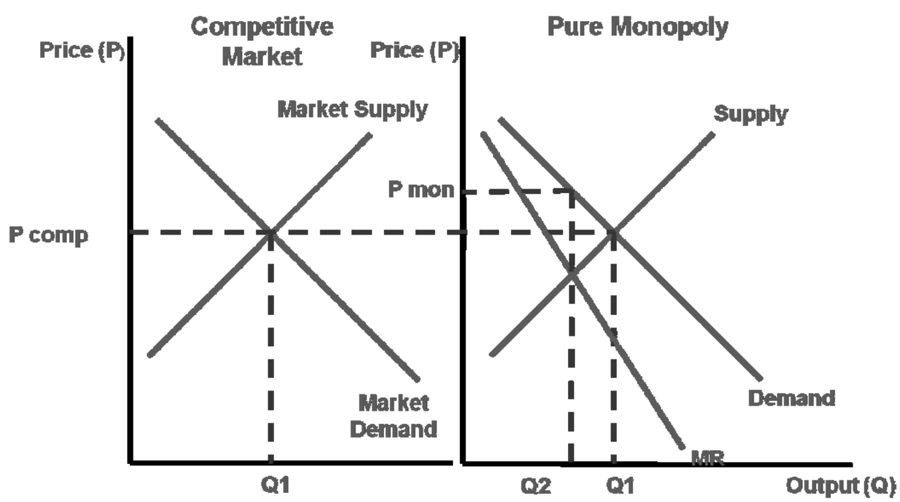
SECTION A *Introductory Business Economics B1 (Paper 8)*

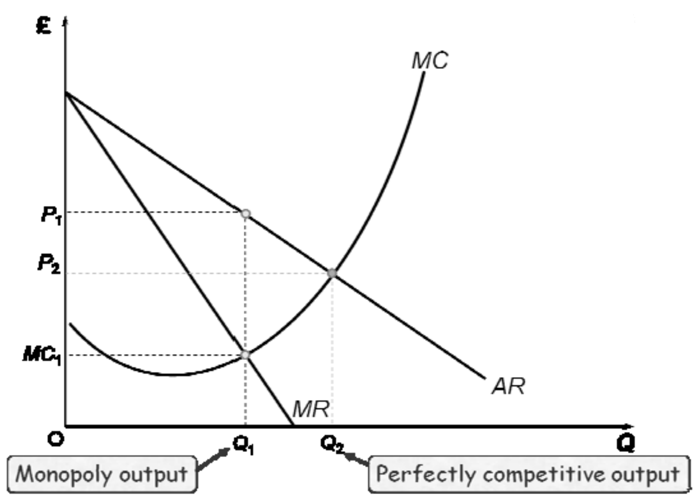
CRIBS

1 (a) Using an appropriate diagram or diagrams, explain how the profit maximising equilibrium levels of output and price differ in an industry which is a monopoly compared to one which is perfectly competitive. [5]

A monopoly is a form of market structure where a firm is the sole supplier of a certain product and can exert power over the setting of price or output. A perfectly competitive market is instead one with several firms too small to have any influence on the ruling market price (they are price takers). The basic model and relevant equilibrium levels are presented in the following diagrams.



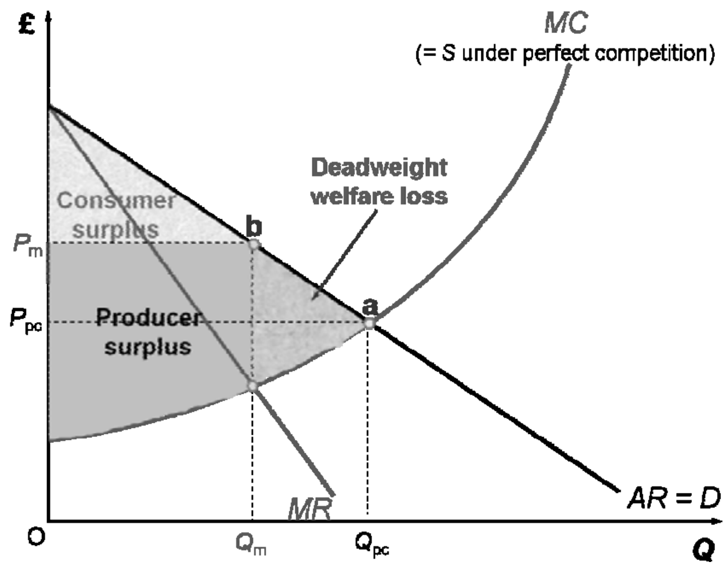
The following graph may be added to clarify the optimisation condition and output and price levels of monopoly and perfect competition.



(b) Why are consumers considered to be worse off when supplied by a monopoly industry rather than by a perfectly competitive industry? [5]

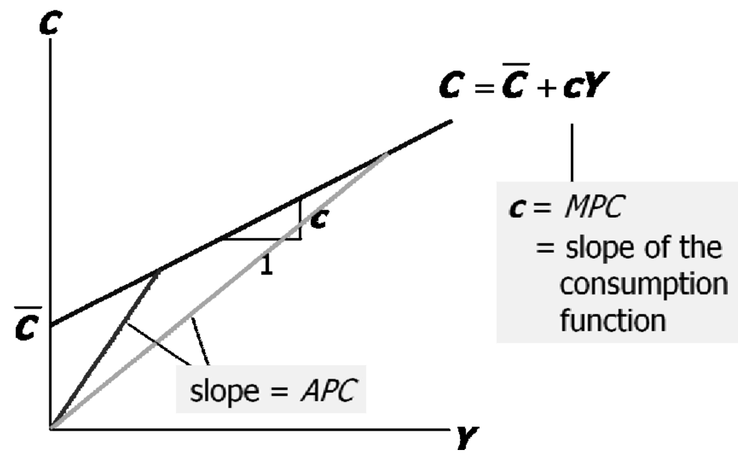
A monopolist can take the market demand curve as its own demand curve and can enjoy some power over the setting of price or output. A lower level of output is produced and consumed at a higher price compared to the position with perfect competition. Thus the standard case against a monopoly is that these businesses can earn abnormal profits at the expense of economic efficiency. The monopolist is extracting a price from consumers that is above the cost of resources used in making the product. Consumers' needs and wants are not being satisfied, as the product is being under-consumed.

The better students will be able to explain deadweight welfare loss under monopoly by means of the following diagram.



