ENGINEERING TRIPOS PART IB

Thursday 7 June 2012 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig. 4.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

pcw04

SECTION A

*Answer not more than **two** questions from this section.*

1 Figure 1 shows a control system in which the output $y(t)$ should track the reference signal $r(t)$ in the presence of a disturbance $d(t)$ at the input to the plant. The plant has a transfer function $G(s) = \frac{1}{s(s+3)}$ and a proportional controller with gain K is used.

(a) Find an expression for $\bar{y}(s)$ in terms of $\bar{r}(s)$ and $\bar{d}(s)$. Hence find the transfer functions from $\bar{r}(s)$ to $\bar{y}(s)$ and from $\bar{d}(s)$ to $\bar{y}(s)$. [4]

(b) Find an expression for the damping ratio of the closed-loop system in terms of K. [3]

(c) What is the steady state error i.e. $\lim_{t\to\infty}(y(t) - r(t))$, for each of the following cases, where $H(t)$ denotes the unit step function:

(i) $r(t) = H(t)$ and $d(t) = 0$, [2]

(ii) $r(t) = 0$ and $d(t) = H(t)$, [2]

(iii) $r(t) = 0$ and $d(t) = e^{-2t}$, [2]

(iv) $r(t) = tH(t)$ and $d(t) = 0$. [3]

(d) How should the controller be modified so that the system is not affected by steady state disturbances as applied in part (c)(ii)? In what way would such a modification change the tracking ability of the system even in the absence of any disturbances? [4]

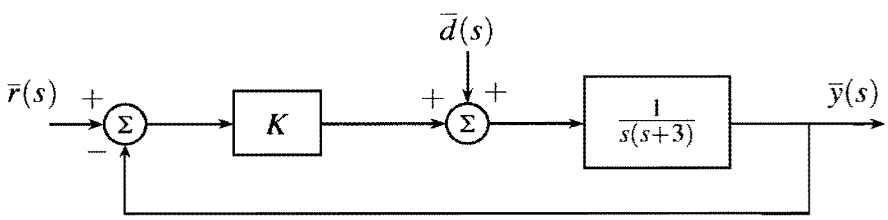


Fig. 1

2 (a) What is the Nyquist stability criterion as it applies to the feedback system shown in Fig. 2 where the controller has a positive gain K and the plant is an asymptotically stable linear system with a transfer function $G(s)$? [3]

(b) For the feedback system shown in Fig. 2, the controller has a gain K, and the plant $G(s)$ is given by

$$G(s) = \frac{1}{s(s^2+4s+2)} .$$

Sketch the Nyquist diagram for the plant, showing any asymptotes. Show how the magnitude of the closed loop frequency response can be determined from the Nyquist diagram. [7]

(c) What is the gain margin of the feedback system if $K = 1$? [2]

(d) For a particular value of K, the phase margin is 45°and occurs at a frequency ω_c. Find the value of ω_c, the corresponding value of K, and the magnitude of the closed loop frequency response at ω_c. [6]

(e) Why is it considered undesirable for the peak magnitude of the closed loop frequency response to be significantly greater than unity? [2]

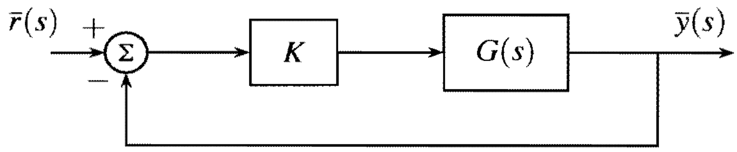


Fig. 2

3 (a) Explain the meaning of the terms *gain margin* and *phase margin* for a feedback control system and state how they can be measured from a Bode diagram. [4]

(b) A controller $K(s)$ and a plant $G(s)$ are connected in a feedback control system as shown in Fig. 3. The plant is asymptotically stable and has a Bode diagram shown in Fig. 4. Find the gain margin and phase margin of the closed loop system when the $K(s)$ transfer function represents a proportional controller and $K(s) = 2$. [5]

(c) The proportional controller is now replaced by a compensator with transfer function

$$K(s) = \frac{20(30+s)}{300+s}.$$

(i) On the supplied copy of Fig. 4, draw the response of $K(s)$ and of the compensated loop, $K(s)G(s)$. You should show any asymptotes and corner frequencies for $K(s)$. [6]

(ii) Determine the new gain margin and phase margin of the compensated loop. [2]

(iii) Comment on the closed-loop behaviour of the system for the two options for $K(s)$ discussed in parts (b) and (c). [3]

