Engineering Tripos Part IB paper 5 – Electrical Engineering 2013

Solutions to paper

1.

(a). This is a class-A amplifier, i.e. the transistor is <u>always</u> on. Typically, ~ 0.7V base-emitter voltage is needed for Si to bias it into conduction.

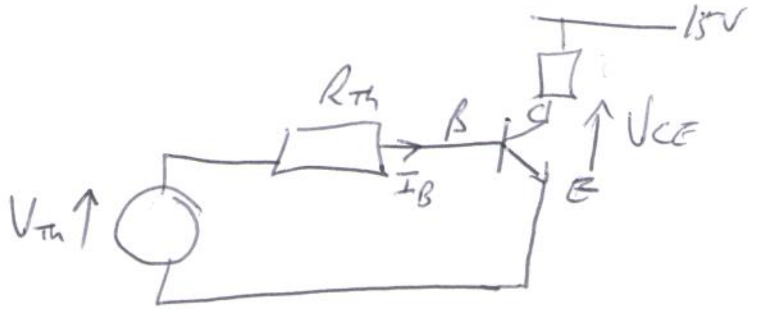
(b) Well, V_{CE} = 7.5V and V_{CC} = 15V => the voltage across resistor R_2 = 7.5V. This voltage is produced by the current I_c flowing through resistor R_2, i.e. I_cR_2 = 7.5V.

From this, we find that $R_2 = 7.5V/I_c = 7.5V/1mA =$ **7.5kΩ.**

The base current, $I_B = I_c/h_{FE}$ = 1mA/200 = **5μA**.

Examiner's note: A number of people thought h_{FE} was the ratio of I_E/I_B

To work out R_1, take the Thevenin equivalent of the input side to the circuit:



From which we find that $\frac{V_{TH}-V_B}{R_{TH}}$ = 5μA

Remember, $V_{TH} = 15\times\frac{10k\Omega}{10k\Omega+R_1}$ and $R_{TH} = \frac{10k\Omega\times R_1}{10k\Omega+R_1}$

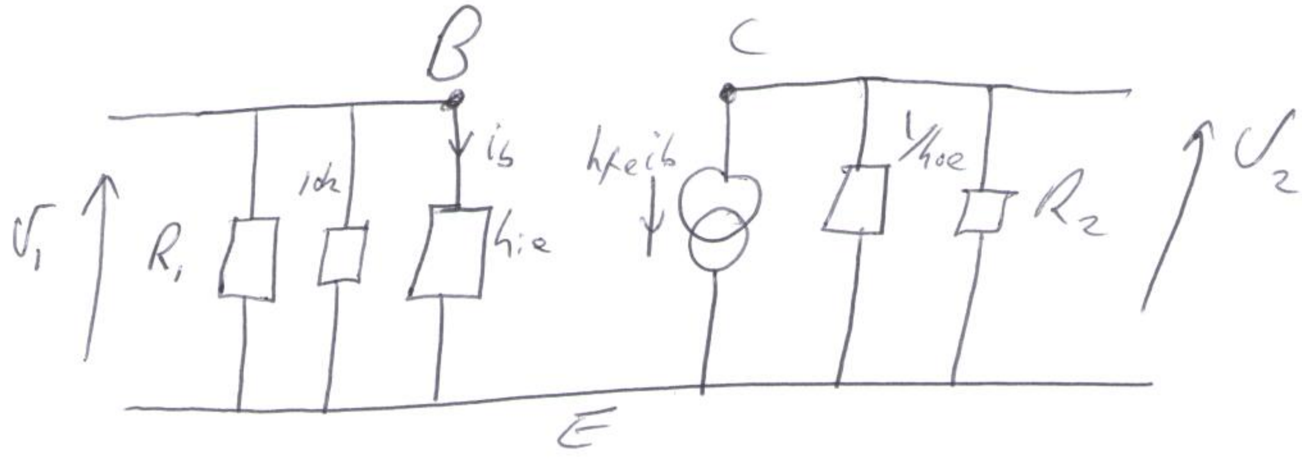
From which we find that

$$\frac{\frac{(15\times 10k\Omega) - V_B\times(10k\Omega + R_1)}{10k\Omega + R_1}}{\frac{10k\Omega\times R_1}{10k\Omega + R_1}} = 5\mu A$$

From which $\boldsymbol{R_1}$ **= 190.67 kΩ**

Examiner's note: Quite a few people treated these two resistors as being a potential divider, which is not valid as they do not carry the same current (due to the base current, I_B).

(c) The small-signal equivalent circuit is as follows:



$v_1 = i_b h_{ie}$

$v_2 = -h_{fe}i_b \times (R_2||1/hoe) = -\frac{h_{fe}i_bR_2}{1+h_{oe}R_2}$

The **gain** $= \frac{v_2}{v_1} = -\frac{\frac{h_{fe}i_bR_2}{1+h_{oe}R_2}}{i_bh_{ie}}$