(c) In what ways does the life cycle model of consumption differ from the Keynesian consumption function model? [10]

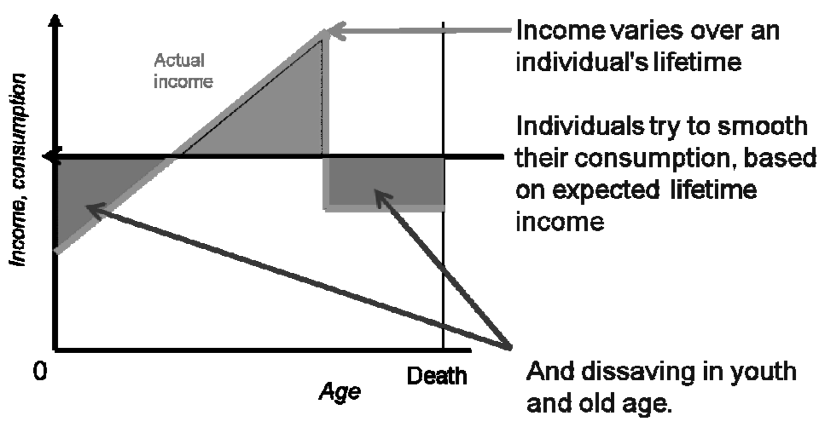
In the Keynesian consumption function current consumption (C) is determined by current personal disposable income (Y) and can be illustrated as follows:



As the diagram shows, as income rises the APC falls (consumers save a bigger fraction of their income). The implications of this model are that:

- The distribution of income will affect total consumption
- Economies may suffer from 'underconsumption' as they grow
- Governments can expand demand through fiscal policy

The life cycle model, developed by Franco Modigliani, is instead based on the notion that consumption is determined by long-run or normal income. It is illustrated in the following diagram:



The implications of this model are that:

- Expectations about future income may change spending patterns today
- Savings rates will be lower when expectation is that future income will be higher
- Savings can be negative
- Governments cannot easily expand demand through fiscal policy

- A temporary decrease in taxes will not increase long-run income and therefore will not lead to an increase in consumption.

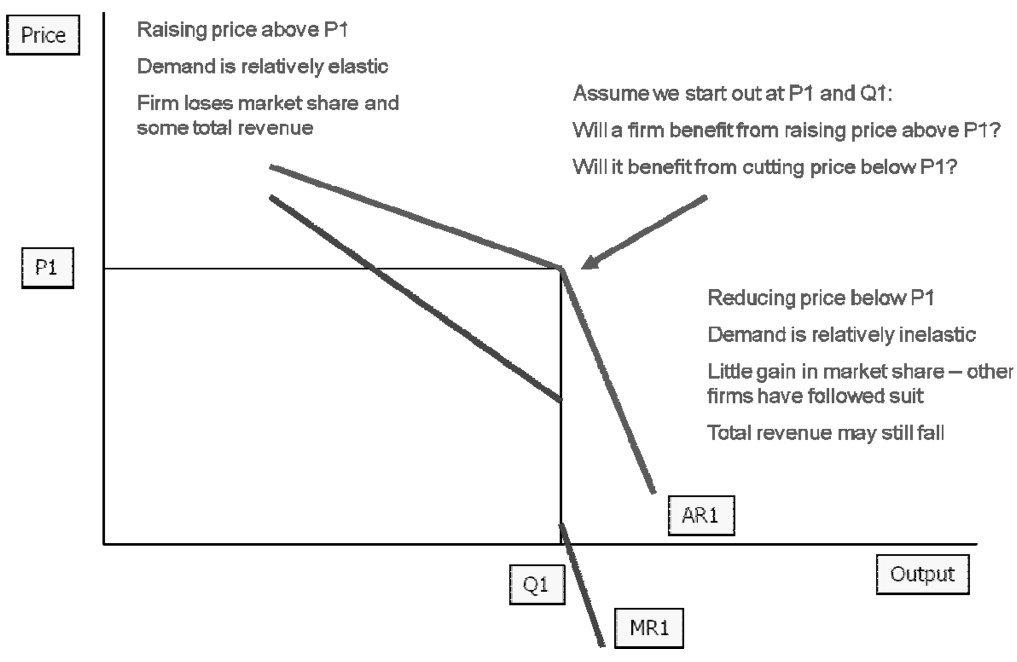
Better candidates may also comment of the limitations of this model (perfect foresight and absence of liquidity constraints).

2 **(a) Illustrate the Sweezy oligopoly model.** [5]

The Sweezy oligopoly model applies to a market where:

- A small number of firms sell similar but slightly differentiated products
- Firms believe that their competitors would follow them if they were to reduce their price below the prevailing market price, and that it is therefore not possible to gain much in the way of extra sales by reducing price (i.e. demand is relatively inelastic below the market price)
- Firms believe that their competitors would not follow them if they were to raise their price above the prevailing market price (i.e. demand is relatively elastic above the market price)
- Key result is price stability

The oligopolist's belief that rivals will match price cuts but not price increases produces a kink in the demand curve.



Better candidates may also derive the implications for profit maximisation (that there will be a range over which changes in marginal costs will have no effect on the optimal level of output).

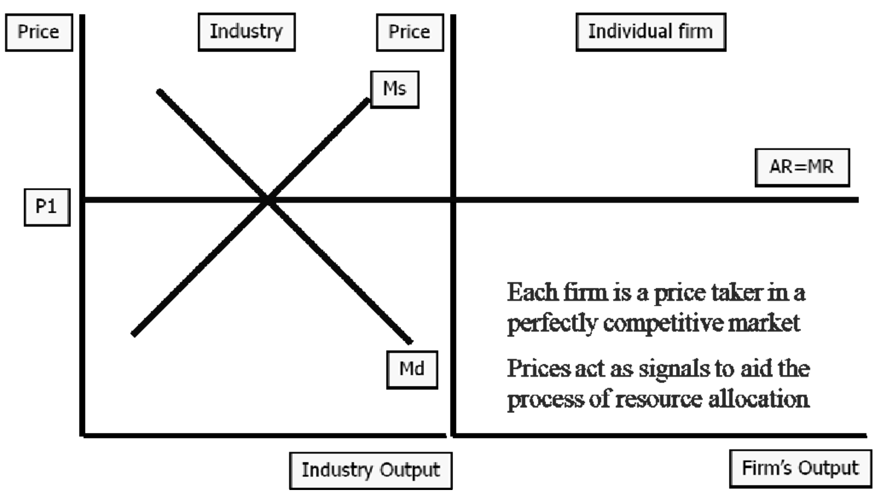
(b) What is a perfectly competitive market? [6]

A perfectly competitive market has the following characteristics:

- There are many suppliers each with an insignificant share of the market
- Each firm is too small to affect prices – each individual firm is assumed to be a price taker
- All firms produce identical output – homogeneous products that are perfect substitutes for each other
- Consumers have perfect information about the prices all sellers in the market charge
- All firms (industry participants and new entrants) have equal access to resources (i.e. technology, other factor inputs) and there is no strategic decision making
- No barriers to entry and exit of firms

Very few industries meet the strict criteria of a perfectly competitive market. Foreign exchange dealing, stock exchanges and agricultural markets are often considered close approximations. E-commerce can favour perfect competition (i.e. improved price transparency; better information on product availability and quality; large number of e-retailers; competition in the market for intermediate products) but also has limitations (i.e. technological barriers; scale economies; strategic behaviour in pricing; language and regulatory barriers). Although perfectly competitive markets are very rare in practice, the *model* of perfect competition provides a point of reference and theoretical benchmark against which we compare and contrast imperfectly competitive markets.

Better students should be able to include the diagram below.

