ENGINEERING TRIPOS PART IB

Friday 8 June 2012 9 to 11.30
9 to 10.30 Foreign Language Option

Paper 8

SELECTED TOPICS

*Answer **one** question from Section A. In addition:*

> *If you are not taking the Foreign Language option, answer **four** questions, taken from only two of Sections B – H. Not more than **two** questions from each section may be answered.*
> *If you are taking the Foreign Language option, answer **two** questions from **one** of Sections B – H.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Data Sheet for Section B (6 pages)
Data Sheet for Section E (2 pages)

STATIONERY REQUIREMENTS
Single-sided script paper
CUED approved calculated allowed

SPECIAL REQUIREMENTS
Engineering Data Book

JR5

You may not start to read questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A *Introductory Business Economics*

*Answer not more than **one** question from this section.*

1 (a) Explain the concept of profit maximisation. [4]

(b) Explain why firms may not seek to maximise profits. [6]

(c) Using the Keynesian Consumption Function Model, explain the potential impact of a temporary increase in income tax on the level of aggregate demand. [5]

(d) Using either the Life Cycle Model or the Permanent Income Model, explain the potential impact of a temporary increase in income tax on the level of aggregate demand. [5]

2 (a) Outline the kinked demand curve theory of oligopoly. [6]

(b) Under what circumstances might monopolies improve consumer welfare? [6]

(c) Outline the accelerator model of investment. [8]

SECTION B *Civil and Structural Engineering*

Answer not more than ***two*** *questions from this section.*

Note Data Sheets at end of paper.

3 An underground railway is to be constructed beneath the centre of Cambridge. The ground conditions are typically 5 m of sand fill overlying Gault Clay, with the water table close to the ground surface. The undrained shear strength of the Gault clay can be taken to be 150 kN m^{-2}, constant with depth. The unit weight of all soils can be taken to be 20 kN m^{-3}.

You are to advise on the relative merits of two options: (i) a cut-and-cover tunnel, and (ii) a bored tunnel. The cut and cover tunnel would be a rectangular, 10 m wide and 6 m high, allowing for two tracks, whereas the bored tunnel would consist of two separate 8 m diameter tunnels. The depth to the bottom of the tunnels would vary between 12 m and 20 m. The tunnel route would be adjacent to a number of historic buildings.

(a) What methods of construction would be suitable for the bored tunnel? Would the tunnels need to be constructed by open-face or closed-face techniques? [5]

(b) Suggest two methods by which the bored tunnels could be lined, briefly describing the processes. [3]

(c) Discuss the advantages and disadvantages of cut-and-cover tunnel construction. [4]

(d) Why does settlement potentially cause damage to masonry structures? How might the damage due to tunnel construction be minimised? [5]

(e) Describe how instrumentation can be used to ensure the success of schemes to prevent damage to overlying structures. [3]

4. A retaining wall, propped at the ground surface, is to be constructed in order to construct an 8 m deep station box for an underground railway, as shown in Fig. 1. The soil profile consists of a stiff clay with a unit weight $\gamma = 20$ kN m^{-3}, an undrained shear strength of 50 kPa and a critical state friction angle $\varphi' = 20°$. The retaining wall will penetrate a distance 8 m into the underlying clay. The water table is below the base of the retaining wall.

Sketch the earth pressure distribution on both sides of the retaining wall and calculate whether a penetration depth of 8 m is sufficient for stability of the wall with no surface surcharge for the following conditions:

(a) assuming short-term undrained conditions; [5]

(b) assuming long-term drained conditions. [5]

A surcharge of 25 kPa is applied to the ground surface. Is the wall stable:

(c) assuming short-term undrained conditions; [5]

(d) assuming long-term drained conditions? [5]

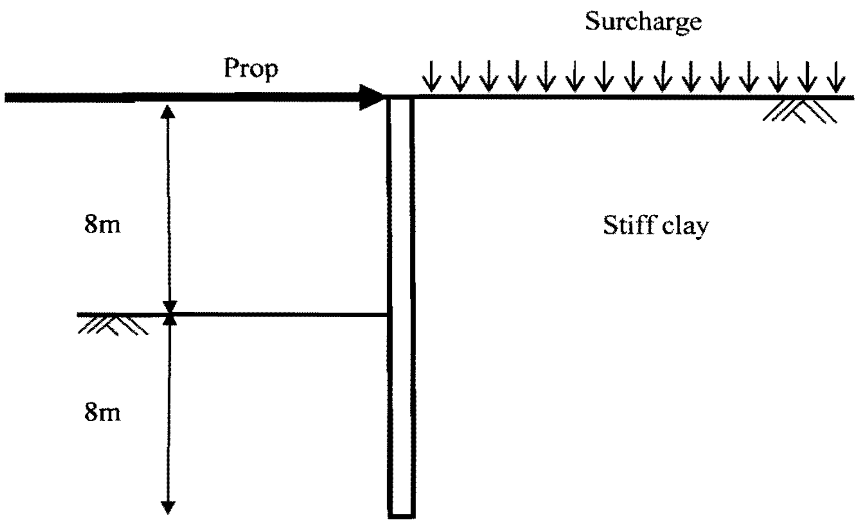


Fig.1

5. A reinforced concrete wall AD supports a 5 m deep excavation and penetrates a further 3 m into the ground, as shown in Fig. 2. Tie rods, at 2 m intervals along the length of the wall, at position B, are connected to remote anchor blocks.

The cube strength of the concrete is 40 MPa and the yield strength of the reinforcing bars is 460 MPa. The earth pressures shown in Fig. 2 are those that act on the wall.

