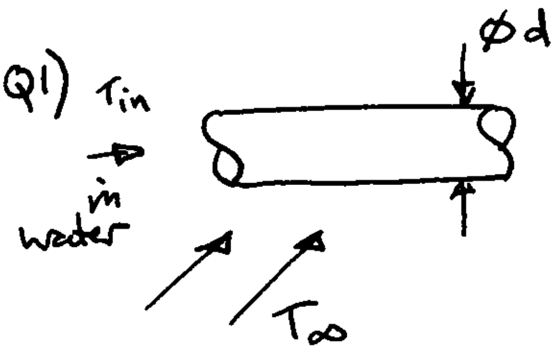
ENGINEERING TRIPOS IB 2012/2013 PAPER 4

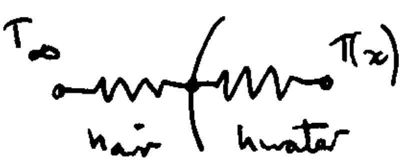
THERMO FLUID MECHANICS

SECTION A

Q1)



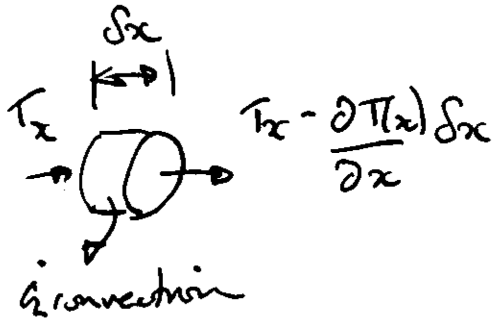
a) i)



- Ignore thermal resistance of the "thin" wall.
- Ignore thickness of "thin" pipe so that A is the same on both sides.

$$\frac{1}{U} = \frac{1}{h_{air}} + \frac{1}{h_{water}} \Rightarrow U = \frac{h_{air} h_{water}}{h_{air} + h_{water}} = \left(\frac{1}{h_{air}} + \frac{1}{h_{water}}\right)^{-1}$$

ii)



Energy balance:

$$\dot{m}c\left[T(x) - \frac{\partial T(x)}{\partial x}\delta x - T(x)\right] = U\pi d\,\delta x\left(T(x) - T_\infty\right)$$

So
$$-\dot{m}c\frac{\partial T(x)}{\partial x}\cancel{\delta x} = U\pi d\,\cancel{\delta x}(T - T_\infty)$$

$$\frac{\partial T(x)}{T - T_\infty} = -\frac{U\pi d}{\dot{m}c}\partial x$$

Integrate $$\int_0^{T(x)} \frac{dT}{T(x)-T_\infty} = \int_0^x \frac{-U\pi d}{\dot{m}c}\,dx$$

$$\ln\left\{\frac{T(x)-T_\infty}{T_{in}-T_\infty}\right\} = \frac{-U\pi d}{\dot{m}c}\,x$$

so $$T(x) = T_\infty + (T_{in}-T_\infty)\exp\left\{\frac{-U\pi d}{\dot{m}c}\,x\right\}$$

iii) Maximum possible temperature drop is $T_{in}-T_\infty$ at $x \to \infty$. At 90% $\theta = 0.1 = \frac{T-T_\infty}{T_{in}-T_\infty}$

so $$U = \frac{\dot{m}c}{\pi d L}\ln 10 = \frac{\dot{m}c}{\pi d L}\,2.3$$

b) i) Estimate h_{water} first

$$\bar{V} = \frac{\dot{m}\,4}{\rho \pi d^2} = 0.056\ ms^{-1}$$

$$Re_{water} = \frac{\rho \bar{V} d}{\mu} = \frac{1000 \times 0.056 \times 0.05}{1.14 \times 10^{-3}} = 2.457 \times 10^3$$

$$Pr_{water} = \frac{\mu C}{\lambda} = \frac{1.14 \times 10^{-3} \times 4.2 \times 10^3}{0.62} = 7.72$$