$= -\frac{h_{fe}}{h_{ie}} \times \frac{R_2}{1+h_{oe}R_2}$ = **-126.3**

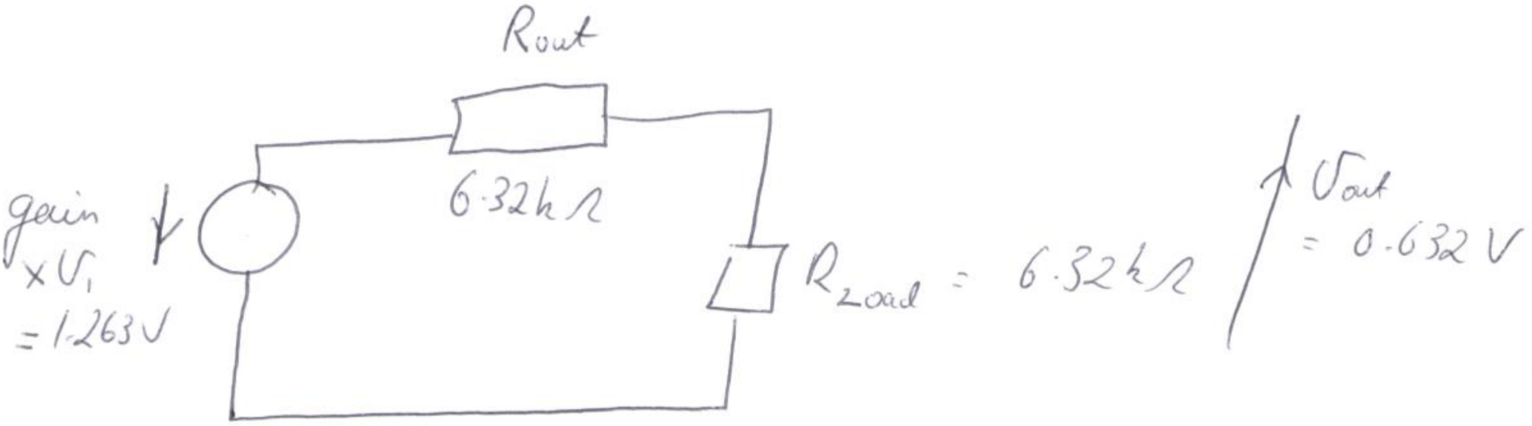
Examiner's note: Quite a few people left out the minus sign here

The output resistance is, by inspection, $R_2||1/hoe = \frac{R_2}{1+h_{oe}R_2}$ = **6.32kΩ.**

(d) By the maximum power transfer theorem, the load resistance should equal the output resistance of the amplifier to maximize the power output.

i.e. $\boldsymbol{R_{Load}}$ = **6.32kΩ**.

To work out the power dissipated in the load, first determine the voltage developed in the amplifier, i.e. :



Power (assuming we are working with peak amplitudes) $= \frac{V^2}{2R_{Load}} = \frac{\left(\left(\frac{1.263}{2}\right)^2\right)}{2\times6.32\times10^3} =$

32 μW

2.

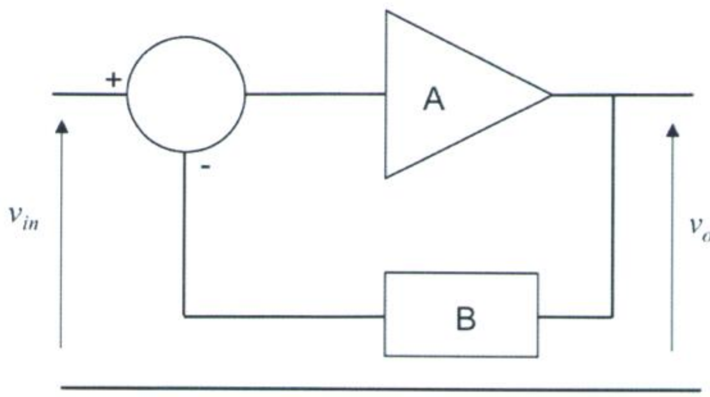
(a) Negative feedback – **Advantages**:

The open-loop gain *(A)* of op-amps is typically very large ($>10^5$), but can vary by several orders of magnitude from one op-amp to the next. This level of variation makes it impractical to construct circuits *without* negative feedback, as it would be impossible to reliably make circuits with predictable behavior. Negative feedback give a more stable and predictable gain, larger bandwidth (range of frequencies that are amplified), larger input resistance (which is good, as it means the op-amp circuit doesn't draw much current from and therefore interfere with the signal source), lower output resistance (also good, as this means very little of the output signal is lost across resistances *within* the op-amp).

Disadvantages:

The disadvantage is lower gain, which however can be overcome by using multiple amplification stages.

(b)



From the figure, $V_{out} = A(V_{in} - BV_{out})$

=> $V_{out} = AV_{in} - ABV_{out}$

=> $AV_{in} = V_{out}(1 + AB)$

=> $\frac{v_{out}}{v_{in}} = \frac{A}{1+AB} = \boldsymbol{gain, G}$

Remember, we have assumed the op-amp is ideal here (apart from having a finite gain)!

(c) Assuming that the result of part (b) holds true, and the op-amp is ideal (and therefore that no current is drawn through the op-amp's inverting terminal to the potential divider formed by R_1 & R_2), then:

$$G = \frac{A}{1+AB}, \qquad B = \frac{R_1}{R_1+R_2} = 0.2$$

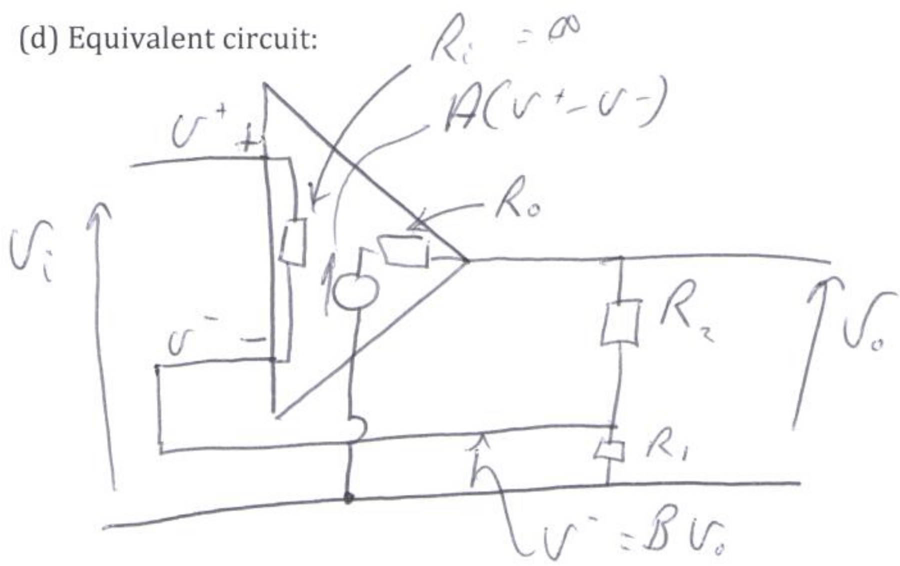
$$\Rightarrow G = \frac{10^5}{1+0.2\times10^5} = 4.99975$$

Now, $1 + AB$ is the factor by which the circuit properties (gain, bandwidth, input and output resistance) change relative to the op-amp's own characteristics, and has the value $1 + 0.2\text{x}10^5 = 20{,}001$.