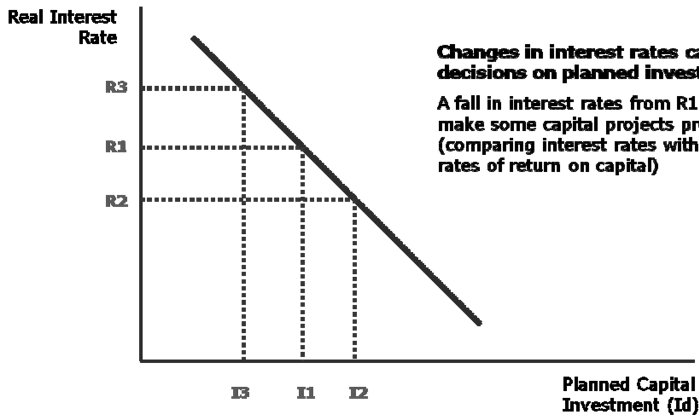


(c) What impact would the following have on the level of investment in the macroeconomy:

(i) a fall in the rate of interest [3]
(ii) reduced growth expectations [3]
(iii) a fall in business confidence? [3]

(i) Interest rates represent the opportunity cost of funds used to finance investment schemes. A fall in the rate of interest will increase the amount of autonomous investment in the macro economy. It decreases the cost of investment relative to the potential yield since planned investment projects on the margin become financially worthwhile.

Better students may include the diagram on the marginal efficiency of capital.



(ii) Reduced growth expectations reduce investment as this would decrease the expected utilisations of existing capital equipment and also decrease the profitability of new investment opportunities at given interest rates.

Better candidates may discuss the Accelerator model (total capital investment in an economy varies directly with the rate of change of output i.e. investment is largely income-induced. Given technological conditions, given relative prices of capital and labour and given size of capital stock needed to produce a given level of output, if the level of output changes, then the desired size of capital stock will also change and net capital investment is the amount by which the required capital stock changes. It follows that the amount of investment depends on the size of the change in output and that when the rate of growth of demand is weak, the size of the capital stock needs to be decreased, depressing demand for capital goods).

(iii) Investment projects involve a degree of risk:

- Revenue streams are uncertain (particularly in industries and markets that are sensitive to cyclical and exchange rate fluctuations)
- Costs are subject to change over time
- There is no guarantee that a project will yield the expected (or required) rate of return.

Changes in business confidence can be induced by changes in subjective expectations (e.g. of future demand, costs, taxation etc.) or external shocks (e.g. 9/11) and have a significant impact on planned capital spending projects. A drop in business confidence can lead to delays in new capital investments being given the go ahead or cancellations of entire projects. Confidence crises may follow bubbles. An important point is that the subjective and possibly irrational views of economic agents matter. They may not be determined by standard economic fundamentals and can lead to self-fulfilling outcomes.

Section B Crib

Q3. a) Stability ratio *N* is given by:

$$N = \frac{\gamma z - \sigma}{s_u}$$

Where γ is the unit weight of the soil, z is the depth of the tunnel axis, σ is the face support pressure and s_u is the undrained shear strength of the soil. If the value of N is greater than 5, the tunnel face will be unstable.

b) In soft clay, $N=10.7 > 5$. The tunnel face is unstable, so closed face tunnelling is essential. An EPB or slurry shield machine would be used with segmental linings bolted together to support the tunnel

In stiff clay, $N = 2.1 < 5$. The tunnel is stable unsupported, so open face techniques could be used with a sprayed concrete lining. Closed face tunnelling might still be used for practical reasons if the tunnel moved between soft and stiff clay frequently.

c) Sand has two features that make tunnelling difficult:

No undrained strength: Sands mobilise strength through friction, so at an open tunnel face where there is no confining stress, the soil has no strength and is unstable, so face support must be provided

High permeability: Water moves rapidly through sands leading to problems with infiltration. Pumping must be used or techniques such as ground freezing to limit water ingress.

d) Tunneling causes ground movements at the surface which can cause distortion of buildings as they follow the ground movements.

Masonry buildings are brittle and have low tensile strength and are hence particularly vulnerable to this damage. In the sagging region directly above the tunnel, the tension is taken by the foundation slabs, which are typically RC and hence have tensile strength. In the hogging regions the building will crack at the top. This is hence the most vulnerable region.

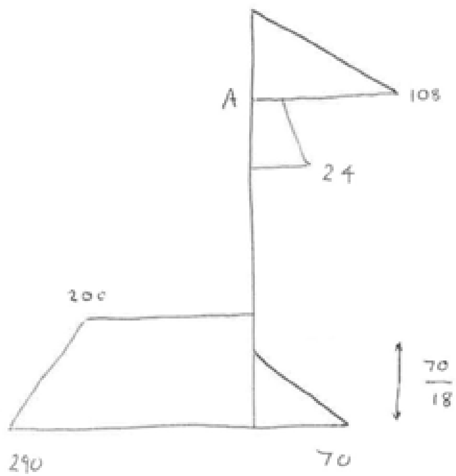
Damage can be minimised by:

Reinforcing buildings using ties
Compensation grouting triggered by instrumentation if settlements become excessive
Jacking of structures to compensate settlements

e) Inclinometers & extensometers measure soil movements
Strain gauges measure building strains
Precise levelling measures settlements

These data can then be used to trigger remedial actions such as compensation grouting.

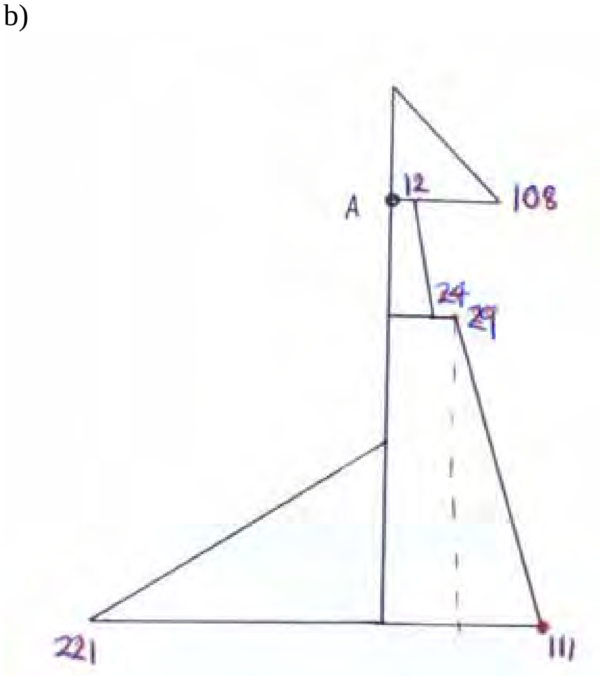
Q4.a)



Taking moments about the prop,
Driving moments = 1633 kNm, Resisting moments = 13121 kNm
Factor of Safety = 8

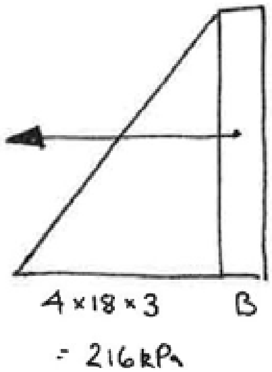
N.B. Very few candidates realised the change from active to passive loading at the prop in the granular surface layer.

b)



Taking moments about the prop,
Driving moments = 6643 kNm, Resisting moments = 6345 kNm
Factor of Safety = 0.96

c)



taking moments about base: Resisting = 576 kNm, driving = 800 kNm

Wall is not adequate, even though it can sustain lateral equilibrium it will rotate and fail.