Databook $Nu_d = 0.023\, Re_d^{0.8}\, Pr^{0.4}$

$$= 0.023 \left(2.457 \times 10^3\right)^{0.8} \times 7.72^{0.4}$$

$$= 26.9$$

$$h_{water} = \frac{Nu_d \lambda_{water}}{d} = \frac{26.9 \times 0.62}{0.05} = 333.1\ Wm^{-2}K^{-1}$$

ii) $h_{air} = 1 \Big/ \left\{ \frac{1}{u} - \frac{1}{h_{water}} \right\}$, $\quad u = \frac{0.11 \times 4.2 \times 10^3 \ln 10}{\pi \times 0.05 \times 30} = 225.74$

$$\text{so } h_{air} = 1 \Big/ \left\{ \frac{1}{225.74} - \frac{1}{333.1} \right\} = 700.4\ Wm^{-2}K^{-1}$$

$$Nu_{air} = \frac{h_{air} d}{\lambda_{air}} = \frac{700.4 \times 0.05}{0.0252} = 1.390 \times 10^3$$

$$Re_{air} = \frac{\rho_{air} V_{air} d}{\mu_{air}}$$

$$V_{air} = \frac{\mu_{air}}{\rho_{air}} Re_{air} = \frac{1.76 \times 10^{-5}}{1.25 \times 0.05} \times \left\{ \frac{1.390 \times 10^{3}}{0.027} \right\}^{1/0.8}$$

$$= 220.8 \text{ ms}^{-1}$$

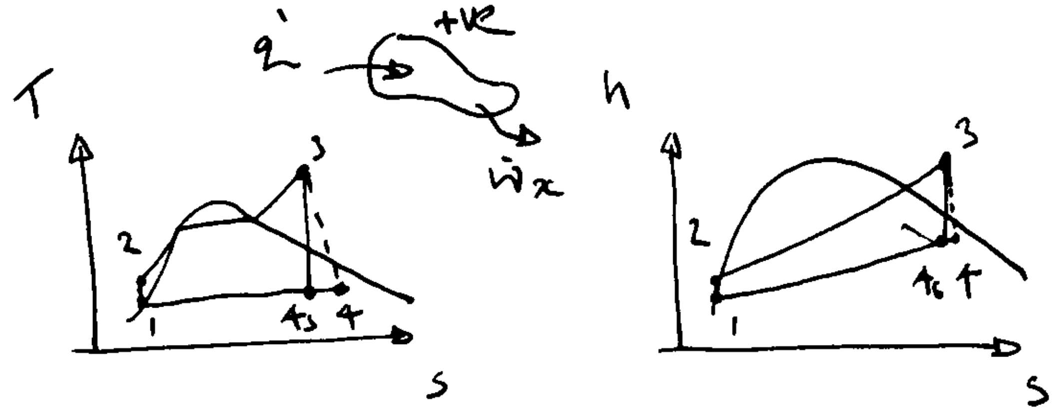
c) Correct answers:

- Lower air temperature (higher density) (or higher pressure)
- Increased surface area - i.e. fins.

N.B. There is an optimum tube diameter, so further analysis is needed to work out whether the tube diameter could be changed, which is beyond the 2 marks allocated.

Q2)

i)



a)

ii) Feed pump: $1 \to 2$ $\quad \Delta h = \int_1^2 v\,dp \approx v\Delta p = 1.004 \times 10^{-3}(0.05 - 30) \times 10^5$

$$-\dot{W}_x = 3 \text{ kJ kg}^{-1}$$

h_1, saturated liquid at 0.05bar $\Rightarrow h_1 = 136.4 \text{ kJ kg}^{-1}$ (table)

$$\Rightarrow h_2 = 139.4 \text{ kJ kg}^{-1}$$

Boiler: $2 \to 3$ $\quad \dot{q}_{in} = \Delta h = h_3 - h_2$

h_3 is 350°C, 30 bar superheated $h_3 = 3120 \text{ kJ kg}^{-1}$ (chart)

Turbine $3 \to 4, 4_s$

$\dot{w}_x = h_3 - h_4 \quad h_{4s} = 2060 \text{ kJ kg}^{-1}$ ($\downarrow \Delta s = 0$ on chart)