- ⇨ R_{in} = 10MΩx20,001 = 200GΩ
- ⇨ R_{out} = 100MΩ/20,001 = 5mΩ
- ⇨ Bandwidth = 10kHzx20,001 =200MHz

The assumptions made are that the op-amp is ideal, apart from having a finite gain.

(d) Equivalent circuit:



Assume that no current flows through the op-amp.

⇨ by potential division,

$$v_0 = \frac{R_1 + R_2}{R_1 + R_2 + R_0} \times A(v^+ - v^-)$$

From the figure, $v^+ = v_{in}$ and $v^- = Bv_0 = \frac{R_1}{R_1+R_2} v_0$

Putting this together, we find that

$$v_0 = \frac{R_1 + R_2}{R_1 + R_2 + R_0} \times A(v_{in} - Bv_o)$$

$$=> v_0\left(1 + AB\frac{R_1+R_2}{R_1+R_2+R_0}\right) = Av_{in}\frac{R_1+R_2}{R_1+R_2+R_0}$$

The gain, $G = \frac{v_0}{v_{in}} = \frac{A}{1+AB\frac{R_1+R_2}{R_1+R_2+R_0}} \times \frac{R_1+R_2}{R_1+R_2+R_0}$

= 4.99975

Output resistance:

The procedure here is to short-circuit the input and apply a test voltage to the output terminals, while determining how much current flows into them. The ratio of the test voltage to the test current is the output resistance.

Summing currents at node x, $\frac{-ABv_{test}-v_{test}}{R_0} + i_{test} = \frac{v_{test}}{R_1 + R_2}$

$$\Rightarrow \quad v_{test}\left(\frac{AB+1}{R_0} + \frac{1}{R_1 + R_2}\right) = i_{test}$$

$$\Rightarrow \boldsymbol{R_{out}} = \frac{v_{test}}{i_{test}} = \frac{1}{\frac{1+AB}{R_0} + \frac{1}{R_1 + R_2}} = \mathbf{4.9999m\Omega}$$

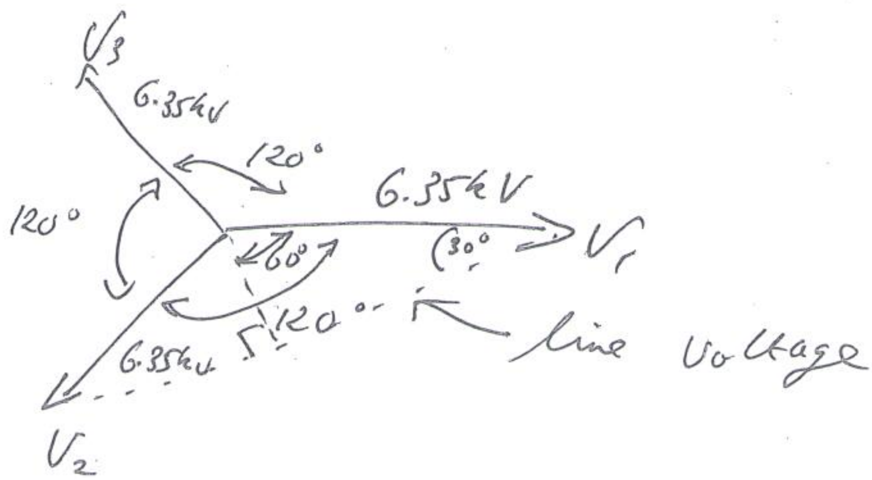
(e) If R_1 and R_2 are reduced by a factor of 100, the gain becomes 4.9997

i.e. not much of a change at all.

Examiner's note: Quite a few people did not think to use the formulas they derived earlier to answer this, and just made up some answer....

i) Electricity is generated as ac because it is easier to generate ac than dc. Three-phase is used as it is the most efficient compromise between efficiency and complexity - more phases would require more wires and more complex windings. Ac is easy to distribute as can be stepped up to high-voltage, low current for transmission then stepped back down for use.

(b)