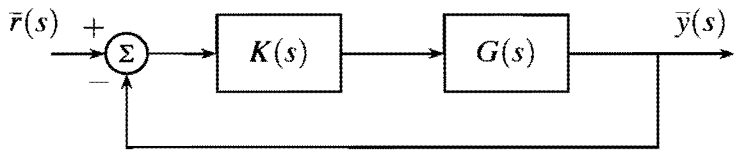


Fig. 3

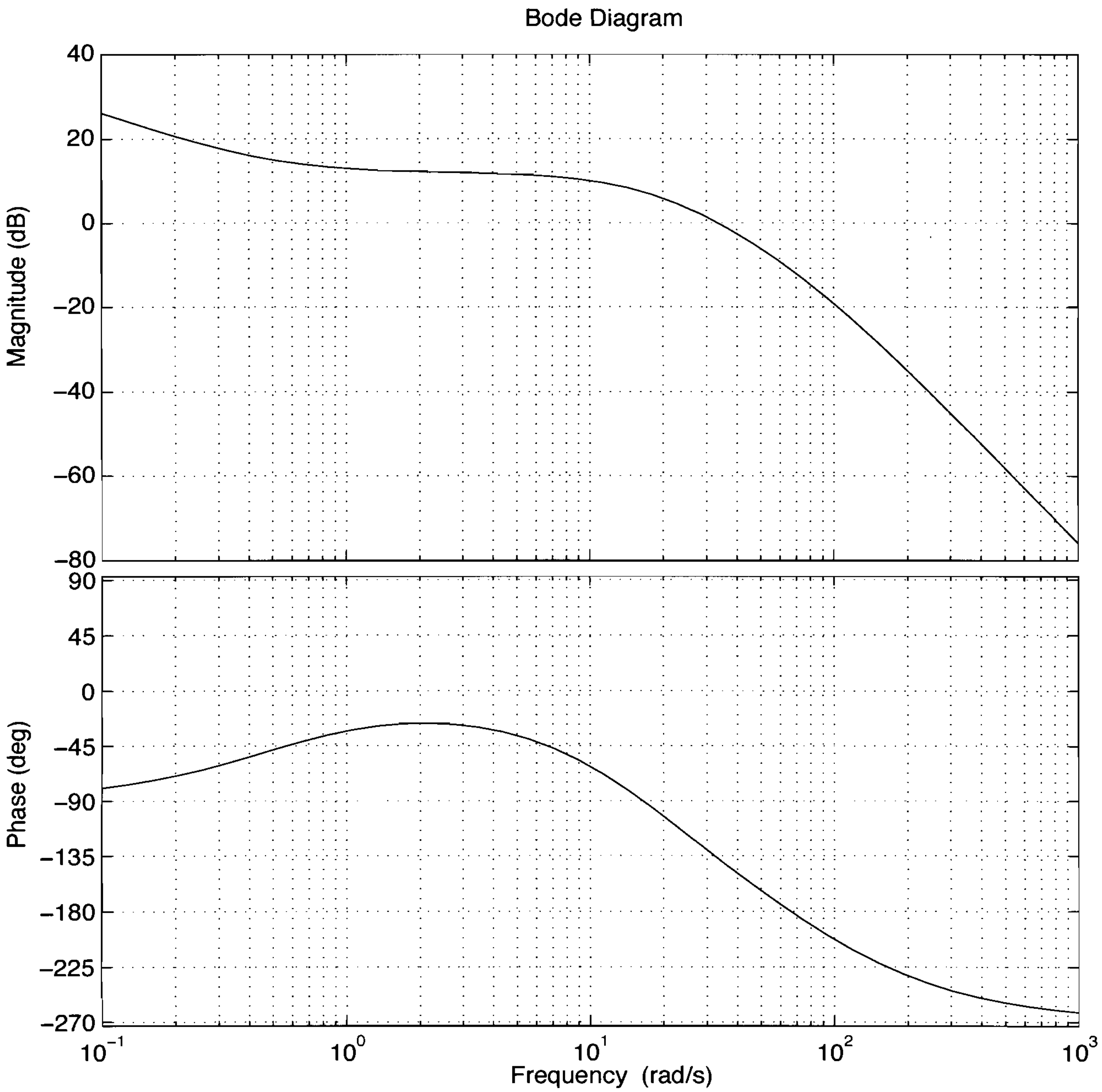


Fig. 4

Note: an additional copy of Fig. 4 is attached at the end of this paper. This should be annotated with your constructions and handed in with your answer to this question.