(a) If the earth pressure is zero at point C, x at point D and is assumed to vary linearly between these points, calculate x. [2]

(b) What tension T will each tie-rod carry? [2]

(c) Calculate the shear force and bending moment at points A, B, C & D and the maximum and minimum bending moments. Sketch the resulting shear force and bending moment diagrams. [6]

(d) If the wall is to be singly reinforced, calculate the minimum effective depth of the section for the most heavily loaded part of the wall. [4]

(e) If the actual effective depth is 50% greater than the value calculated in part (d) above, determine a suitable layout of reinforcement in the wall. [6]

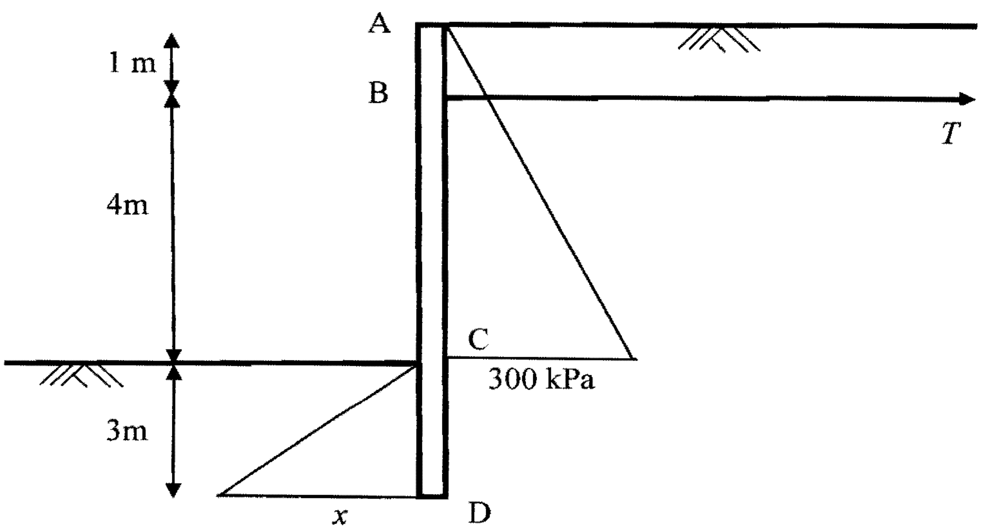


Fig. 2

SECTION C Mechanics, Materials and Design

*Answer not more than **two** questions from this section.*

6 (a) Consider bending vibrations of a wind turbine tower of height L with a uniform cross-section of second moment of area I made of a material with Young's modulus E. Figure 3(a) illustrates an idealisation of the tower as a continuous beam, encastred at its root, whose lowest frequency of vibration f is given by the expression

$$f = \frac{3.52}{2\pi}\sqrt{\frac{EI}{mL^4}}$$

where m is the mass per unit length. Figure 3(b) shows an alternative model where the tower is represented by a light rigid bar of length L and a point mass M at the top of the bar. The base of the bar is connected to the ground via a rotational spring of stiffness k.

(i) One modelling approach aims to match the lowest frequency of bending vibration of the tower and the stiffness associated with a transverse load at the top of the tower for the two idealisations of Figs. 3(a) and 3(b). Find expressions for M and k in terms of E, I, L and m which achieves this. [8]

(ii) An alternative approach matches the mass and lowest frequency of vibration of the two tower idealisations, but not the stiffness. Give an advantage or disadvantage of such an approach, compared to the approach in part (i). [2]

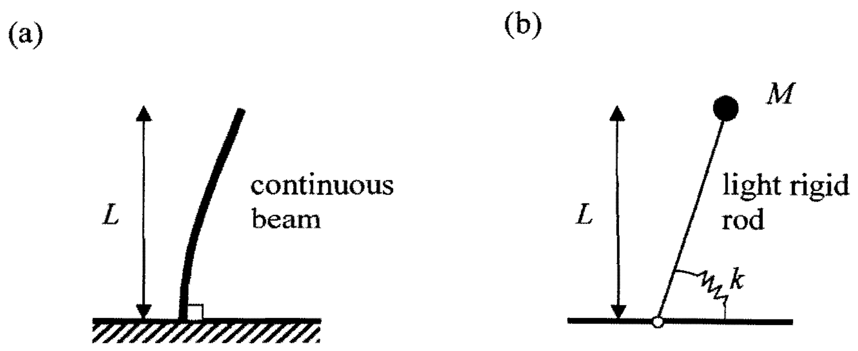


Fig. 3

(b) Define the terms 'cut-in', 'rated' and 'stall' as applied to wind speeds for wind turbines and sketch a typical power versus wind speed characteristic. [3]

(c) Give two methods of controlling the turbine power so that it remains fixed at its rated value between rated and stall wind speeds. [1]

(d) Simplified wind data for an offshore wind turbine is given in the table below. A 'new generation' wind turbine has a cut-in wind speed of 3 m s^{-1}, a stall wind speed of 20 m s^{-1} and a rated wind speed of 14 m s^{-1}, at which it produces a rated output of 5 MW. The turbine operates at variable speed between cut-in and rated wind speeds so as to maintain its optimum tip-speed ratio of 10, at which the power coefficient is 0.45.

Find:

(i) the turbine diameter;

(ii) the rotational speed of the turbine at its rated wind speed;

(iii) the annual energy produced;

(iv) the capacity factor. [6]

Take the power produced by a turbine as $P = 0.5 C_p \rho A v^3$ and the density of air as 1.23 kg m^{-3}.

Wind speed, v (m s^{-1})	Number of days per year
< 2.5	10
7	185
12	100
16	50
>20	20

7 (a) Discuss noise issues relating to wind turbines, including sources of noise, characterisation of noise and ways to manage the impact of noise. [5]

(b) Figure 4(a) illustrates a wind turbine blade subject to a uniform torsional moment intensity Q per unit length of blade. The blade has a thin-walled cross-section, which is uniform along its length. The wall thickness t is uniform around the cross section and along the length of the blade, but is a design variable which can be varied. The area enclosed by the section A_e and the cross-sectional perimeter s are fixed. The blade is made of a material with shear modulus G.

(i) Derive an expression for the grouping of material parameters which should be maximised to minimise blade mass for a given torsional rotation of the tip of the blade relative to its root. [6]

(ii) Why might torsional rigidity be an important design objective? [1]

(c) Figure 4(b) illustrates the cross-section of a blade which is to be made of glass fibre composite with some regions of unidirectional material and other regions of 50% 0° plies and 50% of ±45° plies.