From geometry, line voltage $= 2 \times 6.35kV \times \cos 30°$

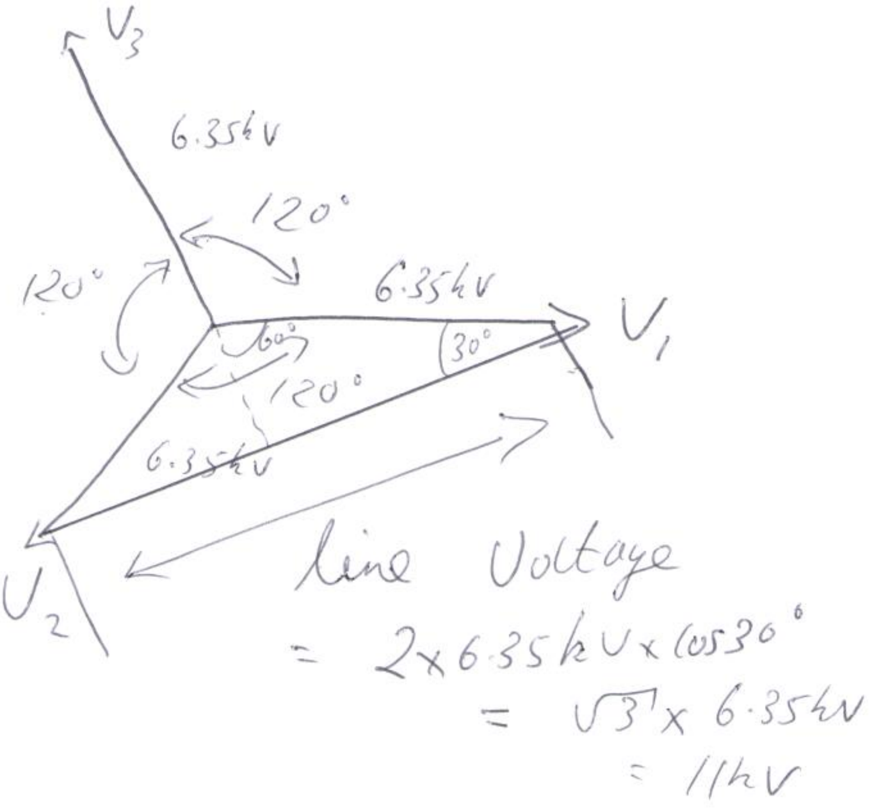
$= 6.35kV \times \sqrt{3}$

$= 11kV$

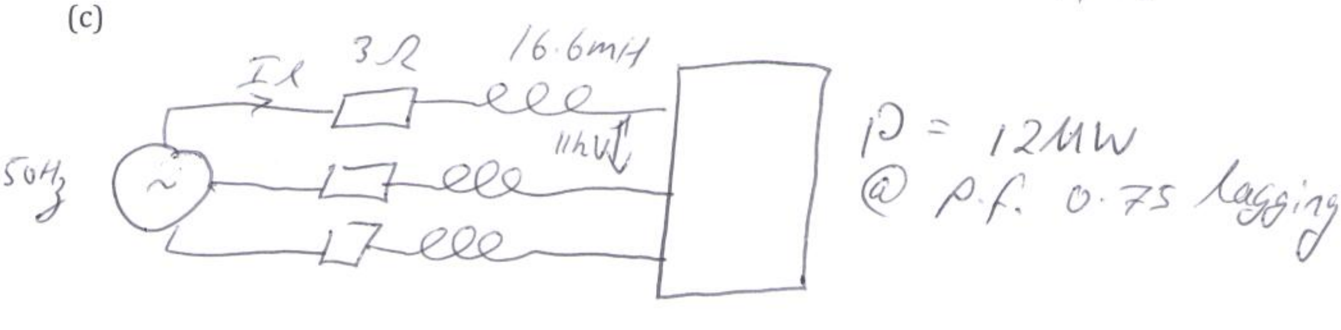
3.

(a) Electricity is generated as ac because it is easier to generate ac than dc – the machines are less complex. Three phase is used in generation as it is the best compromise between efficiency and complexity – more phases would require more wires and more complex windings. AC is used for distribution as the voltage can be stepped up to high values for transmission (power is conserved, so stepping up the voltage reduces the current, minimizing Ohmic losses in the feeder lines), and then stepped down again for use.

(b)



(c)



The power, $P = \sqrt{3}V_{line}I_{line}cos\emptyset$ => $I_{line} = \dfrac{P}{\sqrt{3}V_{line}cos\emptyset}$ = 839.78A

To work out the voltage from the generator, we can make use of the apparent power:

$$S = \sqrt{P_{total}^2 + Q_{total}^2}$$

Where $P_{total} = P_{load} + P_{line}$ and $Q_{total} = Q_{load} + Q_{line}$

$P_{line} = 3I_{line}^2 \times R_{line} = 6.35$ MW

⇨ $P_{total} = 18.35$ MW

As for Q , $Q_{load} = P_{load} \times \tan\phi = 10.58$ MVAR

$Q_{line} = 3I_{line}^2 \times X_{line}$ where $X_{line} = 15.92 \text{x} 10^{-3} \text{x} 2\pi \text{x} 50 = 5\Omega$

=> $Q_{line} = 10.57$ MVAR

⇨ $Q_{total} = 21.15$ MVAR

We can use these figures to work out

S = $\sqrt{(18.35\text{MW})^2 + (21.15\text{MVAR})^2} = 28$ MVA

Now, we also know that S = $\sqrt{3} V_{line} I_{line}$

=> $\boldsymbol{V_{line}} = \frac{28\text{MVA}}{839.78 \times \sqrt{3}}$ = **19.25 kV**

(d) Now add capacitors in parallel with lines at load end. The reactive power *consumed* by the load is 10.58 MVAR. The capacitors, which have a reactance of X_c Ohms, will *generate* reactive power to compensate partially for this. They experience the line voltage, so the three capacitors generate a total reactive power of

$$Q_{capacitors} = -3\frac{V_{line}^2}{X_c}$$

The reactance of the capacitors is $X_c = \frac{1}{2\pi \times 50 \times 25 \times 10^{-6}} = 127.3\ \Omega$

This means that $Q_{capacitors}$ = 2.85 MVAR => total Q for the system = (10.58 – 2.85) MVAR = 7.73 MVAR.

The new power factor is given by the relationship that $Q = P\tan\phi$ => $\tan\phi$ = 7.73/12 => $\cos\phi$ = power factor = 0.84.

To work out the new line current, we can use that

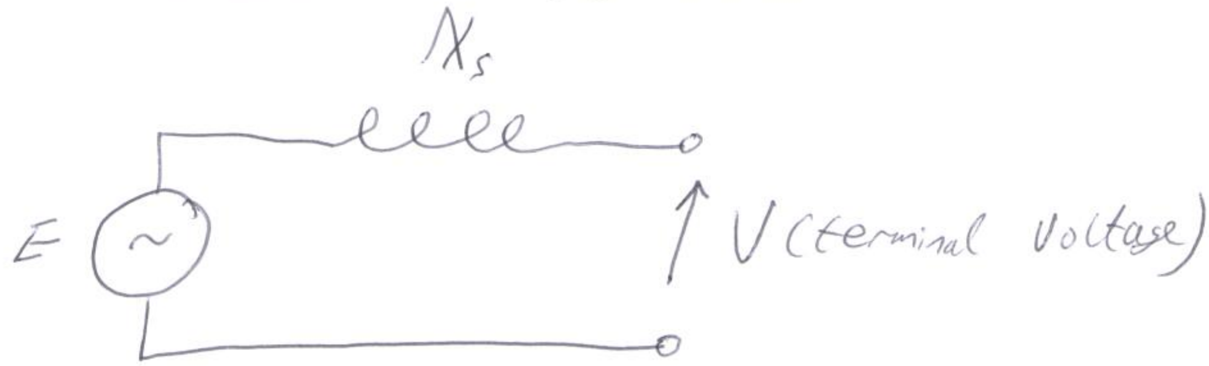
$$P = \sqrt{3}V_{line}I_{line}cos\emptyset \Rightarrow \boldsymbol{I_{line}} = \frac{P}{\sqrt{3}V_{line}cos\emptyset} = \frac{12\times10^6}{\sqrt{3}\times11\times10^3\times0.84} = \mathbf{749.8\ A}$$

The new power loss in the lines = 3x(749.8)^{2}x3 = 5.06 MW.