$\eta = \dfrac{h_3 - h_4}{h_3 - h_{4s}} \quad h_4 = 3120 - 0.85(3120 - 2060)$

$$h_4 = 2219 \text{ kJ kg}^{-1}$$

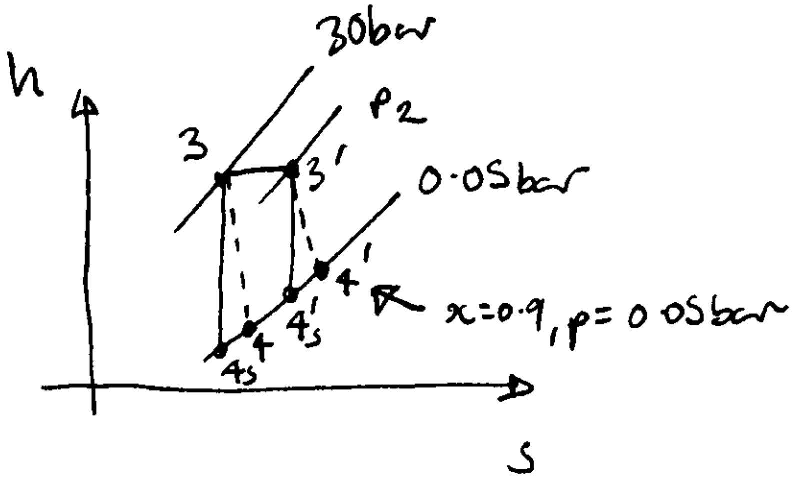
$$\dot{w}_x = 901 \text{ kJ kg}^{-1}$$

$$\eta_{thermal} = \frac{\Sigma \dot{W}_x}{\dot{Q}_{in}} = \frac{901 - 3}{3120 - 139.4} \simeq 30\%$$

ii) Follow up line at 0.05 bar to 2219 kJ kg^{-1}

$\Rightarrow x \simeq 0.86$ which is low enough to cause damage.

b)



Dryness fraction $x=0.9$, at 0.05bar $h_4' = 2320$ kJ kg^{-1} (chart)

$\eta_T = \dfrac{h_3' - h_4'}{h_3' - h_{4s}'} \Rightarrow h_{4s}' = h_3' - \dfrac{(h_3' - h_4)}{\eta}$

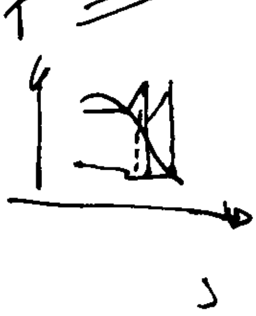
$h_3' = h_3$ as throttle is isenthalpic

so $\underline{\underline{h_{4s}' = 2180 \text{ kJ kg}^{-1}}}$

so $\Delta s = 0$ ↑ gives $p \simeq 12$ bar

so $\underline{\underline{\Delta p \text{ throttle} = 18 \text{ bar}}}$

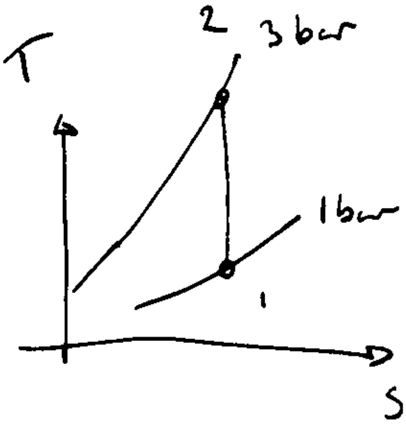
c) $\underline{\underline{\text{reheat.}}}$



New $\eta_{thermal} = \dfrac{h_3' - h_4'}{q_{in}} = \dfrac{3120 - 2320}{3120 - 139.4} = 26.7\%$ (11% drop)

3)

a) i)



$\Delta s = 0$ so $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$= 821.24K$

SFEE $\cancel{\dot{q}}^{0} - \dot{w}_x = \Delta h$

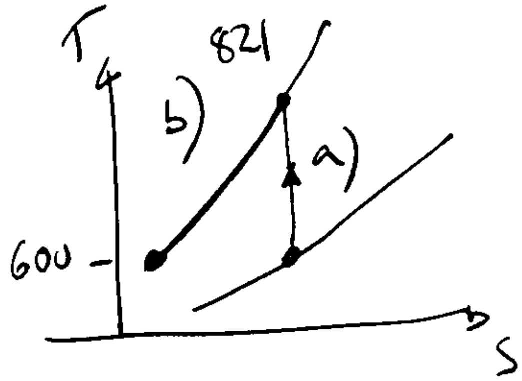
so $-\dot{w}_x = c_p(T_2 - T_1) = 222.3 \, kJkg^{-1}$

ii)

$b_2 - b_1 = -\dot{w}_x + \cancel{\int\left(1 - \frac{T_0}{T}\right)d\dot{q}}^{0} - \cancel{T_0 \Delta s_{irrev}}$

so $\Delta b = -\dot{w}_x$

b) i)