(i) Use Fig. 4(c) to find the Young's moduli of the proposed lay-ups. [2]

(ii) Identify which regions of the cross-section might be appropriate for each of the two lay-ups, explaining your reasoning. [3]

(d) Describe the vacuum infusion process for making composite components, and comment on why it is widely used to make wind turbine blades. [3]

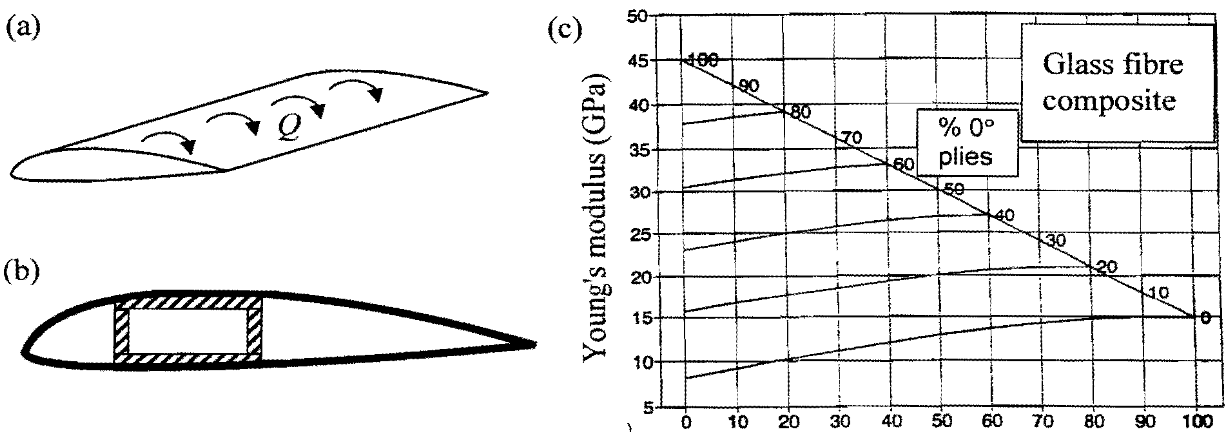


Fig. 4

8 A horizontal axis wind turbine has three blades of radius 20 m and turns with an angular velocity $\omega = 30$ rpm. Under normal operating conditions the aerodynamic loads on each blade may be assumed to be distributed as follows: the normal force intensity F_N varies linearly from 0 at the hub $(r = 0)$ to 1000 N m^{-1} at the blade tip $(r = 20$ m); the tangential force F_T is uniformly distributed along the length of the blade with an intensity of 100 N m^{-1}.

(a) What is the mechanical power generated by the turbine? [3]

(b) Each blade has a structural spar with the box cross-section at the blade root $(r = 0)$ shown in Fig. 5. The wall thickness of the spar is 20 mm. The aerodynamic surface of the blade may be neglected for structural calculations.

What is the maximum stress in the spar cross-section at the blade root due to the aerodynamic loads? [5]

(c) The total mass of each blade per unit length may be assumed to vary linearly from 300 kg m^{-1} at the hub $(r = 0)$ to 0 at the blade tip $(r = 20$ m).

What is the maximum stress in the spar at the blade root due to:

(i) centrifugal loading; [4]

(ii) self-weight loading? [3]

(d) The spar material has a tensile strength of $\sigma_{TS} = 400$ MPa and its fatigue behaviour may be calculated according to Basquin's Law and Goodman's Rule (Materials Data Book) with the constants $\alpha = 0.08$ and $C_1 = 800$ MPa.

What is the expected lifetime in years of the blades, assuming continuous operation under normal loading conditions? Detail any assumptions. [5]

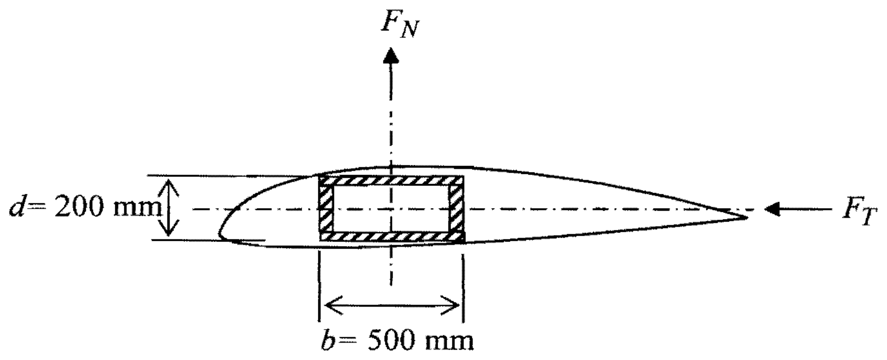


Fig. 5

SECTION D *Aerothermal Engineering*

*Answer not more than **two** questions from this section.*

9 (a) Explain how take off conditions determine the wing area of a passenger aircraft. [3]

(b) Why do passenger aircraft cruise at high altitude? Explain why the cruise altitude should increase during a long distance flight. [4]

(c) Derive the following form of Breguet's range equation for the distance, s, between the start and end of cruise,

$$s = \frac{V \; L/D}{g \; sfc} \ln\left(\frac{W_{start}}{W_{end}}\right)$$

where V is the flight speed, sfc is the thrust specific fuel consumption, L/D is the aircraft lift-to-drag ratio, g is the acceleration due to gravity and W_{start} and W_{end} are the total aircraft weights at the start and end of cruise respectively. State any assumptions made in the derivation of this formula. [5]

(d) Define the terms *thermal efficiency* and *propulsive efficiency*. Show that it is the product of these efficiencies which must be maximised in order to minimise the thrust specific fuel consumption. [4]

(e) Describe how a modern turbofan engine is designed to achieve both high propulsive efficiency and high thermal efficiency. List the factors that limit the upper values of these efficiencies. [4]

10 (a) An aircraft is fitted with two-shaft turbofan engines. At cruise conditions, after the core air stream has passed through the fan, the stagnation temperature T_{023} is 293 K and the stagnation pressure p_{023} is 70 kPa. The core compressor has a stagnation pressure ratio of 25 and an isentropic efficiency of 0.88. Calculate the stagnation temperature T_{03} at exit from the core compressor. Given that the mean blade speed U_m is 350 m s^{-1} and the compressor stage work coefficient $\Delta h_0 / U_m^2$ is not to exceed 0.45, determine the minimum number of core compressor stages required. [5]