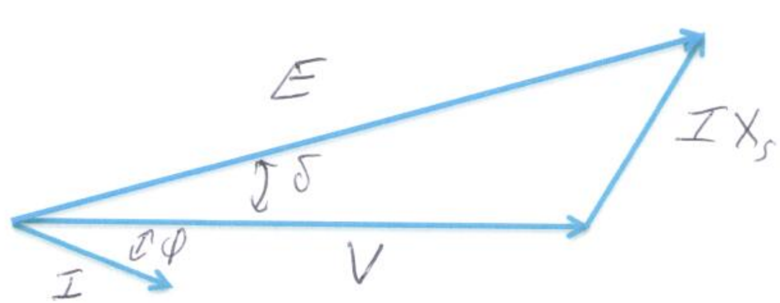
Before, the power loss in the lines was 6.35 MW, so the reduction due to the power factor correction was (6.35-5.06) MW = 1.29 MW, or 20.3%.

4.

(a) There is a rotor and a stator. The rotor has a winding on it, which is typically fed by a DC current, and therefore produces a magnetic field, B_{rotor}. The rotor is free to rotate inside the stator. The stator is surrounded by three-phase windings that are themselves fed with three phase currents. These produce a magnetic field, B_{stator} that has a constant magnitude, but is rotating at an angular frequency ω/p, where p is the number of pole-pairs of the stator magnetic field. These two magnetic fields interact and exert a torque on each other, the strength and direction of which is dictated by the magnitude of the fields, and the angle between, which we call the load angle, δ. The torque on the rotor is proportional to $B_{rotor}B_{stator}\sin\delta$, so if the rotor torque is changed for any reason, the load angle will also change. For a steady torque to be produced, the rotor must spin at the same speed as the stator field. As the rotor spins inside the stator, the stator coils experience an oscillating flux, which induces an emf, E in the stator coils. This can all be represented by the following equivalent circuit:



(b) Phasor diagram:



$p = 3 \Rightarrow \omega_s = 1000$ rpm

The power, $P = T\omega_s$, where T is the torque on the rotor.

$P = 500$ MW

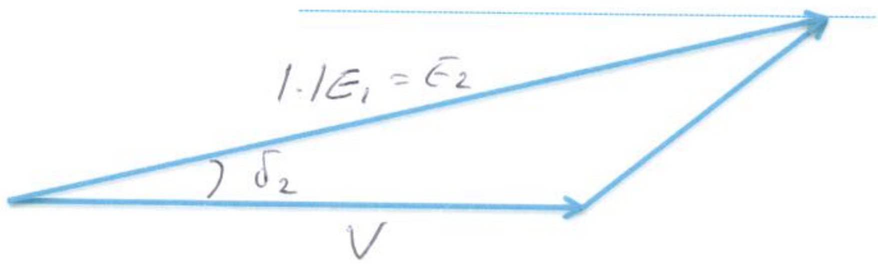
- ⇨ $T = P/\omega_s = \frac{500\times10^6}{2\pi\times1000} = 4.77$ MNm
- ⇨ Using the cosine rule, $E^2 = V^2 + (IX_s)^2 - 2VIX_S\cos(\pi/2 + \phi)$

Also, , $P = \sqrt{3}V_{line}I_{line}cos\emptyset \Rightarrow I_{line} = 9719.7$ A

Substituting into the cosine rule above, we find that E = 27.1 kV/phase = 46.9 kV/line.

From the phasor diagram, $E\sin\delta = IX_s\cos\phi \Rightarrow \delta = sin^{-1}\left(\frac{IX_s cos\phi}{E}\right)$=24.8°

(c) The power remains the same, and the excitation is increased by 10%.



As the power is constant, and it is proportional to the perpendicular height of the above triangle, the load angle must decrease. The perpendicular height is $E\sin\delta$.

$\Rightarrow E_1\sin\delta_1 = E_2\sin\delta_2$.

$\Rightarrow \sin\delta_2 = \frac{E_1}{E_2}sin\delta_1 = \frac{1}{1.1}\times0.419$

$\Rightarrow \delta_2 = 22.4^\circ$

The Maximum excitation can be found from the MVA rating. At 500 MW and 800 MVA,

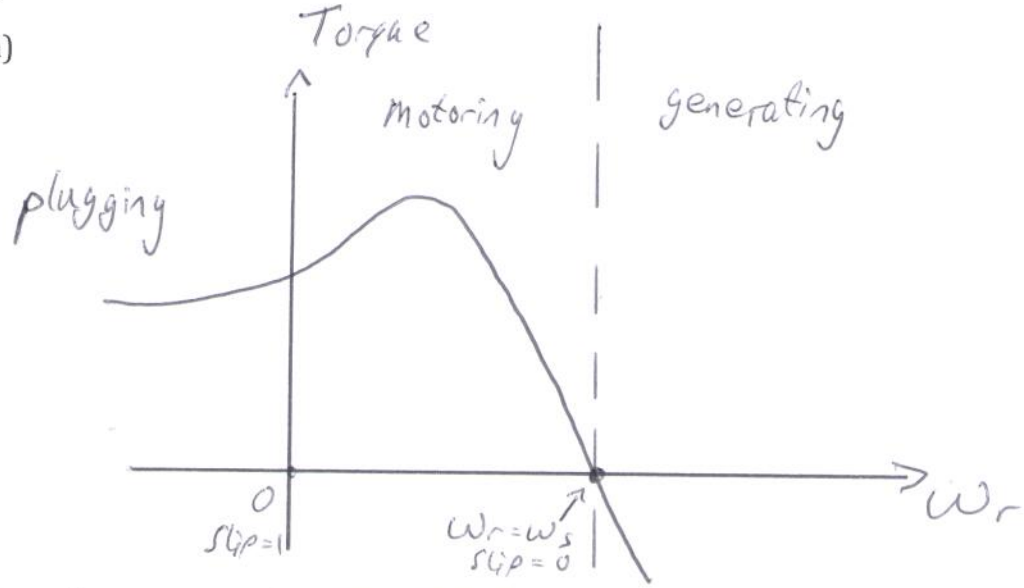
$P = S\cos\phi$, so $\cos\phi = -0.625 \Rightarrow I = \frac{P}{\sqrt{3}V_{line}cos\emptyset} = \frac{500\times10^6}{\sqrt{3}\times33\times10^3\times0.625} = 13996$ A

Then, using the cosine rule, we have that $E^2 = V^2 + (IX_s)^2 - 2VIX_S\cos(\phi)$

$\Rightarrow$ **E = 35 kV.**

5.