$\Delta b = \cancel{-\dot{w}_x}^{0} - \int\left(1 - \frac{T_0}{T}\right)d\dot{q} + \cancel{T_0 \Delta s_{irrev}}^{0}$

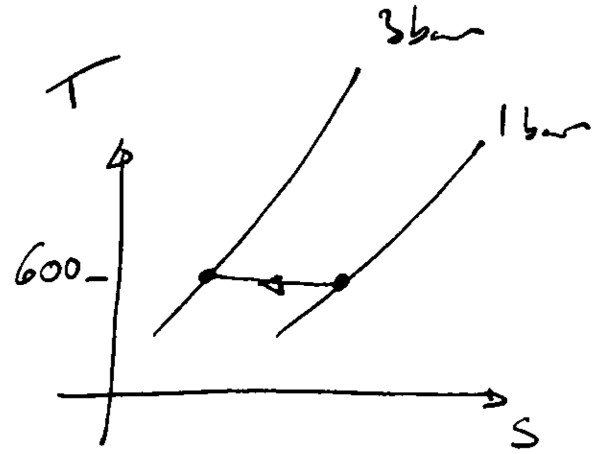
so $\Delta b_{HT} = \Delta b = \Delta h - T_0 \Delta S$

$\Delta S_{2-1} = c_p \ln\left(\frac{T_2}{T_1}\right) - \cancel{R \ln\left(\frac{P_2}{P_1}\right)}^{0}$

so $\Delta b_{HT} = \Delta b = c_p(600 - 821.24) - c_p \ln\left(\frac{600}{821.24}\right)$

$= -127.7 \, kJ \, kg^{-1}$

c) i)



SFEE $\dot{q} - \dot{w}_x = \cancel{\Delta h}^{0}$ so $\dot{q} = \dot{w}_x$

$$\Delta s = \int \frac{d\dot{q}}{T} \Rightarrow \dot{q} = T\Delta s$$

$$\Delta s = -R\ln\left(\frac{p_2}{p_1}\right) \quad \text{so} \quad w_x = \dot{q} = 600 \times -287\ln\left(\frac{3}{1}\right)$$

$$= -189.2 \text{ kJ kg}^{-1}$$

2 methods:

1) $\Delta b_{HT} = \int_1^2 \left(1 - \frac{T_0}{T}\right) d\dot{q} = \dot{q}\left(1 - \frac{300}{600}\right) = \frac{\dot{q}}{2}$.

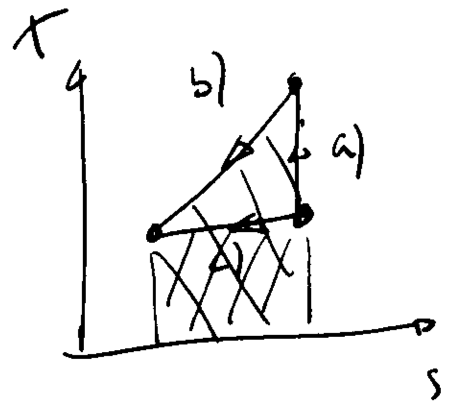
2) $\Delta b = \cancel{\Delta h}^{0} - T_0 \Delta s$

$$\text{so } \Delta b = -\dot{w}_x + [\Delta b_{HT}]$$

$$\text{so } \Delta b_{HT} = \Delta b + \dot{w}_x \qquad \text{etc.}$$

so $\Delta b_{HT} = -94 \text{ kJ kg}^{-1}$

d)



Lower temperature heat transfers in case c)

Difference is the shaft work saved through isothermal compression.

N.B. All processes are reversible

SECTION B

Q4

a) Streamlines are parallel so only pressure gradient in y direction is due to hyrdrostatics. Pressure on free surface is constant (atmospheric) so the pressure is independent of x. Parallel streamlines means y component of velocity is zero everywhere. By continuity, this means x component of velocity is independent of x.

b) Forces on a small element in the flow (recalling dp/dx=0):
$$(\tau + d\tau)\Delta x - \tau\Delta x + \rho g \sin\gamma\, \Delta x dy = 0$$
$$\frac{d\tau}{dy} = -\rho g\ \sin\gamma$$

c) Part (i):

$$\mu\frac{d^2v}{dy^2} = -\rho_0\left(1 - \frac{\alpha y}{t}\right) g \sin\gamma$$
$$\frac{dv}{dy} = \frac{-\rho_0 g \sin\gamma}{\mu}\left(y - \frac{\alpha y^2}{2t}\right) + A$$