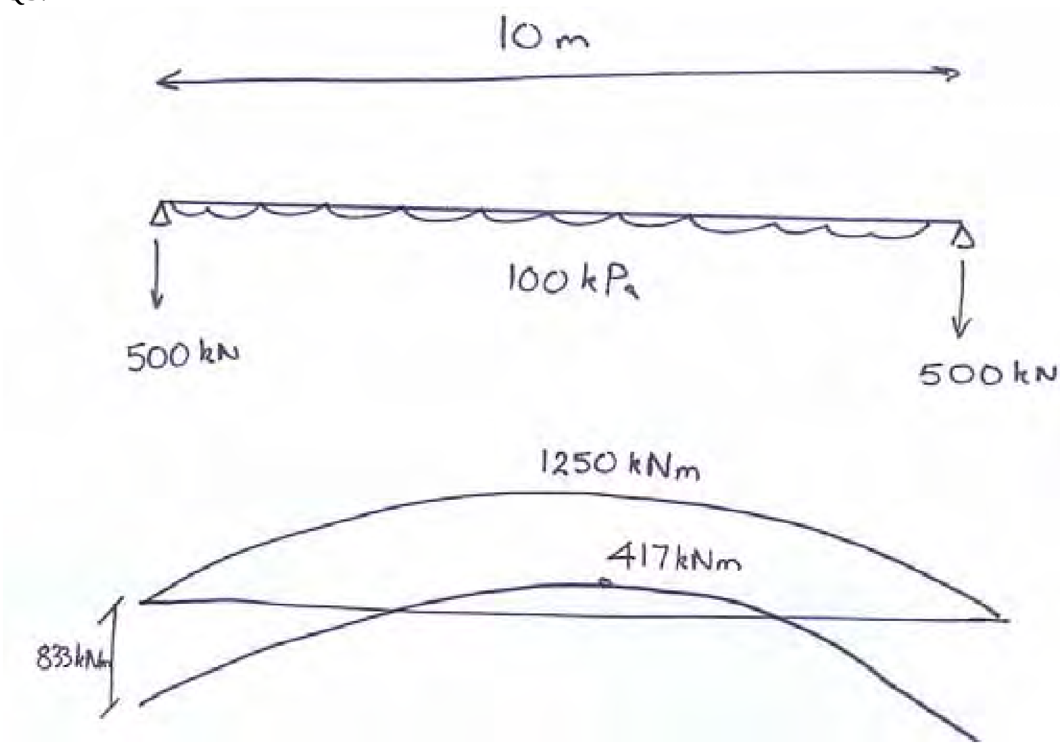
d)
M= 0.87 fy As d (1 – 0.5x)

x= 2.175 (fy/fcu) As/(bd)

Try x = ½
As = 5164 mm^2/m
x=0.43
As = 4935 mm^2/m

e.g. 25 mm dia bars at 100 mm centres or similar.

Q5.



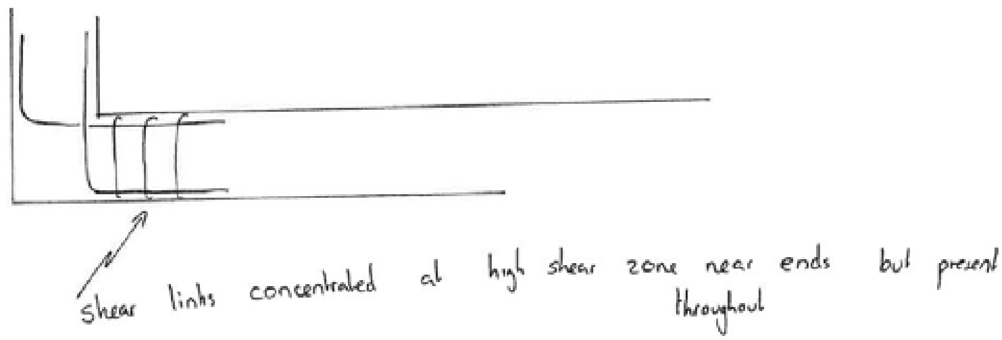
@A Mmax = -833 kNm
@C Mmax = 1250 kNm

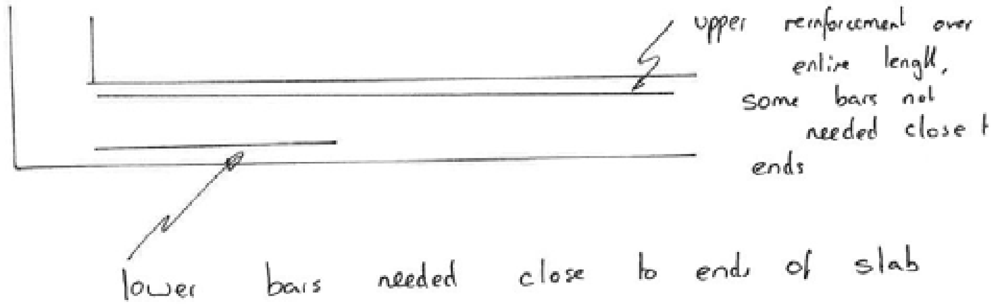
b) $M = 0.15\ f_{cu}\ bd^2$
d > 527 mm

c) @ C
x=0.5 => As = 5206 mm^2/m
x= 0.217 => As = 4380 mm^2/m
e.g. 32 dia bars @ 175 centres

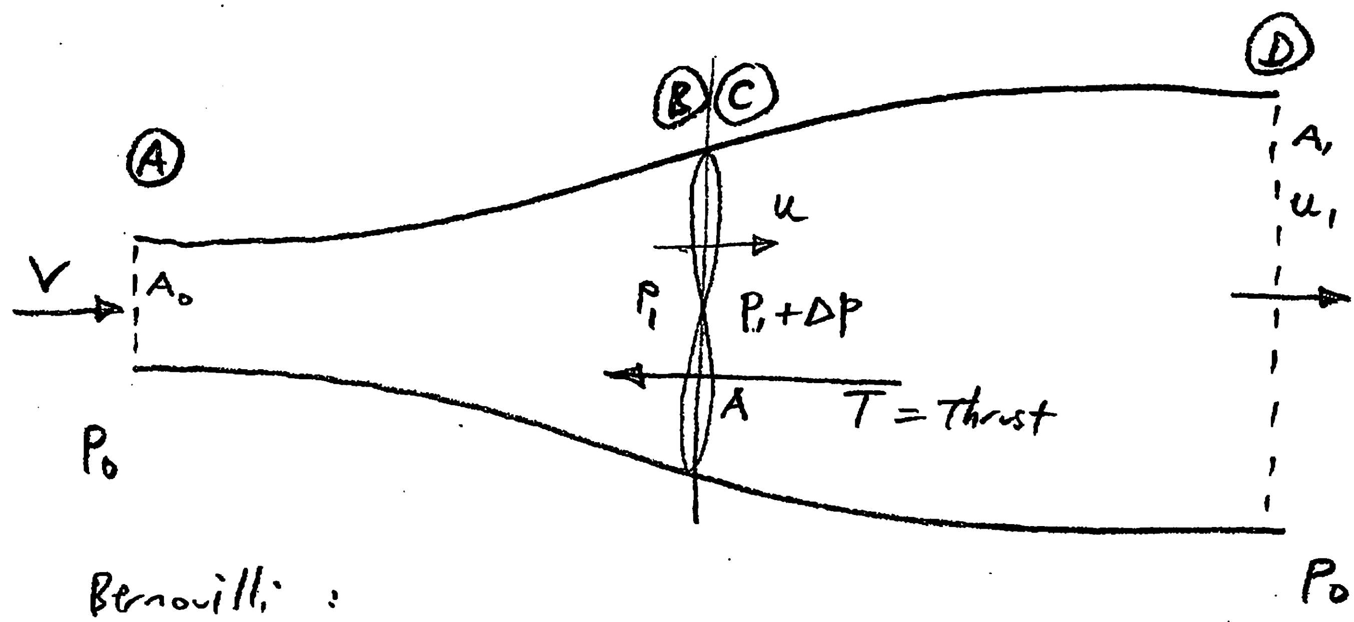
@ A
x=0.5 => As = 3469 mm^2/m
x= 0.145 => As = 2804 mm^2/m
e.g. 20 dia bars @ 100 centres