SECTION B

*Answer not more than **two** questions from this section.*

4 (a) The Fourier transform of a function $f(t)$ is $F(\omega)$. Show that the Fourier transform of the shifted function $f(t-t_0)$ is given by $e^{-j\omega t_0}F(\omega)$. [3]

(b) Using the result in part (a), show that the inverse Fourier transform of $F(\omega)\sin a\omega$ is given by

$$\frac{1}{2j}\left(f(t+a)-f(t-a)\right)$$ [4]

(c) Now consider the Gaussian function $g(t) = e^{-a^2t^2}$ whose Fourier transform is also a Gaussian, $G(\omega)$, where

$$G(\omega) = \frac{\sqrt{\pi}}{a}e^{-\omega^2/(4a^2)}$$

Sketch the derivative, $g'(t)$, of the Gaussian $g(t)$ and give its Fourier transform, $G_1(\omega) = FT(g'(t))$. [You may use standard results.] [5]

(d) We now wish to convolve $g'(t)$ with the rectangular pulse, $p(t)$, of width b and height c, centred on the origin. This convolution is given by $h(t)$ where

$$h(t) = \int_{-\infty}^{+\infty} p(t-\tau)g'(\tau)d\tau$$

(i) Show that the Fourier transform, $H(\omega)$, of this convolution is given by

$$2jc\sin\left(\frac{\omega b}{2}\right)G(\omega)$$ [4]

(ii) Using the result in part (b), find the inverse Fourier transform of $H(\omega)$, showing that it can be written as a difference of Gaussians. Hence give a rough sketch of the convolution $h(t)$ for the two extreme cases, $b \gg 1/a$ and $b \ll 1/a$. [4]

5 (a) A continuous-time signal $x(t)$, which contains no frequency components greater than f_{max}, is sampled at a frequency of f_s. Write an expression for the Fourier transform of the sampled signal in terms of the Fourier transform of $x(t)$. Using this result, explain the concepts of *aliasing* and show how one obtains the minimum value of f_s required to enable exact recovery of the original signal from its samples. [6]

(b) If the signal $x(t)$ is uniformly sampled at intervals of T to produce N samples, $\{x_n\}, n = 0, ..., N-1$, the length-N Discrete Fourier Transform (DFT) of this sequence is a new sequence, $\{X_m\}$, where

$$X_m = \sum_{n=0}^{N-1} x_n \exp(-jnm2\pi/N), \qquad m = 0, ..., N-1$$

A signal is sampled at 4 Hz and 4 samples are recorded to form the $\{x_n\}$. The DFT of this sequence is

$$\{X_m\} = [0, 1, 0, 1]$$

(i) By directly evaluating the inverse DFT, determine the signal samples, $\{x_n\}, n = 0, ..., 3$. [4]

(ii) Write down the frequencies of the non-zero components in the DFT and hence infer the simplest form of the signal $x(t)$ (i.e. assume no aliasing). Verify that the form agrees with the samples found in part (b)(i). [4]

(c) A speech signal with bandwidth 3 kHz is to be sampled, quantised and then transmitted. If the maximum transmission rate is 64 kbit s^{-1} and the signal is sampled at a rate which prevents aliasing, what is the maximum number of quantisation levels that can be used? State any assumptions made.

Estimate the signal-to-noise ratio (SNR) in dB of this quantiser assuming that the speech signal can be approximated as sine waves of maximum amplitude. [6]

6 (a) Explain the concept of modulation in practical communication systems and describe *amplitude* modulation (AM), *phase* modulation (PM), and *frequency* modulation (FM) schemes for analogue signals. Spectral properties of the modulation schemes need not be discussed. [5]

(b) For binary digital signals, describe the modulation systems analogous to AM, PM and FM. [4]

(c) The kth symbol in a message sequence is a_k, where a_k can take one of two values, $+A$ and $-A$. If the symbol period is T, the digital signal can be represented by

$$x(t) = \sum_k a_k p(t - kT)$$

where $p(t)$ is a pulse. This process is *Pulse Amplitude Modulation* (PAM).