BC at free surface is dv/dy=0 at y=t (no shear stress on surface), so:

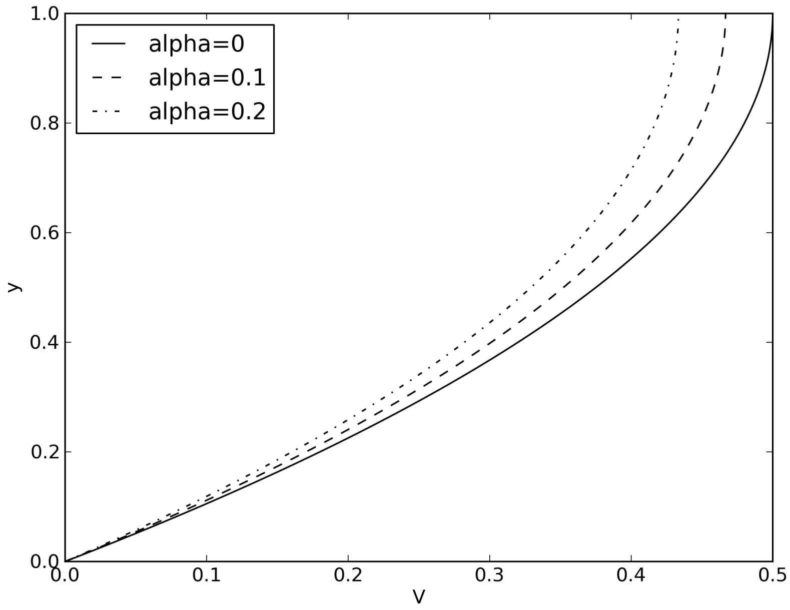
$$0 = \frac{-\rho_0 g \sin\gamma}{\mu}\left(t - \frac{\alpha t}{2}\right) + A$$
$$A = \frac{-\rho_0 g \sin\gamma}{\mu} t\left(1 - \frac{\alpha}{2}\right)$$

Flow rate per unit width is given by
$$Q_w = \int_0^t v dy$$
$$Q_w = \frac{\rho_0 g \sin\gamma}{\mu}\left[\frac{ty^2}{2} - \frac{\alpha y^2 t}{4} - \frac{y^3}{6} + \frac{\alpha y^4}{24t}\right]_0^t$$

$$Q_w = \frac{\rho_0 g\, \mathrm{t}^3 \sin\gamma}{\mu}\left(\frac{1}{3} - \frac{5\alpha}{24}\right)$$

Part (iii):

Curves must satisfy no-slip at the wall and no shear stress on the free surface. The wall shear stress must balance the component of the weight of the fluid down the slope. As alpha is increased, the density of the fluid is reducing and so is the weight, hence the shear stress at the wall reduces. Hence, dv/dy at the wall reduces as alpha increases. The flow rate per unit width also reduces as alpha increases.


alpha=0
alpha=0.1
alpha=0.2
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.1
0.2
0.3
0.4
0.5
V

Q5

a) Bends: Flow turning around bends means pressure is lower on the inside of the bend (curved streamlines). Upstream and downstream of the bend, the streamlines are parallel so the pressure is uniform. This means the flow on the inside of the bend accelerates going into the bend (pressure falling) and decelerates leaving the bend (pressure increasing). The opposite happens on the outside of the bend. The regions of deceleration (or adverse pressure gradient) will thicken boundary layers and may cause separation. This will cause a reduction in stagnation pressure due to mixing and viscosity.

   Diffuser: Flow is decelerating, pressure is increasing. Likely boundary layer separation.

   Contraction: Accelerating flow keeps boundary layers thin (and no separation) so viscous effects are small compared to the bends and diffuser.

b) Reduction in stagnation pressure is due to bends and diffuser:

$$\Delta p_0^{lost} = \frac{1}{2}\rho V_0^2 (0.5 + 4\times 0.5 + 4\times 0.5 + 0.5 + 0.5)$$

$$\Delta p_0^{lost} = \frac{1}{2}\rho V_0^2 \times 5.5 = \frac{11}{4}\rho V_0^2$$

c) Part (i):

$$V_2 = 0.9V_0 + 0.1\alpha V_0 = V_0(0.9 + 0.1\alpha)$$

[also $V_2^2 = V_0^2(0.81 + 0.18\alpha + 0.01\alpha^2\ )$]

Part (ii):