d)


shear links concentrated at high shear zone near ends but present throughout


upper reinforcement over entire length, some bars not needed close t ends
lower bars needed close to ends of slab

6 (a) Control volume for air passing through a turbine of swept area A



Bernoulli :

(A) → (B) $$P_0 + \tfrac{1}{2}\rho V^2 = P_1 + \tfrac{1}{2}\rho u^2$$

(C) → (D) $$P_0 + \tfrac{1}{2}\rho u_1^2 = P_1 + \Delta P + \tfrac{1}{2}\rho u^2$$

Subtract ∴ $$\Delta P = \tfrac{1}{2}\rho\left(u_1^2 - V^2\right)$$

Thrust $$T = -\Delta P\, A = \tfrac{1}{2}\rho A\left(V^2 - u_1^2\right)$$

Momentum flux through C.V.

$$\dot m V - \dot m u_1 = T$$

$$\therefore \quad \dot m\,(V - u_1) = \tfrac{1}{2}\rho A\,(V - u_1)(V + u_1)$$

$$\therefore \quad \dot m = \tfrac{1}{2}\rho A\,(V + u_1)$$

but $\dot m = \rho A u$ $\quad \therefore \quad u = \tfrac{1}{2}\,(V + u_1)$

introduce induction factor a defined by

$$u = V(1-a)$$

$$\therefore \quad V(1-a) = \tfrac{1}{2}(V + u_1)$$

$$\therefore \quad u_1 = 2V - 2aV - V = V(1-2a)$$

$$\text{Power} = T \cdot u \qquad \text{(force} \times \text{velocity)}$$

$$= \tfrac{1}{2}\rho A\left(V^2 - u_1^2\right)u$$

$$= \tfrac{1}{2}\rho A V^3\left(1-(1-2a)^2\right)(1-a)$$

$$= \tfrac{1}{2}\rho A V^3\left(1-1+4a-4a^2\right)(1-a)$$

$$= \tfrac{1}{2}\rho A V^3 \; 4a(1-a)^2$$

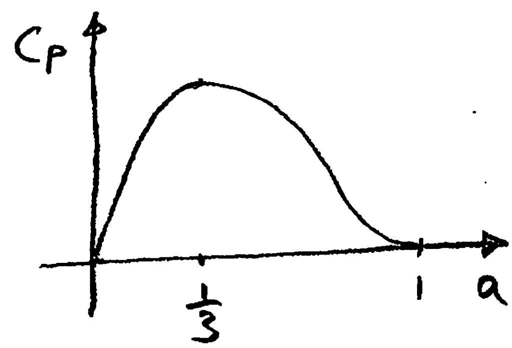
Power in the flow across an area A = P_{ref}

$$P_{ref} = \tfrac{1}{2}\dot{m}V^2 \qquad \text{rate of flow of K.E.}$$

$$= \tfrac{1}{2}\rho A V^3 \qquad \text{in free flow}$$

Power coefficient $\quad C_p = \dfrac{P}{P_{ref}}$

$$C_p = 4a(1-a)^2$$



This is maximum when

$$\frac{1}{4}\frac{dC_p}{da} = 0$$

$$\therefore \quad (1-a)^2 - 2a(1-a) = 0$$

$$\therefore \quad (1-a)(1-a-2a) = 0$$

$$\therefore \quad (1-a)(1-3a) = 0$$

$a=1$ is a minimum

$a=\frac{1}{3}$ is a maximum

$$C_{P(a=\frac{1}{3})} = 4 \cdot \frac{1}{3} \cdot \left(1-\frac{1}{3}\right)^2 = \frac{16}{27} = 0.59$$

This is the Betz Limit.

[For full marks an answer need to have:
- a good sketch showing the C.V.
- clear application of Bernoulli, momentum and power flow
- correct use of induction factor
- correct differentiation to obtain $a = \frac{1}{3}$]

(b) Thrust for $a = \frac{1}{3}$

$$T = \frac{P}{u} = \frac{\frac{1}{2}\rho A V^3 \, 4a(1-a)^2}{V(1-a)}$$

$$= \frac{1}{2}\rho A V^2 \, 4a(1-a)$$

$$= \frac{1}{2}\rho A V^2 \, 4 \, \frac{1}{3}\left(1-\frac{1}{3}\right)$$

$$= \frac{4}{9}\rho A V^2$$

(Or, if (a) was omitted use momentum flow:

$$\text{Thrust} = \dot{m}\left(V - \frac{V}{3}\right) \qquad \left(\frac{V}{3} \text{ given}\right)$$

