ENGINEERING TRIPOS PART IB

Wednesday 6 June 2012 2 to 4

Paper 5

ELECTRICAL ENGINEERING

*Answer not more than **four** questions.*

*Answer at least **one** question from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Graph Paper for Question Four.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

ajf03

SECTION A

*Answer at least **one** question from this section.*

1 (a) The circuit diagram of a simple inverting amplifier is shown in Fig. 1. What is the gain, input impedance and output impedance of the circuit if the operational amplifier is ideal? [2]

(b) A more complicated inverting amplifier circuit is shown in Fig. 2. The operational amplifier is ideal. Show that the gain, G, of this circuit is given by

$$G = -\left(\frac{R_3}{R_4} + 2\right)$$ [5]

(c) An amplifier is required with a gain of −50 and an input impedance of 240 kΩ, and it is to be made with an ideal operational amplifier.

(i) If the circuit of Fig. 1 is to be used to produce this amplifier, calculate the required value of R_2. [2]

(ii) If the circuit of Fig. 2 is to be used to produce this amplifier, calculate the required value of R_4. [2]

(iii) Which circuit would you recommend using for this application? Justify your answer. [2]

(d) In practice, you must use one of the non-ideal operational amplifiers listed in Table 1 to make your chosen circuit. Which of the four types of available operational amplifier is most suitable for making the circuit you have selected in part (c)? Explain your answer quantitatively. [3]

(e) The gain of all of the operational amplifiers listed in Table 1 varies with frequency as $A = A_0/(1 + \mathrm{j}f/f_c)$ where $f_c = 10\ \mathrm{Hz}$. Estimate the high frequency cut-off of your circuit with the operational amplifier that you have selected in part (d). [4]

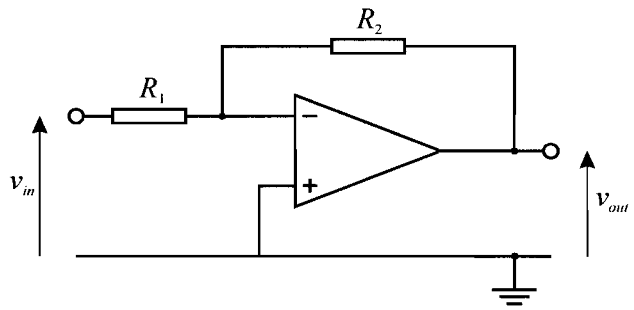

Fig. 1

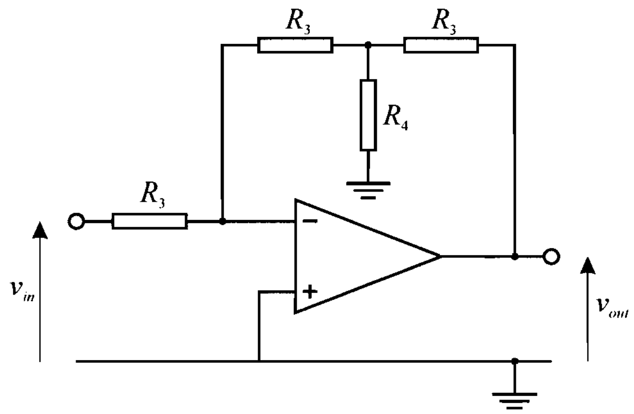

Fig. 2

Operational Amplifier	A_0	**Output Resistance**	**Input Resistance**
Type A	10^6	10 Ω	100 kΩ
Type B	10^6	100 Ω	1 MΩ
Type C	10^6	1 kΩ	10 MΩ
Type D	10^6	10 kΩ	100 MΩ

Table 1

2 (a) Explain what is meant by the *common mode rejection ratio* (CMRR) of a differential amplifier and why it is important. [4]

(b) Calculate the current that will flow through the load resistance Z in the circuit of Fig. 3 if the bipolar junction transistor (BJT) has a potential difference between its base and emitter, V_{BE}, of 0.6 V when it is turned on. You may assume that the dc current gain is infinite. [4]

(c) The circuit should present a constant small-signal impedance, Z_c, looking into the collector of the transistor at terminal X, not taking the load resistance, Z, into account. The bipolar junction transistor has small signal parameters of $h_{ie} = 1.5\ \text{k}\Omega$, $h_{fe} = 250$, $h_{oe} = 200\ \mu\text{S}$ and h_{re} is negligible. If R_{12} represents the effective resistance of R_1 and R_2 in parallel, then the full expression for Z_c is

$$Z_c = \frac{1}{h_{oe}}\left\{1 + R_3 h_{oe} + \frac{h_{fe}R_3 - h_{oe}R_3^2}{R_3 + h_{ie} + R_{12}}\right\}$$