(a)

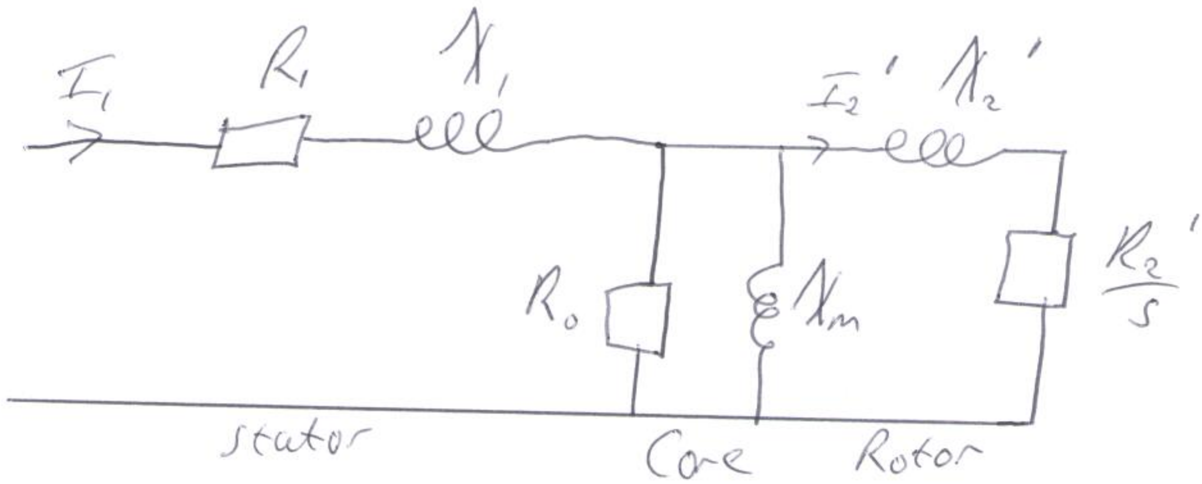


At synchronous speed, there is no relative motion between the stator field and the rotor, so the magnetic flux threading the rotor does not change => no rate of change of flux => no emfs induced in the rotor => no current flowing in rotor => no rotor magnetic field => no torque.

(b) 4-pole machine => synchronous speed = 60x50/2 = 1500 rpm.

=> slip = $\frac{\omega_s - \omega_r}{\omega_s} = \frac{1500-1350}{1500} = 0.1$

(c)



R_1 = Stator resistance

R_2' = Rotor resistance referred to the stator

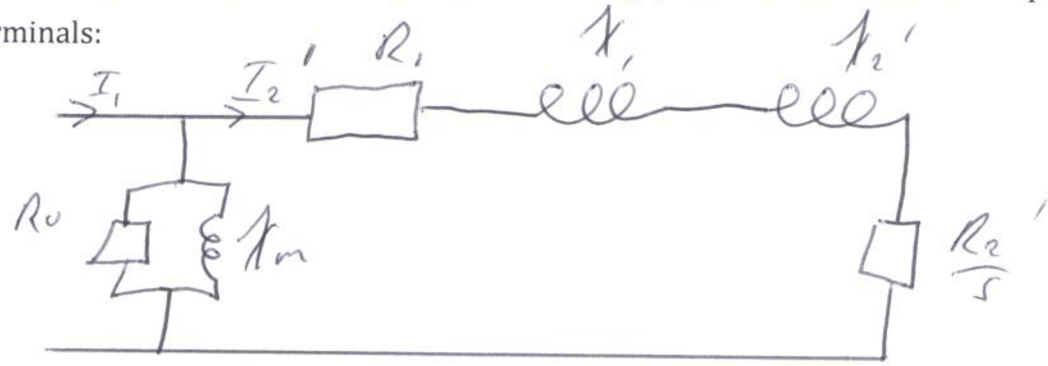
X_1 = Stator leakage reactance, to take account of flux not coupled to the rotor

X_2' = Rotor leakage reactance, referred to the stator.

X_m = Magnetising reactance

R_0 = Resistance of the magnetic core as seen by the eddy currents (the electric currents that flow as a result of the induced emfs)

As the magnetizing branch impedance is so large, we can move it to the input terminals:



From which we can say that $I_2' = \frac{V}{R_1 + \frac{R_2'}{s} + jX_1 + jX_2'} = \frac{600/\sqrt{3}}{2+30+j} = 10.83$ A

From the data book,

$T = \frac{3I{'}_2^2 R{'}_2}{\omega_S s} = 67.19$ Nm

Efficiency is defined as $\frac{power\ out}{power\ in}$

The power out = $T_{out} x \omega_r$

$T_{out} = T - T_{loss}$

Tloss = $\frac{P_{loss}}{\omega_r}$ = 2.8 Nm

⇨ T_{out} = (67.19 – 2.8) Nm = 64.39 Nm

⇨ $P_{out} = Toutx\omega_s x(1-s)$ = 9150.4 W

As for the power in, $P_{in} = 3VI_1\cos\phi$

Now, $I_1 = I_2' + I_m$

= 10.8 – 0.34j + 0.69 – 1.73j

= 11.49 -2.07j, which has a magnitude of 11.67 A

- ⇨ $\tan\phi = -\frac{2.07}{11.49} \Rightarrow \cos\phi = 0.98$
- ⇨ $P_{in} = 3\times\frac{600}{\sqrt{3}}\times 11.67\times 0.98 = 11.8857$ kW
- ⇨ ***Efficiency =*** $\frac{\mathbf{9150.4}}{\mathbf{11885.7}}\times\mathbf{100\%}$ ***= 77%***

6.