(b) The high-pressure turbine drives the core compressor and has an isentropic efficiency of 0.85. The stagnation temperature T_{04} at the inlet to the high-pressure turbine is 1650 K. Determine the stagnation temperature T_{045} and the stagnation pressure p_{045} at entry to the low-pressure turbine. [4]

(c) The low-pressure turbine has an isentropic efficiency of 0.90. The stagnation pressure ratio across the low-pressure turbine is 8.5 and atmospheric pressure is 28.7 kPa. Calculate the velocity V_9 of the core jet. [6]

(d) The cruise speed of the aircraft is 240 m s^{-1} and the temperature rise for the core flow passing through the fan $(T_{023} - T_{02})$ is 41 K. If the core jet velocity is equal to the bypass jet velocity, calculate the bypass ratio of the engine assuming that the fan is isentropic. [5]

For the purposes of this question, neglect the mass flow rate of the fuel. Assume that the combustion products behave as a perfect gas with the same properties as air. Neglect any losses in the ductwork and the propelling nozzles.

Take $\gamma = 1.4$, $R = 287$ J kg^{-1}K^{-1} and $c_p = 1010$ J kg^{-1}K^{-1} for air.

11 (a) A turbofan engine is fitted to an aircraft that cruises at Mach 0.80, at an altitude where the pressure and temperature are 26 kPa and 223 K, respectively. The engine has a total mass flow rate $\dot{m}_a$ of 400 kg s^{-1} and the jet velocity is 350 m s^{-1} for both the core and the bypass streams. Determine the flight velocity, the net thrust and the gross thrust from one engine. Assuming no loss in the inlet ducting, calculate the stagnation pressure p_{02} and stagnation temperature T_{02} at the engine inlet. [5]

(b) The non-dimensional thrust $\widetilde{F}$ and the non-dimensional mass flow $\widetilde{m}$ for the engine are defined as

$$\widetilde{F} = \frac{F_G + p_a A_N}{p_{02} A_N} \qquad \text{and} \qquad \widetilde{m} = \frac{\dot{m}_a \sqrt{c_p T_{02}}}{p_{02} A_N}$$

where A_N is the total nozzle exit area, p_a is the ambient pressure and F_G is the engine gross thrust. Explain carefully the significance of using the term $F_G + p_a A_N$ in the expression for $\widetilde{F}$. If the total nozzle exit area is 2.8 m^2, calculate the values of $\widetilde{F}$ and $\widetilde{m}$ for the engine at cruise. [5]

(c) Use the relationship between net thrust F_N and gross thrust F_G to show that

$$\frac{F_N}{p_a A_N} = \left\{ \widetilde{F}\left(\frac{p_{02}}{p_a}\right) - 1 \right\} - \widetilde{m} M \sqrt{\gamma - 1}\left(\frac{p_{02}}{p_a}\right)\left(\frac{T_{02}}{T_a}\right)^{-0.5}$$

where M is the flight Mach number and T_a is the ambient temperature. [4]

(d) The engine is operated at a new cruise condition at higher altitude where the pressure and temperature are 18 kPa and 216 K, respectively. The flight Mach number at this condition is 0.85. Assuming that the non-dimensional operating point is unchanged, calculate the total mass flow through the engine and the net thrust at the new cruise condition. [6]

For the purposes of this question, $\gamma = 1.4$, $R = 287$ J kg^{-1}K^{-1} and $c_p = 1010$ J kg^{-1}K^{-1} for air. It can be assumed throughout that the propelling nozzles of the engine are choked.

SECTION E *Electrical Engineering*

Answer not more than ***two*** *questions from this section.*

Note the Data Sheet at end of paper

12 (a) Compare and contrast the use of aluminium, aluminium/copper and aluminium/silicon for application as the metallisation system in state-of-the-art integrated circuits. [7]

(b) As device dimensions continue to shrink, multilevel interconnects are increasingly required to reduce the delay time. Estimate the delay time associated with two parallel aluminium wires of length 2.0 mm and of square cross section of side 0.35 μm, separated by a 200 nm silicon dioxide layer. Assume the resistivity of aluminium is 2.7 μΩ cm and that the relative dielectric constant for silicon dioxide is 3.9. [6]

(c) Explain what is meant by a "self-aligned structure" including its benefits for transistor manufacture and explain why poly-silicon is often used as the gate contact material in such a process. [7]

13 (a) Explain the difference between a metal, a semiconductor and an insulator in terms of their band structures. [2]

(b) Explain with the aid of diagrams how conduction occurs in a doped semiconductor. What controls the number of carriers? [3]

(c) Derive the relationship between the electrical conductivity and the mobility of a conductor. Then use the random walk model of electron scattering to relate the electrical conductivity to the effective electron mass, m*, and the time between collisions, τ. [7]

(d) The mobility of a particular semiconductor is 0.5 $m^2 V^{-1}s^{-1}$, its scattering limited velocity is 1.25×10^5 m s^{-1} and its effective electron mass m* is 0.2. This material, doped to a donor density of 4×10^{21} m^{-3}, is used to make a depletion mode FET, operating at a supply voltage of 0.5 V.

(i) At what source-drain length does the conduction become scattering limited?

(ii) What is the corresponding transit time? [5]

(e) Calculate the scattering time between collisions. Compare this to the transit time and comment on the result. [3]

The electronic charge = 1.6×10^{-19} C. Mass of electron = 9.104×10^{-31} kg.