(i) State the conditions that the pulse $p(t)$ should satisfy. If $p(t)$ is a unit amplitude rectangular pulse of width T centred on the origin, sketch the PAM signal for the message sequence $[-A, A, A, A, -A, A, -A, -A, A, A]$. [3]

(ii) The sequence in part (c)(i) is now modulated using *Binary Phase Shift Keying* (BPSK) with a carrier frequency of f_c. Sketch the transmitted signal if the carrier frequency is $1/T$ Hz. [3]

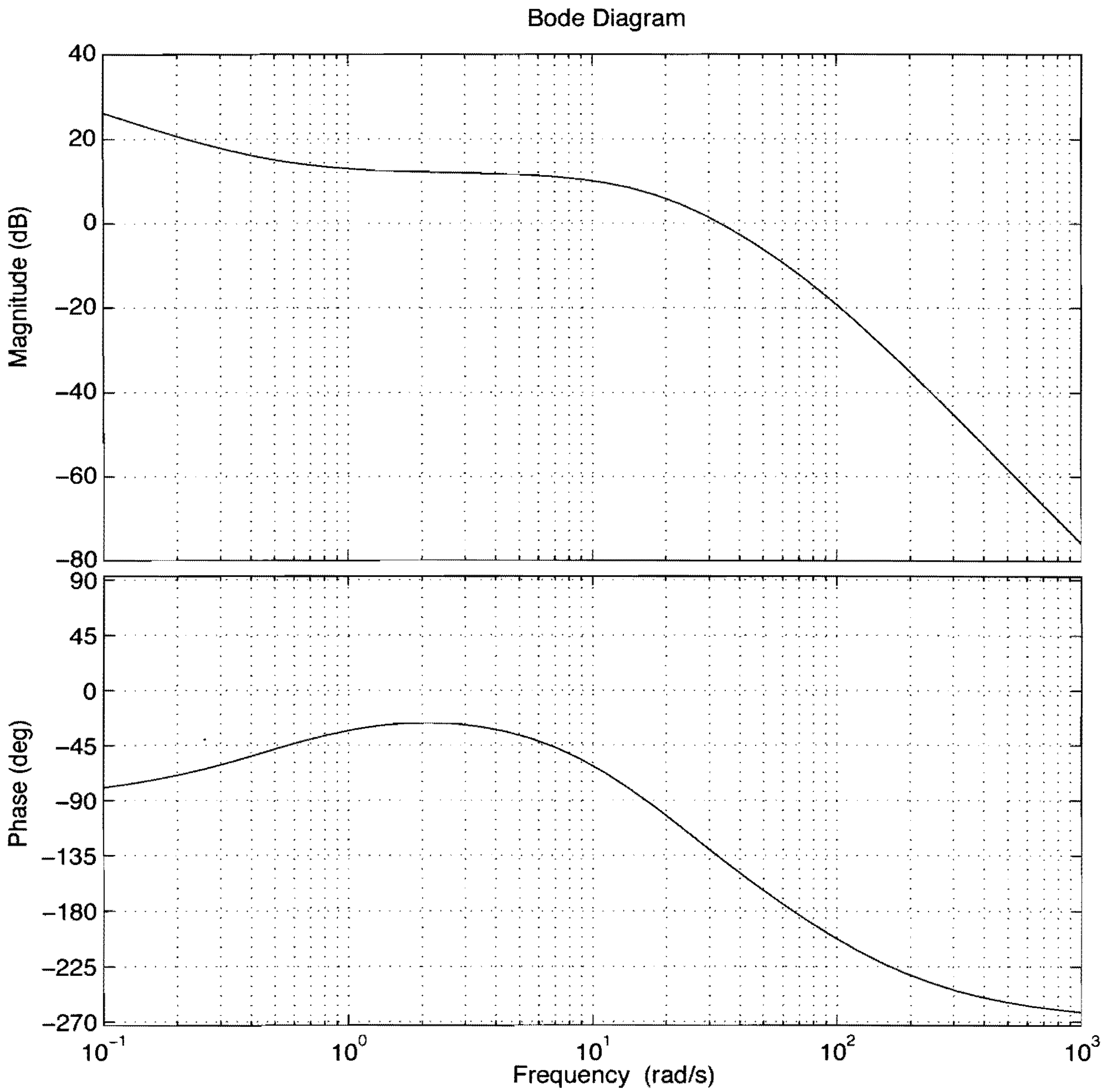
(iii) The modulated signal in part (c)(ii) is transmitted over a channel that introduces Gaussian noise of zero mean and variance σ^2. The received signal is demodulated and sampled at pulse peaks giving the sequence y_k. A value of $+A$ is used as an estimate of a_k if the observed value $y_k > 0$ and a value of $-A$ if the observed value $y_k < 0$. Show that the probability of error, P_e, is given by

$$P_e = Q(A/\sigma)$$

where $Q(x) = 1 - \Phi(x)$, where $\Phi(x)$ is the cumulative Gaussian distribution function. [5]

END OF PAPER

Candidate Number:



Copy of Fig. 4. This should be annotated with your constructions and handed in with your answer to question 3.