SFME from location 1 to location 2. Assume no forces on upper and lower sides of control volume:

$$p_1 - p_2 = \rho V_2^2 - 0.1\rho\alpha^2 V_0^2 - 0.9\rho V_0^2$$

Part (iii):

For mainstream: $p_{01} = p_1 + \frac{1}{2}\rho V_0^2$ and $p_{02} = p_2 + \frac{1}{2}\rho V_2^2$

$$p_{02} - p_{01} = (p_2 - p_1) + \frac{1}{2}\rho V_0^2(0.81 + 0.18\alpha + 0.01\alpha^2 - 1)$$

$$p_{02} - p_{01} = \frac{1}{2}\rho V_0^2(0.18\alpha^2 - 0.36\alpha + 0.18 - 0.19 + 0.18\alpha + 0.01\alpha^2)$$

$$p_{02} - p_{01} = \frac{1}{2}\rho V_0^2(0.19\alpha^2 - 0.18\alpha - 0.01)$$

If the stagnation pressure rise must equal that in part (b):

If the stagnation pressure rise equals the drop calculated in part (b):

$$5.5 = 0.19\alpha^2 - 0.18\alpha - 0.01$$
$$0 = 0.19\alpha^2 - 0.18\alpha - 5.51$$
$$\alpha = \frac{0.18 \pm \sqrt{0.18^2 + 4\times 0.19\times 5.51}}{0.38}$$

the positive root is α=5.88

Part (iv):
The injected jet could be positioned so as to add momentum into a boundary layer that might otherwise separate. For example, at the exit of bends on the inner wall, or in the diffuser.

Q6

a) $D_L = f(\rho, V, d, \mu)$ for incompressible flow

$$\frac{D_L}{0.5\rho V^2 d} = f\left(\frac{\rho V d}{\mu}\right)$$

i. e. drag coefficient is a function of Reynolds number only.
Reynolds number is the ratio of inertia forces (scale as ρV^2) to viscous forces (scale as $\mu V/d$). Drag coefficient is the ratio of the drag force (per unit length) to the drag that would be caused by full stagnation pressure acting uniformly over the frontal area.

b) Part (i):
At low Reynolds numbers, viscous forces dominate:

$$D_L \sim \frac{\mu V}{d} d = \mu V$$

so the drag coefficient scales as

$$C_D \sim \frac{\mu V}{\rho V^2 d} = \frac{1}{Re}$$

Part (ii):
Above a Reynolds number of about 10, the drag is dominated by form drag. The boundary layers are laminar and can stand very little adverse pressure gradient before separating. From the leading edge stagnation point, the surface pressure falls continuously until the 90 degree point. For a potential flow, the surface pressure would then rise again back to the stagnation value at 180 degrees. However, almost as soon as the pressure starts to rise, the laminar boundary layer separates (at about 90 degrees). The separation points remain approximately fixed (apart from the fluctuations caused by vortex shedding – part (c)) until the sudden drop in drag coefficient between Re=10^5 and 10^6. Here the boundary layers have become turbulent just before the laminar separation point. The turbulent boundary layer can withstand the adverse pressure gradient for longer so that separation occurs after 90 degrees – the wake is thinner and the drag is reduced. At higher Reynolds numbers, the drag rises again as transition moves further forward on the cylinder and the greater wall shear stress of the turbulent boundary layer (compared to the laminar one) increases the drag.

c) Part (i):
If the Reynolds number is the same for the real chimney and for the test, so will be the drag coefficient. We assume that the properties of air (density, viscosity) are the same in each case:

$$\left(\frac{\rho V d}{\mu}\right)_{real} = \left(\frac{\rho V d}{\mu}\right)_{model}$$

$$V_{real} d_{real} = V_{model} d_{model}$$

$$D_{L_{real}} = D_{L_{model}} \frac{V_{real}^2 d_{real}}{V_{model}^2 d_{model}} = 300 \times \frac{(7^2 \times 2)}{93.3^2 \times 0.15} = 22.5\ N/m$$

Part (iii):
Caused by vortex shedding. The wake is made to oscillate from side to side, this means the

$\frac{f}{V/d} = f(Re)$ by same dimensional analysis arguments as part (a), but with frequency as the dependent variable.

$$\left(\frac{f}{V/d}\right)_{real} = \left(\frac{f}{V/d}\right)_{model}$$

$$f_{real} = f_{model} \frac{V_{real}/d_{real}}{V_{model}/d_{model}} = 120 \times \frac{7/2}{93.3/0.15} = 0.675\ Hz$$