14 (a) Describe what is meant by 'scaling' in CMOS integrated circuits. In simple scaling, if a feature length scales by a factor *k*, how much do the following properties scale by: device density, transit time, voltage, power per chip, wafer diameter? Make clear any assumptions needed for your answer. [6]

(b) What are the main factors that are presently limiting scaling? [4]

(c) Draw a typical carrier velocity vs. electric field curve for a III-V semiconductor such as GaAs, labelling the various regions and giving typical numerical values. [2]

(d) Derive the transit time for an InGaAs FET with source-drain distance of 0.05 μm, if the mobility is 4 $m^2 V^{-1} s^{-1}$, the scattering limited velocity is 2×10^5 m s^{-1} and V_{SD} is 0.5 V. Comment on the result. [3]

(e) Calculate a suitable dopant density so that the transistor will operate with a gate voltage swing of 0.1 V. Assume that the dielectric constant for InGaAs is 1.5×10^{-10} F m^{-1} and the channel depth is 0.02 μm. [5]

SECTION F *Information Engineering*

*Answer not more than **two** questions from this section.*

15 (a) Derive the formula for performing 1-dimensional linear interpolation to obtain the intensity x_p of a pixel at location p, where $a < p < b$ and x_a and x_b are the intensities of pixels located at a and b respectively. [5]

(b) Extend this formula to 2-dimensional bi-linear interpolation to obtain the pixel intensity $x_{p,q}$ at location (p,q) in terms of the four pixels at locations (a,c), (a,d), (b,c) and (b,d). [5]

(c) For fun, it is required to adjust the locations of the features of a facial image. Describe the image-processing technique to achieve this and explain why a Gaussian function is often used to produce the displacement field. Derive an expression for the displacement field $\mathbf{d}(s,t)$ at arbitrary position (s,t) in the image. The maximum of the displacement is centred at location (p,q) and the field is such that the pixel at (p,q) in the modified image comes from the location (u,v) in the original image, while the other pixels around it are displaced less than this. [5]

(d) Explain why it is necessary to control the standard deviation (spread) of the Gaussian function so that it maintains the sense of the image after the image has been distorted (i.e. features should not be distorted in an unrecognisable way). Hence determine how the deviation should relate to the maximum displacement required, $|(u-p,\ v-q)|$. [5]

16 (a) A grey scale image, $I(x,y)$, is to be smoothed and differentiated as part of the edge detection process.

(i) Why is smoothing required? Describe a smoothing filter used in practice, and explain the effects of increasing the size of the filter. [3]

(ii) Give an expression for computing the intensity of a smoothed pixel, $S(x,y)$, in terms of two discrete 1D convolutions. [3]

(iii) Derive expressions for computing the spatial derivatives after smoothing. Include details of the two 1D convolutions required. [4]

(b) Image features such as *blob-like* shapes can be localised in an image by filtering with a *band-pass* filter over multiple scales.

(i) Show how different resolutions of the image can be represented efficiently in an *image pyramid*. Your answer should include details of the implementation of smoothing to avoid convolution with large filters. [4]

(ii) What is meant by *band-pass filtering* and how is this implemented for this purpose. [2]

(iii) Show how image features such as *blob-like* shapes can be localised in an image and describe how to determine an appropriate scale and feature size.

[4]

17 (a) A mobile phone application is used to match an image of a planar target such as a book cover by searching images of known planar objects in its database. A finite number of interest points are extracted automatically from each image. Each feature is to be characterised by its SIFT descriptor.

(i) Give details of how the SIFT descriptor is computed. What image structure is the descriptor attempting to describe? [3]

(ii) How are the descriptors used to match image features to features of known objects in the database? [2]

(iii) How is the target recognised? [3]

(b) The matching scheme is to be extended to describe the similarity of images with different categories of objects using histograms of *visual words*.

(i) What is meant by *visual words* and how can they be computed from SIFT descriptors of interest points in the image? [3]

(ii) Show how an arbitrary image can be represented by a vector of fixed length. [5]

(iii) How can the category of the object in the image be assigned? [4]

SECTION G *Engineering for the Life Sciences*

*Answer not more than **two** questions from this section.*

18 The cornea of the eye is largely comprised of the natural material collagen.

(a) Describe the structure and self-assembly of collagen. [4]

(b) How is collagen in the cornea organized? How does this differ from the collagen organization in the sclera? How does this relate to the function of the cornea? [4]

(c) Describe LASIK surgery in relation to the cornea. [3]

(d) Both the collagen density and the collagen fibril diameter can influence different mechanical properties of a soft collagenous tissue such as the cornea.

(i) Consider a hypothetical disease in which the collagen fibril diameter is unchanged, but in which the collagen density increases. What effect does this change have on the elastic modulus of the tissue? What effect does this change have on the intrinsic permeability? How do the elastic modulus and intrinsic permeability relate to the function of the cornea?

(ii) Now consider a different hypothetical disease, in which the collagen density overall does not change, but for which the diameter of individual collagen fibrils is decreased. What effect does this change have on the elastic modulus and intrinsic permeability, and how does this differ from the change in part (d)(i) above?

[9]

19 (a) Describe, with an appropriate diagram, how the confocal optics in a Scanning Laser Ophthalmoscope (SLO) allows depth sectioning of the fundus. [4]

(b) A light beam is focused at a thin confocal aperture of diameter d. The beam cross-section is a disc of radius r given by:

$$r^2(z) = r_0^2\left(1 + \left(\frac{\lambda z}{\pi r_0^2}\right)^2\right)$$

where z is the axial distance from the confocal aperture, r_0 is the radius at the focal point, and λ is the wavelength of light. Assume that the lens is placed in air.

(i) What is the approximate radial resolution of the lens, $\Delta x = 2r_0$, in terms of the numerical aperture NA and wavelength λ ? [5]

(ii) The axial resolution Δz is defined as the range in z over which at least half of the beam cross-sectional area would pass through the aperture. Show that:

$$\Delta z = \frac{2\lambda}{\pi (NA)^2}\sqrt{\frac{1}{2}\left(\frac{d\,\pi\,NA}{\lambda}\right)^2 - 1}$$

and evaluate Δz for $NA = 0.4$, $\lambda = 800$ nm and $d = 20$ μm. [4]

(iii) First by increasing NA alone, then by decreasing d alone, find the minimum achievable Δz (in each case keeping other values as in (ii)). Also list in each case the values of NA and d which you used. [2]

(iv) What are the problems with the improvements outlined in (iii)? [2]

(c) What factors, other than lens resolution and aperture size, limit the radial resolution and the depth range of an SLO? [3]

20 (a) This question regards the mechanisms of visual processing.