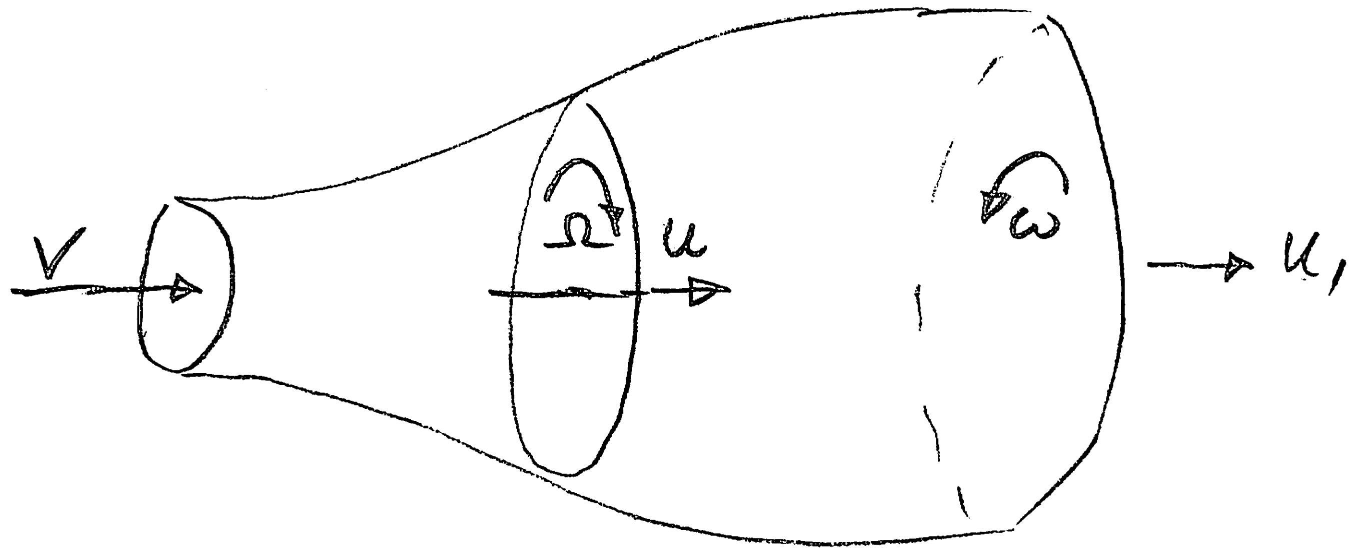
$$= \rho A u \frac{2}{3} V$$

$$u = V(1-a) = \frac{2}{3}V$$

$$\therefore \text{Thrust} = \frac{4}{9}\rho A V^2$$)

(c) Axial induction factor $a = 1 - \frac{u}{V}$

Angular induction factor $a' = \frac{\omega}{2\Omega}$ where ω is the angular velocity of the wake and Ω is the angular velocity of the blades

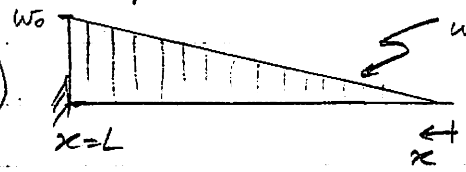


[Note Ω & ω are in opposite directions – worth a bonus!]

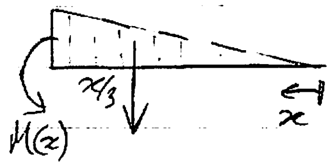
It is clear that there is rotational energy in the wake so this energy is not being extracted. Also the angular momentum of the wake affects the available torque.

1B Paper 8, 2013

7a) i)



$w(x) = \frac{w_0 x}{L}$, $w_0 = \frac{2W}{L}$ (total load W)



$$M = \frac{wx}{2} \cdot \frac{x}{3} = \frac{2Wx}{L^2} \cdot \frac{x}{2} \cdot \frac{x}{3} = \frac{Wx^3}{3L^2}$$

$$= \frac{WL}{3}\left(\frac{x}{L}\right)^3$$ [3]

ii) Skin thickness $\ll d$ $\therefore$ $I(x) = 2 \times \frac{A}{2} \cdot d^2 = \frac{A_0 x}{L}\left(\frac{d_0 x}{L}\right)^2 = A_0 d_0^2 \left(\frac{x}{L}\right)^3$

$$\sigma_{max} = \frac{Md}{I} = \frac{WL}{3}\left(\frac{x}{L}\right)^3 \cdot \frac{d_0 x}{L} \cdot \frac{1}{A_0 d_0^2 (x/L)^3} = \frac{Wx}{3A_0 d_0} = \frac{WL}{3A_0 d_0}\left(\frac{x}{L}\right)$$

Maximum at $x = L \Rightarrow \sigma_{max} = \frac{WL}{3A_0 d_0}$ [3]

iii) $m = \int_0^L \rho A \, dx = \rho \int_0^L \frac{A_0 x}{L} \, dx = \frac{\rho A_0 L}{2}$

But $A_0 = \frac{WL}{3\sigma_{max} d_0}$ where $\sigma_{max} = \sigma_f$ at failure

$\therefore$ $m_\sigma = \frac{\rho L}{2} \cdot \frac{WL}{3\sigma_f d_0} = \frac{\rho W L^2}{6\sigma_f d_0}$ [3]

7b) i) $\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{WL}{3}\left(\frac{x}{L}\right)^3 \cdot \frac{1}{E} \cdot \frac{1}{A_0 d_0^2 (x/L)^3} = \frac{WL}{3EA_0 d_0^2}$ i.e. $\frac{d^2y}{dx^2} = C$

$\Rightarrow \frac{dy}{dx} = Cx + D$ but $\left.\frac{dy}{dx}\right)_{x=L} = 0$ $\therefore$ $\frac{dy}{dx} = C(x - L)$

$\Rightarrow y = C\left(\frac{x^2}{2} - Lx\right) + E$ but $y(L) = 0$ $\therefore$ $y = C\left(\frac{x^2}{2} - Lx + \frac{L^2}{2}\right)$

$\delta = y(0) = \frac{CL^2}{2} = \frac{WL}{3EA_0 d_0^2} \cdot \frac{L^2}{2} = \frac{WL^3}{6EA_0 d_0^2}$ [5]

ii) $A_0 = \frac{WL^3}{6E\delta d_0^2}$ $\therefore$ $m_\delta = \frac{\rho L}{2} \cdot \frac{WL^3}{6E\delta d_0^2} = \frac{\rho W L^4}{12\delta E d_0^2}$

If design is stiffness limited, $M_\delta > M_\sigma$ or $\frac{M_\delta}{M_\sigma} > 1$

$$\Rightarrow \frac{\rho W L^4}{128 E d_o^2} \cdot \frac{6 \sigma_F d_o}{\rho W L^2} > 1 \quad \text{i.e.} \quad \frac{\sigma_F L^2}{2\delta E d_o} \quad \text{or} \quad \frac{\sigma_F}{2E(\delta/L)(d_o/L)} > 1$$

[3]

For GFRP, $\sigma_F/E = \frac{56 \times 10^6}{28 \times 10^9} = 0.002$; For bamboo, $\frac{\sigma_F}{E} = \frac{36 \times 10^6}{18 \times 10^9} = 0.002$

So for same design (δ/L and d_o/L), both materials are stiffness limited. The design choice depends on other factors, such as:

> Bamboo has lower density, leading to overall weight-saving in blades, which may reduce the size of the turbine tower required and/or reduce transportation costs

> Bamboo has lower embodied carbon (energy), reducing impact of blades on life-cycle energy.

> Bamboo may have shorter lifetime / maintenance costs.

Q8. (a) From $P=0.5C_p\rho Av^3$ and substituting $P=2.5\times10^6$, $\rho=1.23$ kgm^{-3}, $C_p=0.38$ and $v=12$ ms^{-1} gives the swept area A as 6191 m^2 = $\pi d^2/4$ from which d=88.8 m.

Using $\lambda=\omega R/v$ and with $\lambda=8$, R=d/2=44.4 m and v=12 gives $\omega=2.16$ rads^{-1}.

(b) To operate at constant optimum tip-speed ratio the angular speed of the turbine must be kept proportional to the wind speed. Thus, if the turbine rotates at 2.16 rads^{-1} when v=12 ms^{-1}, at the most probable wind speed of 6 ms^{-1} it will rotate at half that speed, so 1.08 rads^{-1}. A 12 pole induction generator (so p=6) connected directly to the f=50 Hz grid has a speed which is very close to its synchronous speed given by $2\pi f/p$ giving 52.4 rads-1. Therefore the required gearbox ratio is 52.4/1.08=48.5.

