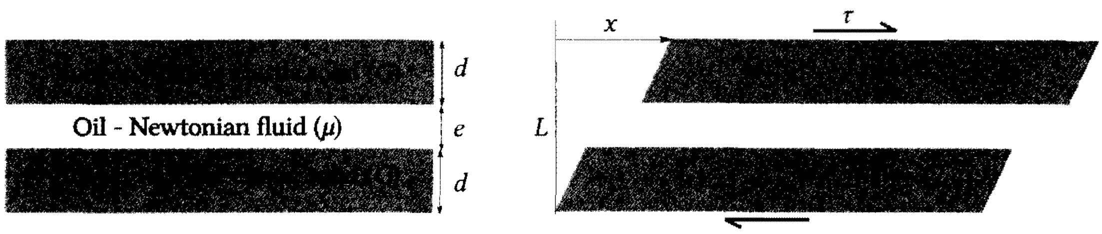


Fig. 7

6 A gear wheel made of plain carbon steel (0.2 wt% carbon) is to be carburised in a carbon rich atmosphere at 1000 °C before quenching to room temperature.

(a) Briefly explain the process of carburisation and how it modifies the properties of the component. [3]

(b) The coefficient of diffusion D of carbon in steel is 2.45×10^{-11} m^{2}s^{-1} at 900 °C, and 4.14×10^{-11} m^{2}s^{-1} at 950 °C. Find the value of the coefficient of diffusion at 1000 °C. [4]

(c) The expression for the carbon concentration $C(x,t)$ in the steel is given by:

$$C(x,t) = (C_s - C_0)\left[1 - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right] + C_0$$

where x is the distance below the surface, t is the time elapsed, C_0 is the initial carbon concentration in the steel and C_s is the concentration of carbon in the steel at the surface during the carburisation. The error function erf is defined in the Materials Databook. Show that the expression for $C(x,t)$:

(i) satisfies the boundary conditions at all times; [2]

(ii) satisfies the 1D unsteady diffusion equation. [5]

(d) Estimate the value of the carbon concentration C_s at the steel surface during the carburisation process. Briefly explain your answer. [2]

(e) Figure 8 shows the Vickers Hardness Hv of the quenched steel as a function of the carbon content. Calculate the duration of the carburisation process such that the Vickers Hardness is 800 at 1mm below the surface after quenching. [4]

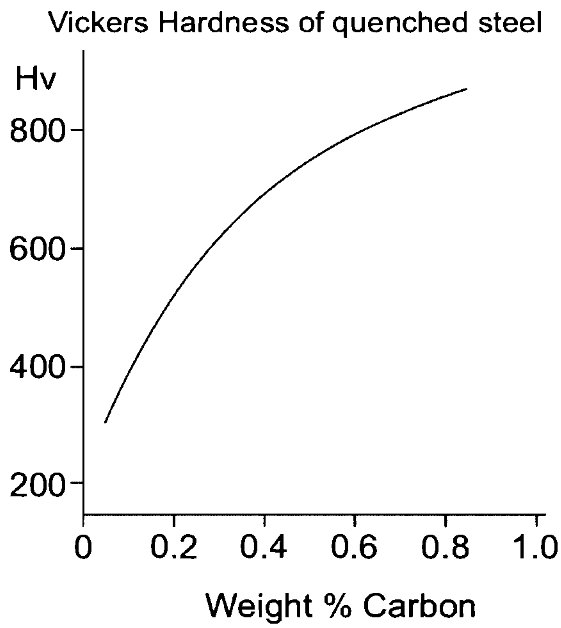


Fig. 8

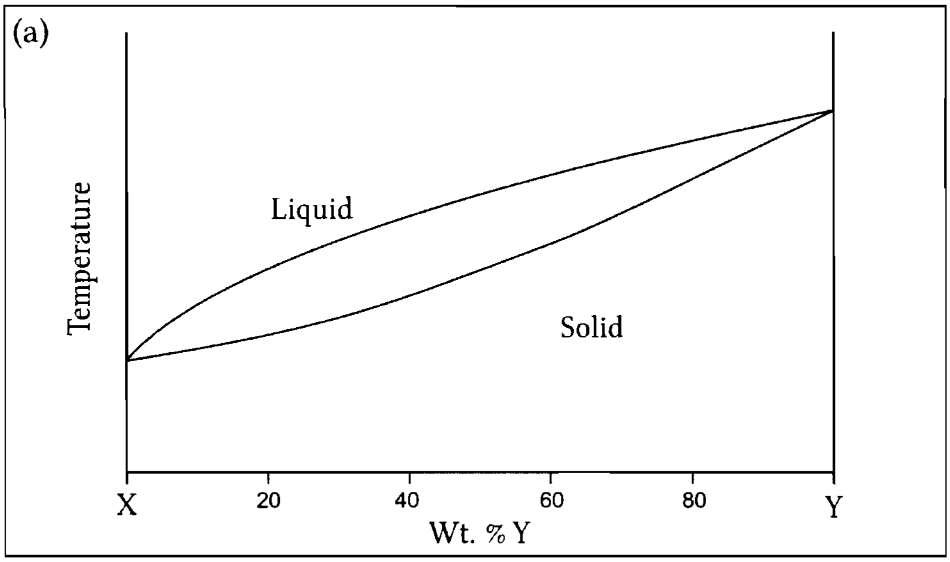
END OF PAPER

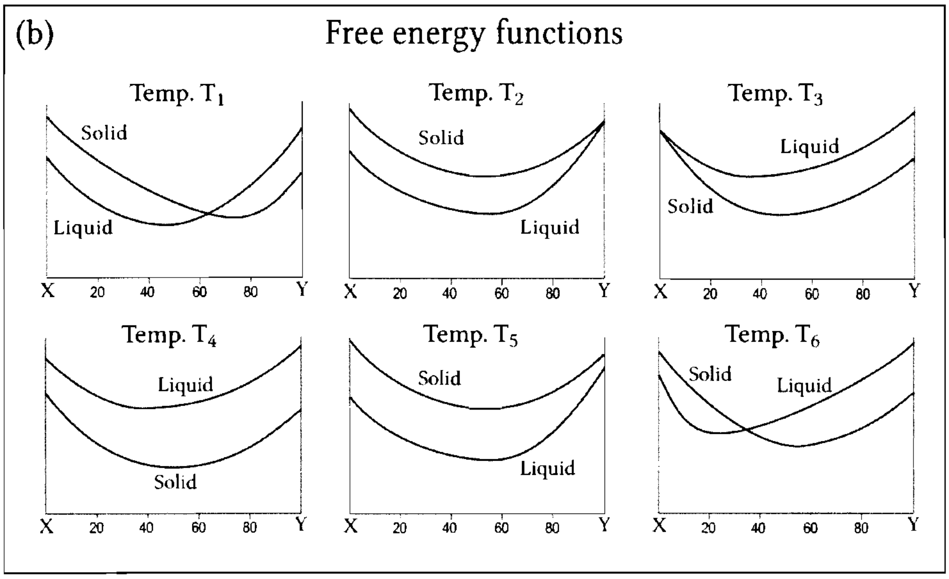
THIS PAGE DELIBERATELY LEFT BLANK

CANDIDATE No. _______________

ENGINEERING TRIPOS PART IB

Tuesday 5 June 2012, Paper 3, Question 4.





Phase diagram and free energy function plots for Question 4.

(to be returned with your solution)

dds4