(i) Describe the differences between ganglion cell, simple cell and complex cell receptive fields.

(ii) What is colour opponency and why might it be a useful way for the visual system to code colour?

(iii) Describe the evidence that the visual system processes different spatial frequencies within an image independently.

(iv) Describe the cues to object depth that are missing from a photograph.

[10]

(b) This question regards optimality in visual processing.

(i) Describe the source of the resolution limit of compound eyes, including the corresponding mathematical formula. [3]

(ii) Derive the formula for the response of a neuron, r, as a function of the stimulus (contrast level of light), s. Assume that the neuron encodes this stimulus with an information theoretically optimal tuning curve, with negligible noise in its response. Assume that its response lies between r_{min} and r_{max} and is a monotonically increasing function of the stimulus, and that the stimulus is uniformly distributed between s_{min} and s_{max}. In your derivation:

- explain what makes a tuning curve optimal in this case (i.e. what is the quantity that the tuning curve needs to optimise),
- state what is the response distribution of the neuron in this case,
- explain the steps taken to derive the final formula for the tuning curve. [5]

(iii) Describe the conditions under which two line segments may appear to belong to the same physical object despite not being connected, and how this reflects statistically optimal computations in the brain. [2]

SECTION H Manufacturing, Management and Design

*Answer not more than **two** questions from this section.*

21 (a) List and describe four sources of invention that could be classified as technology push, and four sources of invention that could be classified as market pull. Provide one example for each of the sources. [6]

(b) Describe the four generic ways in which a market may be segmented using a perceptual map. [4]

(c) List the two characteristics of a well-defined market segment. [2]

(d) Sketch three different perceptual maps that could be used to segment the markets for a low cost electrical bicycle. Explain your choice of axes used in each of your perceptual maps. [8]

22 A team of university researchers have developed a novel device for detecting very small quantities of explosives in the air with very high levels of reliability. The researchers believe that this device could be used to improve the effectiveness of airport security systems.

(a) Describe who would be the potential stakeholders in the market for such an airport security device. [6]

(b) Describe how interviews and observations could be used to understand the needs of customers and users of such a device. [6]

(c) The device combines standard and bespoke components, and the reliability of the device is highly dependent on the quality of the components and the assembly process. It is anticipated that the market for this device will be around 100 units per year. Discuss the issues that would need to be considered in designing a production system for this device. [8]

23 (a) List the four tests an invention must satisfy in order for it to be patentable. [2]

(b) Explain why it is desirable to have more than one claim in a patent. [4]

(c) Provide one strength and one weakness for each of the following types of business model:

(i) product;

(ii) service;

(iii) service enabled by a product;

(iv) product with consumables. [4]

(d) The website of the UK semiconductor firm ARM plc states that its business model "*[..] involves the designing and licensing of IP [intellectual property] rather than the manufacturing and selling of actual semiconductor chips*".

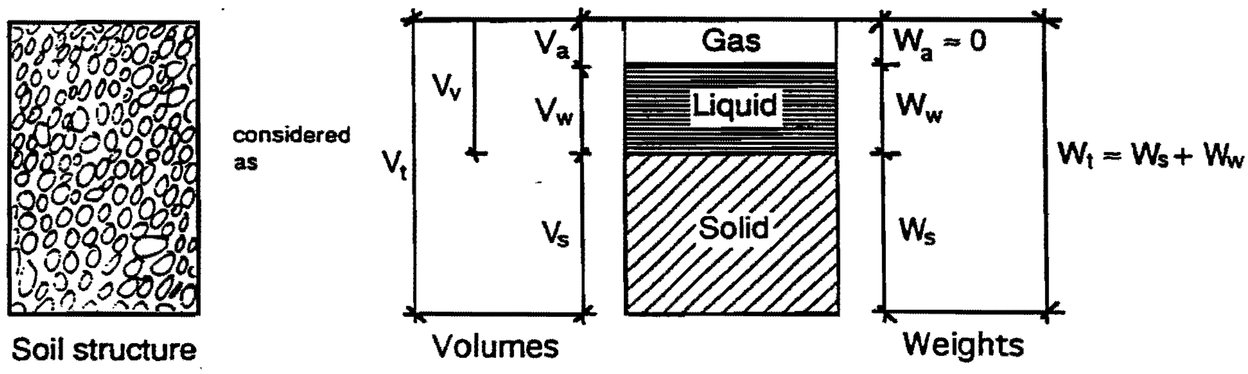
(i) Describe what is meant by 'licensing of IP' in this context. [2]

(ii) Discuss the relative merits of a business model based around licensing of IP in comparison to a business model that is focused upon the manufacturing and selling of semiconductor chips. [8]

END OF PAPER

Data sheet – Soil Mechanics

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w G_s / e)$
Unit weight of water	$\gamma_w = 9.81$ kN/m^3 (although we assume 10 kN/m^3)
Unit weight of soil	$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e} \right) \gamma_w$
Buoyant (effective or submerged) unit weight	$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w$ (soil saturated)
Unit weight of dry soil	$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e} \right) \gamma_w$
Relative density	$I_d = \frac{(e_{max} - e)}{(e_{max} - e_{min})}$

where e_{max} is the maximum voids ratio achievable in the quick tilt test (for sands), and e_{min} is the minimum voids ratio achievable by vibratory compaction (for sands).