Typical gearbox ratios for a single stage are 3 – 4, and so 3 stages will most likely be needed. Alternatively, use a generator with more poles to bring the gearbox ratio down from its rather high value.

Assuming lossless operation the turbine torque will be given by P/ω. At the wind speed of 6 ms^{-1}, and since the power is proportional to wind speed cubed, $P=2.5/2^3=0.3125$ MW = 312.5 kW and with $\omega=1.08$ rads^{-1} the torque is 312.5/1.08=289 kNm. The generator torque will be reduced by the gearbox ratio and is therefore 289/48.5=5.96 kNm.

(c) To simplify the calculation, recognize that the induction generator will be operating on the steep part of the torque-speed characteristic such that the slip s is very small and negative. Thus, R_2'/s dominates the induction generator impedance and so R_1, X_1 and X_2 may all be ignored. Thus, by Ohm's law:

$I_2'=V/(R_2'/s)$

Substituting this into the databook equation for the torque of an induction machine:

$T=3I_2'^2R_2'/(s\omega s)$ gives the simplified torque-slip equation:

$T=3V^2s/(\omega_sR_2')$

The generator is star-connected and so V=6.6 kV/√3=3.81 kV, ω_s has been calculated in part (b) as 52.4 rads^{-1}, $R_2'=1\ \Omega$ and T=-5960 Nm from part (b). Thus slip s can be found as -0.00717. The generator input current is approximately $Vs/R_2'=27.3$ A. A more exact (but not required) answer can be found by including R_1, X_1 and X_2' in this calculation.

(d) Variable-speed operation of wind turbines means that the tip-speed ratio can be kept at its optimum value by keeping ω/v constant. This in turns maximizes the power coefficient C_p at all wind speeds up to rated, meaning that the wind turbine extracts the maximum possible energy from the wind.

DFIGs use a power electronic controlled slip-frequency voltage supply connected via slip-rings to the rotor in order to mimic the method of speed control of adding extra rotor resistance, but without the ensuing power loss. Other benefits of this technique are that the slip-frequency converter only has to be fractionally-rated, and that the phase of the slip-frequency voltages can be controlled so that the induction generator contributes to the demand for reactive power.

D

9 ~~10.~~ (a) Stage loading: $\psi = \frac{\Delta h_0}{U^2} = \frac{c_p \Delta T_0}{U^2}$

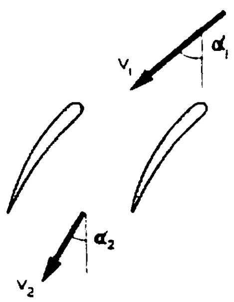
Flow coefficient: $\phi = \frac{V_x}{U}$

The favorable pressure gradient in turbines means a large pressure drop (hence a large stagnation enthalpy drop) can be achieved in a single stage. In compressors, a large ψ would result in boundary layer separation due to the adverse pressure gradient. If ψ is too high in a turbine, the efficiency will be compromised, but this may be offset by the weight savings of fewer stages. If ψ is too high in a compressor then the compressor will stall or surge and the engine will not function – much more serious! [5]

(b) Compressors: flow turning is always towards the axial direction such that there is an increase in the flow area and thus a reduction of flow velocity and increase in pressure (flow diffusion). Turning limited to ~30 degrees to avoid B/L separation.

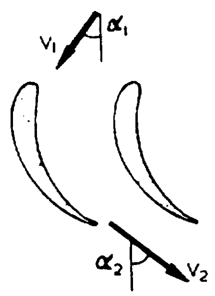
Turbines: flow turning is towards the tangential direction so that (usually) there is a reduction in flow area, acceleration of the flow and a drop in pressure. [Impulse (rotor) blades have equal (relative) flow angles at inlet an exit and all the pressure drop is therefore across the stators.] Flow turning in a single blade row may be 90 degrees or more.

Compressor blade row



Area increase: pressure rise

Turbine blade row



Area decrease: pressure drop

[5]

(c) Blade speed: $U = \Omega r = \frac{6000}{60} \times 2\pi = 282.7\,\text{m/s}$

For a single stage: $\Delta T_{0.\max} = \frac{\psi U^2}{c_p} = 0.40 \times 282.7^2 / 1005 = 31.8\,K$

For compressor: $\Delta T_0 = \frac{T_{01}}{\eta_c}\left[\left(\frac{P_{02}}{P_{01}}\right)^{(\gamma-1)/\gamma} - 1\right] = 297.8\,\text{K}$

∴ No. Stages $= 297.8 / 31.8 \quad = 9.36$ [10 stages] [5]

(d) Flow area $= 2\pi r l_{out} = 2\pi r l_{in} \times \frac{\rho_{in}}{\rho_{out}}$

$\therefore$ $$l_{out} = l_{in} \times \frac{P_{01}}{P_{02}} \times \frac{T_{02}}{T_{01}} = 15 \times \frac{1}{10} \times \frac{585.6}{288} = \underline{\underline{3.05\,\text{cm}}}$$ [3]

(e) For turbine: $$U = \sqrt{\frac{\Delta h_0}{\psi}} = \sqrt{\frac{1005 \times 297.8}{2.0}} = 386.8\,\text{m/s}$$

$\therefore$ $$r = \frac{U}{\Omega} = \frac{386.8}{2\pi(6000/60)} = \underline{\underline{0.615\,\text{m}}}$$ [2]

۱Q (a) Stagnation pressure and temperature are the pressure and temperature attained by bringing a flow to rest adiabatically and reversibly without work transfer and at constant altitude. (Note that, strictly, the reversible condition is not required in the definition of stagnation temperature for ideal gases.)

$$T_0 = T(1+\frac{\gamma-1}{2}M^2) = 222.85(1+0.2\times0.85^2) = \underline{\underline{255.05\,K}}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)} = 0.260\left(\frac{255.05}{222.85}\right)^{3.5} = \underline{\underline{0.417\,\text{bar}}}$$ [4]

(b)

$$\text{Dimensionless mass flow} = \frac{\dot{m}_a \times (\text{characteristic velocity})}{A_N P_{02}}$$

$$= \frac{\dot{m}_a\sqrt{c_p T_{02}}}{A_N P_{02}}$$

This must be a function of the engine condition, which is most obviously represented by

$$\frac{T_{04}}{T_{02}}$$ [4]

[More formal Dim. Anal. Method is also acceptable.]