(i) Draw the small-signal equivalent circuit that would be used to find this expression for Z_c (but *do not attempt to derive the equation for yourself*). [3]

(ii) Calculate Z_c for this circuit. [2]

(iii) A given batch of BJTs will have a range of values for h_{fe}. Calculate the acceptable range of values for h_{fe} if the circuit must present an impedance of Z_c that is within ±10% of the value you have calculated in part (ii). [3]

(d) Draw a diagram to show how the circuit of Fig. 3 could be used to produce a differential amplifier circuit with a very high CMRR. Explain how the very high CMRR is achieved. [4]

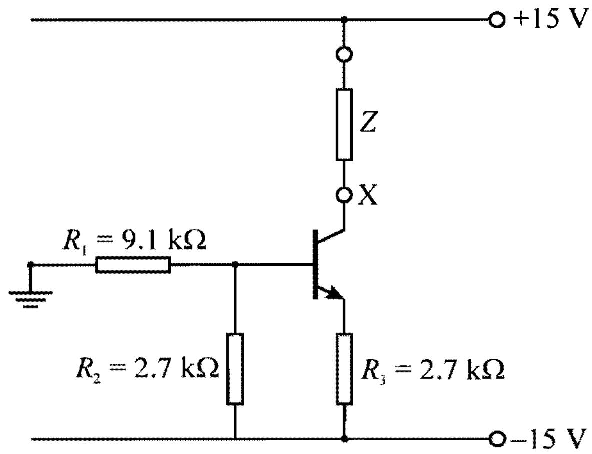

Fig. 3

SECTION B

*Answer at least **one** question from this section.*

3 (a) Explain briefly the advantages of using the *per-unit* system for analysing power supply networks. [2]

(b) A town is connected to a three-phase power supply. It collectively behaves as a balanced load, and consumes 70 MW of real power with a power factor of 0.85 lagging. If the line voltage is 9.9 kV, calculate the line current drawn by the town. [3]

(c) Power is supplied to the town using the power supply network shown in Fig. 4. This consists of a generator rated at 500 MVA and 22kV with a reactance of 0.2 per unit (pu) which is connected to a 132 kV transmission line via a 250 MVA, 22 kV / 132 kV step-up transformer with a reactance of 0.1 pu. The transmission line itself has an impedance of (10 + j15) Ω per phase. A 125 MVA, 132 kV / 11 kV transformer with a reactance of 0.1 pu is then used to step down the voltage for supply to the town.

(i) Calculate the real power dissipated in the 132 kV transmission line. [4]

(ii) Calculate the generator excitation line voltage. [7]

(iii) Calculate the fault current that would flow at the point marked F in Fig. 4 if a symmetrical three-phase short circuit to ground occurs at this point. [4]

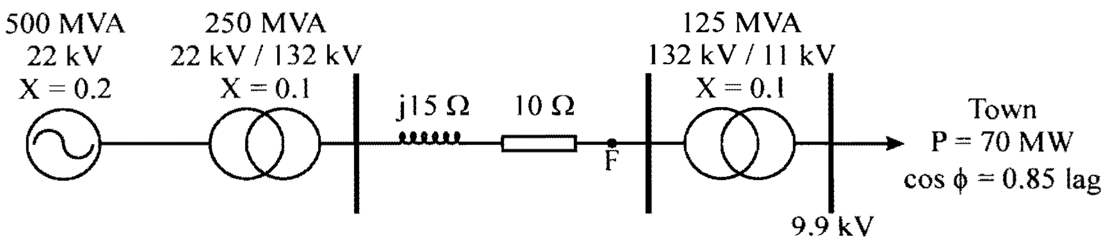

Fig. 4

4 An a.c. synchronous generator is rated at 600 MVA. It is star-connected to a 50 Hz, 22 kV three-phase infinite bus and has a synchronous reactance of 1.5 Ω and negligible stator resistance. It is driven by a turbine with a rated output power of 500 MW, and is operating at its rated MVA with a lagging power factor of 0.75.

(a) Draw a labelled phasor diagram for one phase of the generator. [4]

(b) Calculate the stator current, generator excitation voltage and the rotor load angle. [5]

(c) If the excitation is increased by 5%, calculate the new load angle and power factor. [5]

(d) *Using the detachable sheet at the end of this paper which should be detached and handed in with your answers*, draw a labelled operating chart for the generator if the maximum excitation is 60.6 kV (line) and determine the range of power factors at which the rated MVA could be delivered. [6]

5 (a) The equivalent circuit of one phase of a three-phase induction motor is shown in Fig. 5. Explain the physical significance of each of the six components in the equivalent circuit. Why must the slip be included in the term R_2'/s ? [4]

(b) In a particular induction motor, R_i and X_m can be ignored.