Classification of particle sizes

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D equivalent diameter of soil particle

D_{10}, D_{60} etc particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

Stress components

Principle of effective stress (saturated soil):

total stress	σ =	effective stress σ' +	pore water pressure	u
	τ =	τ' +		0

and

σ_v = vertical stress

σ_h = horizontal stresss

τ = shear stress

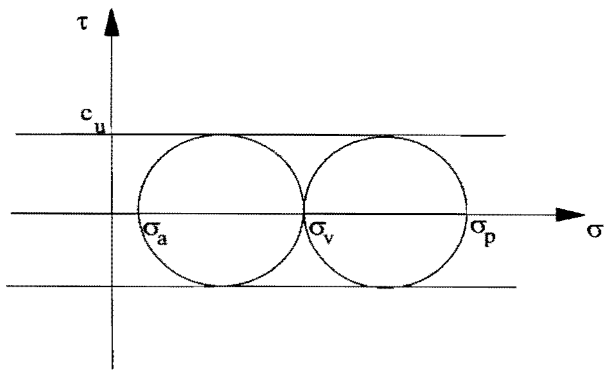
Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the *undrained shear strength* c_u which is mobilized when the shear stress $\tau = c_u$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, σ_a and σ_p respectively, are given by

$$\sigma_a = \sigma_v - 2\,c_u$$

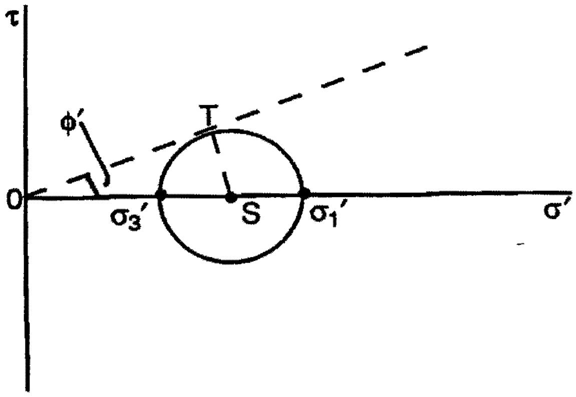
$$\sigma_p = \sigma_v + 2\,c_u$$

where σ_v is the total vertical stress.



Strength of sands

Mobilised angle of shearing ϕ'
where $\tau = \sigma' \tan \phi'$



$$\sin\phi' = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

$$\therefore \phi' = \sin^{-1}\left[\frac{\left(\sigma_1'/\sigma_3'\right) - 1}{\left(\sigma_1'/\sigma_3'\right) + 1}\right]$$

Earth pressure coefficient K: $\sigma_h' = K\sigma_v'$

Active pressure: $\sigma_v' > \sigma_h'$ $\quad \therefore \sigma_1' = \sigma_v'$

$\sigma_3' = \sigma_h'$

$K_a = (1 - \sin\phi'/1 + \sin\phi')$

Passive pressure: $\sigma_h' > \sigma_v'$ $\quad \therefore \sigma_1' = \sigma_h'$

[We assume principal stresses $\quad \sigma_3' = \sigma_v'$

are horizontal and vertical] $\quad K_p = (1 + \sin\phi')/(1 - \sin\phi') = \dfrac{1}{K_a}$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left(\sigma_1'/\sigma_3'\right)_{max}$

at critical state ϕ'_{crit} after large strains.

Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi' = \phi'_{max}$ and

$$\phi'_{max} = \phi'_{crit} + \phi'_{dilatancy}$$

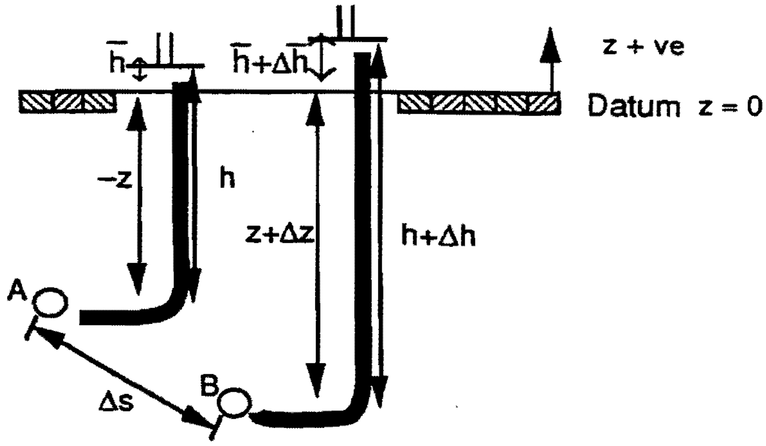
where ϕ'_{crit} is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero $(\phi'_{dilatancy} -> 0)$ as $\phi'_{max} -> \phi'_{crit}$.

ϕ'_{crit} is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values (±2°):

	ϕ'_{crit}	ϕ'_{max}	
feldspar	40°		
quartz	33°	53°	($I_d = 1$, and mean effective stress OS < 150 kPa)
mica	25°		

Seepage

Excess pore water pressure



Head $h = u/\gamma_w$

Potential $\bar{h} = h + z$

Total pore water pressure head at A: $u = \gamma_w h = \gamma_w (\bar{h} - (-z))$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta\bar{h} + \Delta z)$

[Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w (\bar{h} + \Delta\bar{h})$]

Hydraulic gradient A–B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s} = -\dfrac{\Delta\bar{u}}{\gamma_w \Delta s}$

Darcy's law $\quad v = ki$

v = average or superficial seepage velocity

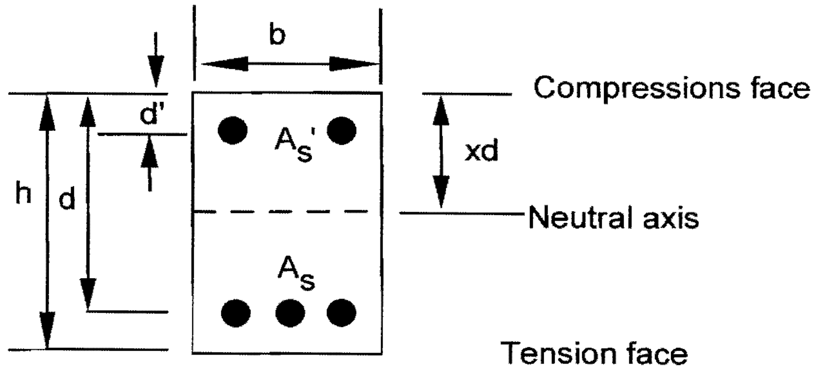
k = coefficient of permeability

Typical permeabilities

$D_{10} > 10$mm	:	non-laminar flow
10 mm > D_{10} > 1μm	:	$k \cong 0.01(D_{10}\text{in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



Design Stresses

Cube strength for concrete f_{cu}. At failure in bending, stress in concrete = $0.4f_{cu}$ over whole area of concrete in compression.