(a)

The characteristic impedance, Z_0 is the ratio E/H for an electromagnetic wave. For a wave in a transmission line, this is the same as V_F/I_F, or the ratio of voltage to current for a unidirectional wave (i.e. in the absence of any reflections). From the capacitance and inductance per unit length, we can also define $Z_0 = \sqrt{\frac{L}{C}}$.

For a wave in free space, there are no voltages or currents, but there are E and H fields, so the ratio E/H can be written in a more fundamental way as $\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} =$ 376.7 Ω.

The physical meaning of characteristic impedance is that it tells us something about how *easily* the wave propagates in a medium, and also what happens when it encounters a discontinuity between several media – we can use impedance to determine reflection coefficients.

Examiner's note: Very few people answered this part of the question well

(b)

$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{400\times10^{-9}}{400\times10^{-12}}} = 100\Omega.$

Phase velocity $= \frac{1}{\sqrt{LC}} = 2.5\text{x}10^8$ m/s

This is 5/6 of the speed of light, and is relatively standard for a transmission line. There is no upper limit on phase velocity, but there is on the *group velocity* (dω/dk) – which cannot be faster than the speed of light. *Nobody knew this.*

(c)

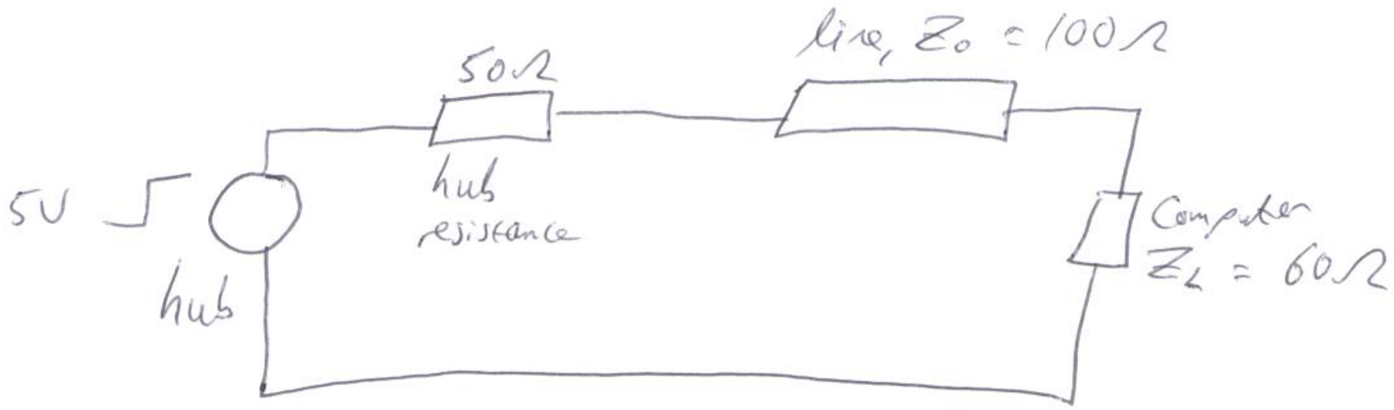
$c = f\lambda \Rightarrow \lambda = c/f$

= 2.5m

⇨ the wavelength is 2.5 m, meaning that the maximum length of cable before transmission line effects become important, which is roughly $\lambda/16$, is around 15.6 cm. The distance between computers and hubs is much larger than this, which means that when designing such systems, we must consider transmission line effects.

Examiner's note: Quite a few people just thought that this would be impractically short rather than just saying that as typical cables are longer than this, we must take TL effects into account.

(d) The time taken for a pulse to travel down the line from the hub to the computer is $\frac{10\ m}{2.5\times10^{8} m/s} = 40\ ns$



The first voltage pulse has an amplitude of 5Vx$\frac{100\Omega}{50\Omega+100\Omega} = 3\frac{1}{3}$ V

The reflection coefficient at the right hand side $= \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60-100}{60+100} = -0.25$

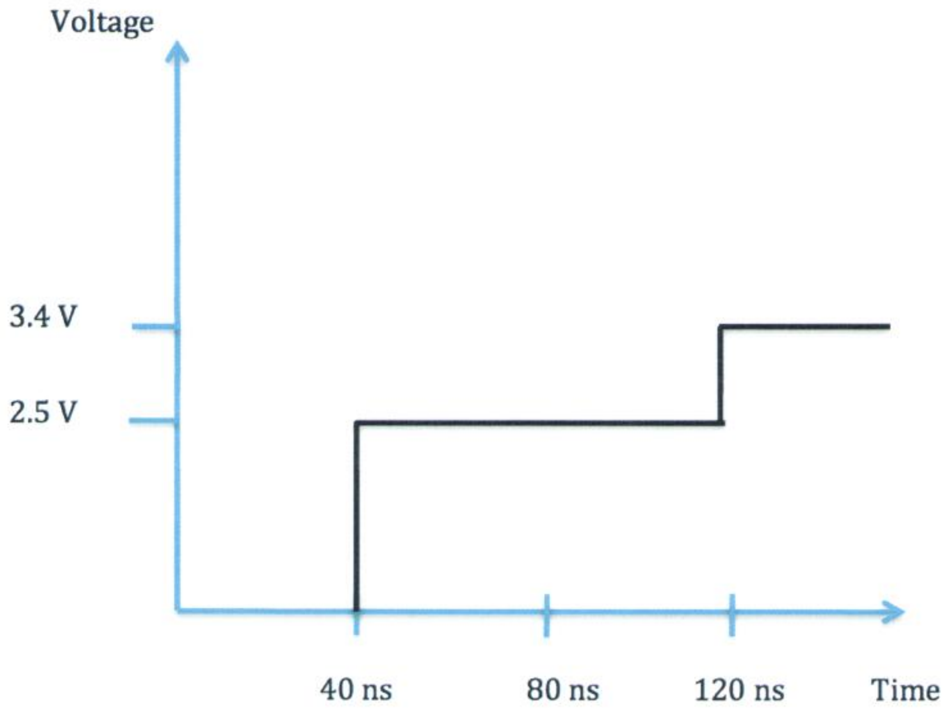
The reflection coefficient at the left hand side $= \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50-100}{50+100} = -1/3$

This means that $3\frac{1}{3}$ V propagates down the line from the hub towards the computer. There, ¼ gets reflected, so a pulse of 5/6 V travels back towards the hub. At the computer, there is interference between the incident and reflected wave, giving a net voltage of

$V = V_F + V_B = \frac{10}{3} - \frac{5}{6} = 2.5$ V.