(i) By considering the power input to the equivalent circuit of the motor, the power losses and the conservation of power, show that the motor's output torque is given by

$$T = \frac{3V_{ph}^2}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2} \frac{R_2'}{s\omega_s}$$

where ω_s is the synchronous speed of the motor. [7]

(ii) Derive an expression for the slip at which the torque developed by the motor is maximised. [5]

(iii) Derive an expression for the slip at which the power output from the motor is maximised. [4]

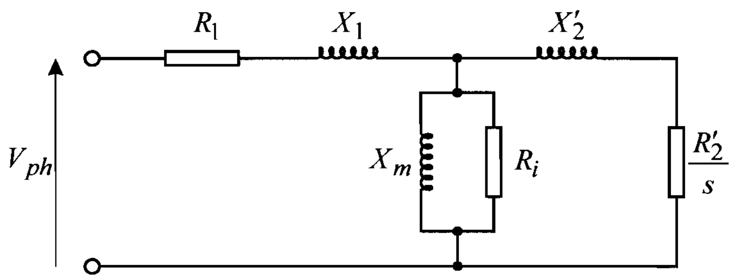

Fig. 5

SECTION C

*Answer at least **one** question from this section.*

6 An ultraviolet laser is to be used to locally remove a 50 nm thick layer of a conductor from the surface of a plastic substrate onto which it has been deposited. The ultraviolet laser operates at a wavelength of 300 nm and produces pulses of light which last 5 ms. Each pulse has an energy of 25 mJ. The light beam is circular in cross section with a radius of 0.5 mm. The whole system operates in air.

(a) Calculate the maximum electric field intensity and maximum magnetic field intensity produced by the laser beam in air. [5]

(b) The conductor has a relative permittivity of 5, a relative permeability of 1, and a conductivity of $1\times10^{6}\ \Omega^{-1}\ \text{m}^{-1}$.

(i) Calculate the amount of energy which enters the conductor from the first pulse. [5]

(ii) Calculate the thickness of conductor in which 99% of the energy which enters the conductor that you calculated in part (i) is absorbed. Comment briefly on your result. [6]

(c) If the laser beam is to be focussed to a point on the surface of the conductor to maximise the power intensity, calculate the minimum radius of the beam at the focal point if breakdown of air is to be avoided, which occurs at a field of 3 MV m^{-1}. You may assume that there are no reflections or energy losses due to the focussing optics. [4]

7 The *Telegrapher's Equations* for a lossless transmission line are

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t} \quad \text{and} \quad \frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t}$$

where L is the inductance per unit length and C is the capacitance per unit length.

(a) Starting from the Telegrapher's Equations, show that a travelling electrical wave will propagate along a lossless transmission line with a velocity $v = 1/\sqrt{LC}$. [4]

(b) A simple sinusoidal electrical signal propagating along a lossless transmission line in the absence of reflections may have its voltage and current expressed as

$$V_F = \Re\{\overline{V}_F \exp[\mathrm{j}(\omega t - \beta x)]\} \quad \text{and} \quad I_F = \Re\{\overline{I}_F \exp[\mathrm{j}(\omega t - \beta x)]\} .$$

Using these expressions and the Telegrapher's Equations, show that the characteristic impedance of the transmission line is given by $Z_0 = \sqrt{L/C}$. [4]

(c) A lossless coaxial cable has a characteristic impedance of 50 Ω and a capacitance per unit length of 100 pF m^{-1}. It is used to transmit a sinusoidal signal at a frequency of 100 MHz.

(i) Calculate the velocity with which an electrical signal will propagate along the transmission line. How would the design of the coaxial cable need to be changed to increase this velocity? [3]

(ii) The coaxial cable is to be terminated with a resistive load so that a standing wave is produced in the cable with a voltage standing wave ratio (VSWR) of 1.5. Calculate the required load resistance to achieve this. [5]

(iii) Assuming that the coaxial cable has a non-zero length, what is the minimum length that the cable must be if the input impedance to the cable is to be the same as the load resistance? [4]

END OF PAPER

ENGINEERING TRIPOS PART IB
ELECTRICAL

CANDIDATE NUMBER: __________

Graph paper for question 4 which should be handed in with your solutions. Additional copies are available from the Invigilator.