Tensile yield stress of steel f_y. At failure in bending, stress in bars in tension = $0.87f_y$, stress in bars in compression = $0.75f_y$.

Design Equations

Moment capacity of singly reinforced beam

$$M \leq 0.15\, f_{cu} bd^2$$
$$M = 0.87 f_y A_s d(1 - x/2)$$
$$x = 2.175(f_y / f_{cu}).(A_s / bd) \quad (\leq 0.5 \text{ to avoid over reinforcement})$$

Moment capacity of doubly reinforced beam

$$M = 0.15 f_{cu} bd^2 + 0.75 f_y A'_s (d - d')$$
$$0.87 f_y A_s = 0.75 f_y A'_s + 0.2 f_{cu} bd$$

Shear capacity of all beams

Total shear capacity $V = (v_c + v_s)bd$

Where, $v_c = 0.68(100\,A_s/bd)^{0.33}.(400/d)^{0.25}$ (N/mm^2)

and $v_s = 0.87 f_y\, A_{sq}/(bs)$

in which s = shear link spacing, A_{sq} is total area of all shear bars in a link and A_s is the total area of effective longitudinal *tension* steel at the section.

Standard bar sizes

Diameter (mm)	6	8	10	12	16	20	25	32	40	50
Area (mm^2)	28	50	78	113	201	314	491	804	1256	1963

Available steel types

Deformed high yield steel $f_y = 460$ N/mm^2

Plain mild steel $f_y = 250$ N/mm^2

Lap and anchorage lengths 40 bar diameters

Density of reinforced concrete: $24\ kN/m^3$

Reinforcement areas per metre width

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540
Areas calculated to 3 significant figures according to B.S.I recommendations										

April 2010

Part IB Data Sheet: Electrical Engineering Elective Transistor Design Summary Sheet

Gauss's Theorem

$\varepsilon_o \varepsilon_r E_1 - \varepsilon_o \varepsilon_r E_2$ = charge per unit area enclosed between upper surface 1 and lower surface 2.

FET Design Summary

- $\tau_t \to$ switching time as 1st approx. (scattering limited transit time).
- $\tau_{eff} = \tau_t + R_{load} C_{eff(output)} \to$ switching time as 2nd approx.
- $L = v_s \tau_t$ (source-drain spacing).
- $I_{sat} = e N v_s W d_s = e N W L d_s / \tau_t$
- Aspect ratio W/L (technology?).
- $(1/2) e N (d_s)^2 / \varepsilon_o \varepsilon_r =$ (Max Gate Voltage)
- $E'_{peak} = e N d_s / \varepsilon_o \varepsilon_r < E_{breakdown}$
- Minimum Drain Source Voltage $\sim E_s L$ (E_s is the field required to reach limiting velocities).

Mutual Conductance

$$g_{mo} \sim I_{sat} / V_{gate(max)}$$

Mutual conductance reduces with frequency as $g_m(\omega) \approx g_{mo} / (1 + j\omega \tau_t)$;

$$v_{out} = g_m(\omega) R \left(1 + j\omega R C_{eff(out)}\right) \approx g_{mo} R / \left[1 + j\omega \left(\tau_t + R C_{eff(out)}\right)\right]$$
$$= g_{mo} R / [1 + j\omega \tau_{eff}]$$

Capacitances for FET

Parallel Plate Capacitance: $\varepsilon_o \varepsilon_r$ Area/spacing

Used for rough estimates of parasitic capacitance.

Effective Capacitances for FET

$$C_{eff(out)} \to C_{gate/drain} + C_{drain/source} + C_{load};$$

$$C_{eff(in)} \to M C_{gate/drain} + C_{gate/source\,(proximity)} + C_{gate/source\,(electronic)};$$

$$C_{electronic} = g_{mo} \tau_t \quad ; \quad M = \left(1 + g_{mo} R_{load}\right)$$

Time Constants for FET

$\mu = e\tau / m^*$ relates mean free time τ and mobility.

Transit time τ_t over distance L and scattering limited velocity v_s are related by $\tau_t = L / v_s$.

$v_{out} \approx g_{mo} R / [1 + j\omega \tau_{eff}] = g_{mo} R / [1 + j\omega / (2\pi f_t)]$

$1/(2\pi f_\tau) = \tau_t + R C_{eff(out)} = \tau_{eff}$ The transition frequency is f_t.

10% to 90% rise time is $T = 2.2\, \tau_{eff} = (2.2/2\pi)(1/f_t) = 0.35/f_t$.

Electrical Engineering Elective:
Tunnel Barrier Design Summary Sheet

Schrodinger's Equation

Complex Wave $\Psi = A\ \exp(-j2\pi f\,t + j2\pi x/\lambda) = A\exp(-j\omega t)\exp(jkx);$
<momentum> $\Psi = p\Psi = (h/\lambda)\Psi = -j(h/2\pi)\partial\Psi/\partial x;$

<Total energy> $\Psi = E\Psi = hf\Psi = j(h/2\pi)\partial\Psi/\partial t$
$(h/2\pi) \rightarrow \hbar;\ \ h = 6.625 \times 10^{-34}$ J/s.

Schrodinger's equation:-

$$E\Psi = (1/2m)[-j\hbar\partial/\partial x]^2\Psi + e\phi\Psi$$

Tunneling (Rectangular barrier $\varepsilon\phi$)

Propagating waves outside barrier with incident kinetic energy $U_{incident} = (\hbar k_i)^2/2m^*$

Evanescent waves inside barrier: $-(\hbar k)^2/2m^* = (\hbar k_i)^2/2m^* - e\phi$

Technology Design Summary

Diffusion

Constant Surface Concentration:

$$C(x,t) = C_s\ \text{erfc}\left[\frac{x}{2(Dt)^{1/2}}\right]$$

Constant Total Dopant:

$$C(x,t) = \frac{S}{(\pi Dt)^{1/2}}\exp\left[\frac{-x^2}{4Dt}\right]$$

W I Milne
May 2011