The 5/6 V pulse that travels back towards the hub gets partially reflected, the reflected amplitude being 5/6 x 1/3 = 5/18, with a 180° phase shift. Remember that the original 5/6 V pulse was also with a 180° phase shift. This pulse reaches the computer, and 5/18 x ¼ = 5/72 V gets reflected, with a 180° phase shift. Therefore, after this reflection, the net voltage at the computer is 2.5 V +5/6 V - 5/72 V = 3.40 V.

Therefore, the plot of Voltage at the computer versus time looks like:



7.

(a)

The gain, G, of an antenna is a measure of its directionality. It is defined as the factor by which its maximum radiated intensity exceeds that of an isotropic antenna if they emit equal power from an equal distance.

Examiner's note: A number of people incorrectly said Gain is the amount by which a signal is amplified before being sent to an antenna.

(b)

(i) $\lambda = c/f = \frac{3\times10^{8} m\, s^{-1}}{1.2\times10^{9}\, s^{-1}} = 0.25$ m

(ii)

The power received $= \frac{P_{transmitted}}{4\pi r^2} \times G \times Area$

⇨ $P_{transmitted} = \frac{P_{received}}{G \times Area} \times 4\pi r^2 = 169.3$ W

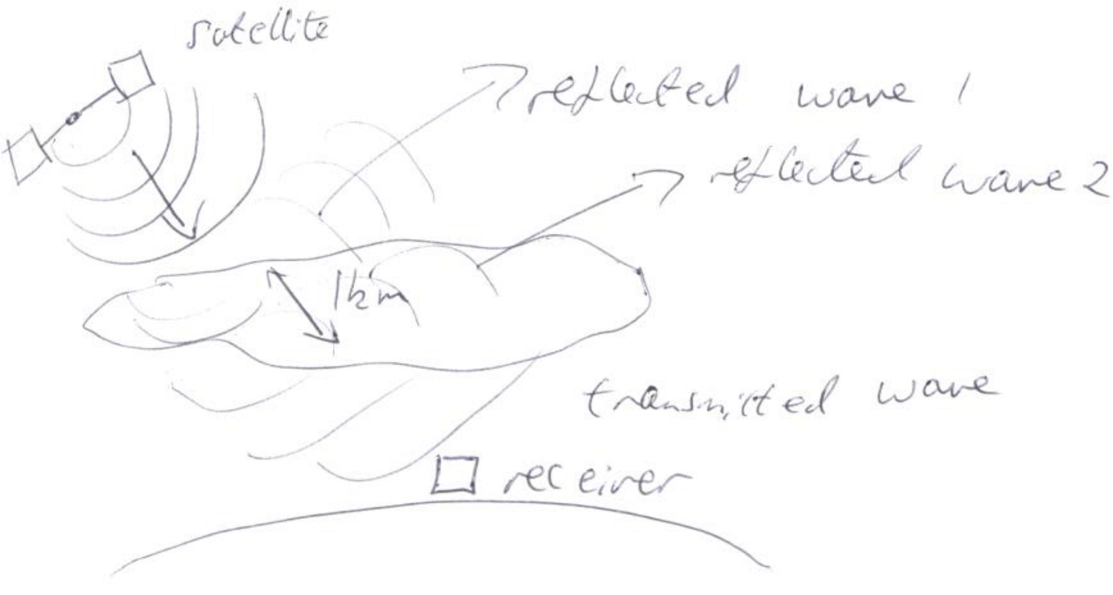
(iii) the voltage in the receiver:

$P_{received} = \frac{V^2}{R}$ ... assuming rms quantities.

⇨ $V = \sqrt{P_{received} \times R} = \sqrt{10^{-15}\, W \times 75\Omega} = 0.28\mu V$

(c)

Consider the power reflected:



Assumption: air and vacuum have the same characteristic impedance, Z_0.

The wave from the satellite will be reflected from each side of the cloud, and will be attenuated due to absorption within the cloud.

The reflection coefficient at each interface with the cloud has a magnitude

$\rho = \left|\frac{Z_0 - Z_C}{Z_0 + Z_C}\right|$

Now, Z_0 = 376.7 Ω, and $Z_C = \frac{Z_0}{\sqrt{\varepsilon_r}}$ = 307.6 Ω.

- ⇨ $|\rho|$ =0.1
- ⇨ Transmitted amplitude of EM wave = $P_{incident} \times (1-|\rho|^2)$
- ⇨ Amplitude reaching the bottom of the cloud, of thickness, d

  = $P_{incident} \times (1-|\rho|^2) \times e^{-\alpha d}$

  There will be a further reflection at the bottom of the cloud, giving a net transmitted amplitude of = $P_{incident} \times (1-|\rho|^2)^2 \times e^{-\alpha d}$

  The transmitted power is therefore $P_{incident} \times ((1-|\rho|^2)^2 \times e^{-\alpha d})^2$

= $P_{incident} \times (1-|\rho|^2)^4 \times e^{-2\alpha d}$

= $10^{-15} \times 0.99^4 \times e^{-2}$

= 1.3×10^{-16} W

Examiner's note: This question was well-answered, but a few people did not realize that as the amplitude of the wave decays as $e^{-\alpha d}$, the power will decay as $e^{-2\alpha d}$.