(c) At test: $\tilde{F} = \frac{F_G + P_a A_N}{P_{02} A_N} = \frac{2.5\times10^5 + 1.013\times10^5\times2.5}{1.013\times10^5\times2.5} = 1.987$

$\therefore$ In flight: $F_G = \tilde{F}\times P_{02}A_N - P_a A_N = 1.987\times0.4.17\times10^5\times2.5 - 0.260\times10^5\times2.5$
$= 142.1\,\text{kN}$

To get the net thrust we need the mass flow rate and flight velocity:

$$\dot{m}_a = \dot{m}_{a,test} \times \frac{P_{02}}{P_{02,test}} \times \sqrt{\frac{T_{02,test}}{T_{02}}}$$

$$= 800 \times \frac{0.417}{1.013} \times \sqrt{\frac{288.15}{255.01}} = 350\,\text{kg/s}$$

$$V = M\sqrt{\gamma RT} = 0.85 \times \sqrt{1.4 \times 287 \times 222.85} = 254.4\,\text{m/s}$$

Net thrust:

$$F_N = F_G - \dot{m}_a V$$

$$= 142.1 \times 10^3 - 350 \times 254.4 = \underline{\underline{53.1\,\text{kN}}}$$

[7]

(d) Mass flow rate of fuel causes temperature rise of "air". Hence

$$\dot{m}_f LCV = \dot{m}_a (T_{04} - T_{03}) = \dot{m}_a T_{02} \left(\frac{T_{04}}{T_{02}} - \frac{T_{03}}{T_{02}} \right)$$

The bracketed terms are a function of the fixed engine condition.

$$\therefore \quad SFC = \frac{\dot{m}_f}{F_N} = SFC_{test} \times \frac{\dot{m}_a}{\dot{m}_{a,test}} \times \frac{T_{02}}{T_{02,test}} \times \frac{F_{N,test}}{F_N}$$

$$= 0.3 \times \frac{350}{800} \times \frac{255.05}{288.15} \times \frac{250}{53.1} = \underline{\underline{0.547\,\text{kg / hr / kg}}}$$

[5]

11. (a) $\eta_{th} = \frac{\Delta \dot{KE}}{\dot{m}_f LCV}$ $\eta_p = \frac{F_N \times V}{\Delta \dot{KE}}$

$$SFC = \frac{\dot{m}_f}{F_N} = \frac{\dot{m}_f \times V}{\eta_p \Delta \dot{KE}} = \frac{V}{\eta_p \eta_{th} LCV}$$ [4]

(b) Neglect mass flow rate of fuel:

$$\eta_p = \frac{F_N V}{\Delta \dot{KE}} = \frac{\dot{m}_a (V_j - V) V}{\dot{m}_a 0.5 (V_j^2 - V^2)} = \frac{2V}{V + V_j}$$

$$V = M\sqrt{\gamma RT} = 0.85\sqrt{1.4 \times 287 \times 226.7} = 256.5 \text{ m/s}$$

∴ $$\eta_p = \frac{2 \times 256.5}{256.5 + 400} = \underline{\underline{78.15\%}}$$ [4]

(c) Increasing thrust at fixed altitude would require higher T_{04} and would shorten engine life. Increased thrust is achieved at lower altitude due to higher air density. [1]

(d) Keep fixed C_L in order to maintain optimum (*L/D*) – note there will be a slight variation of (*L/D*) due to Mach number effects, but this is offset by drag on dead engine etc.

$$C_L = \frac{L}{0.5 \rho V^2 A}$$ *L/A* = Weight /*A* is fixed

∴ $$V' = V\sqrt{\frac{\rho}{\rho'}} = 256.5 \times \sqrt{\frac{0.287}{0.4} \times \frac{241.5}{226.5}} = \underline{224.3 \text{ m/s}}$$ [3]

(e) Assume (*L/D*) unchanged

$$\frac{s'}{s} = \frac{V'}{V} \times \frac{SFC}{SFC'} = \frac{V'}{V} \times \frac{V}{V'} \times \frac{\eta'_p \eta'_{th} LCV'}{\eta_p \eta_{th} LCV} = \frac{\eta'_p}{\eta_p}$$

To get the new propulsive efficiency requires the new jet velocity. We know, however, that, since (*L/D*) is assumed the same, the thrust from each engine must increase by 4/3.

∴ $$\dot{m}'_a (V'_j - V') = \frac{4}{3} \dot{m}_a (V_j - V)$$

$$\therefore \quad V_j' = V' + \frac{4}{3}\frac{\dot{m}_a}{\dot{m}_a'}(V_j - V) = 224.3 + \frac{4}{3} \times \frac{1}{1.212} \times (400 - 256.3) = 382.4\,\text{m/s}$$

$$\therefore \quad \eta_p' = \frac{2 \times 224.3}{224.3 + 382.4} = 73.9\%$$

[8]

Hence $\frac{s'}{s} = \frac{0.739}{0.782} = 0.946$ (i.e., <u>remaining range reduced by 5.4%)</u>

AJW Jan 2013

12 (a) Volume of 1 mole of silicon is

$$\frac{29\text{gm/mole}}{2.33} = 12.446\text{cm}^3\text{/mole}$$

Volume of 1 mole of SiO_2

$$\frac{60.00}{2.20} = 27.27\text{cm}^3\text{/mole}$$

Since 1 mole of Si is converted to 1 mole of SiO_2

$$\frac{\text{Thickness of Si} \times \text{Area}}{\text{Thickness of SiO}_2 \times \text{Area}} = \frac{\text{Vol of 1mol of Si}}{\text{Vol of 1 mol of SiO}_2} = \frac{12.446}{27.27}$$

$$\therefore \quad \frac{\text{Thickness of Si}}{\text{Thickness of SiO}_2} = 0.456$$

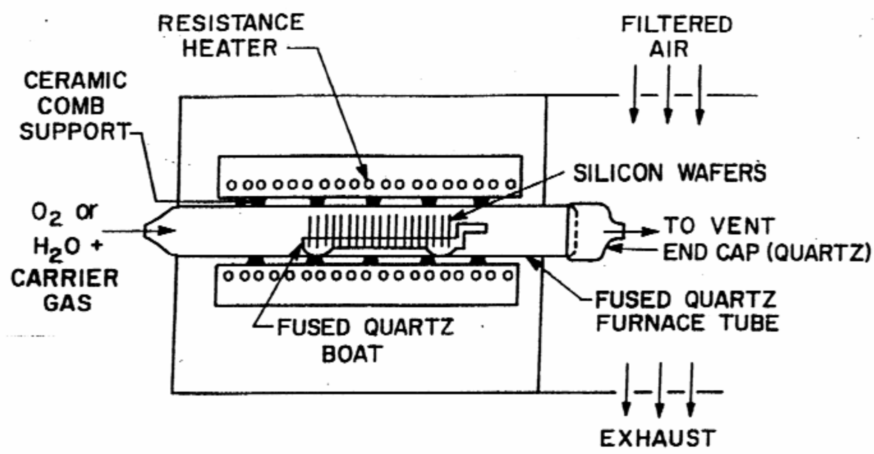
$\therefore \quad 0 \cdot 456\text{d}$ of Silicon will be consumed.

(b) Silicon Dioxide

Successful MOSFET operation depends upon a good quality gate insulator. Silicon dioxide is the standard insulator and it is also used in many of the different process steps as a barrier layer. It is therefore vital to have a reliable method of producing such material.

Oxidation of silicon is achieved by heating silicon wafers in a furnace in the presence of an oxidising atmosphere such as oxygen or water vapour. The two common approaches are:
Wet oxidation where the oxidising atmosphere contains water vapour. Temperature is approx 900-1100 C. This leads to a fast growth rate – depends on temperature but is of order of 0.5 microns per hour at 1100 C.

Dry oxidation where the oxidising atmosphere is pure oxygen. Growth temperatures have to be in the region of 1200 C to achieve acceptable growth rates of 0.1 microns/hour.



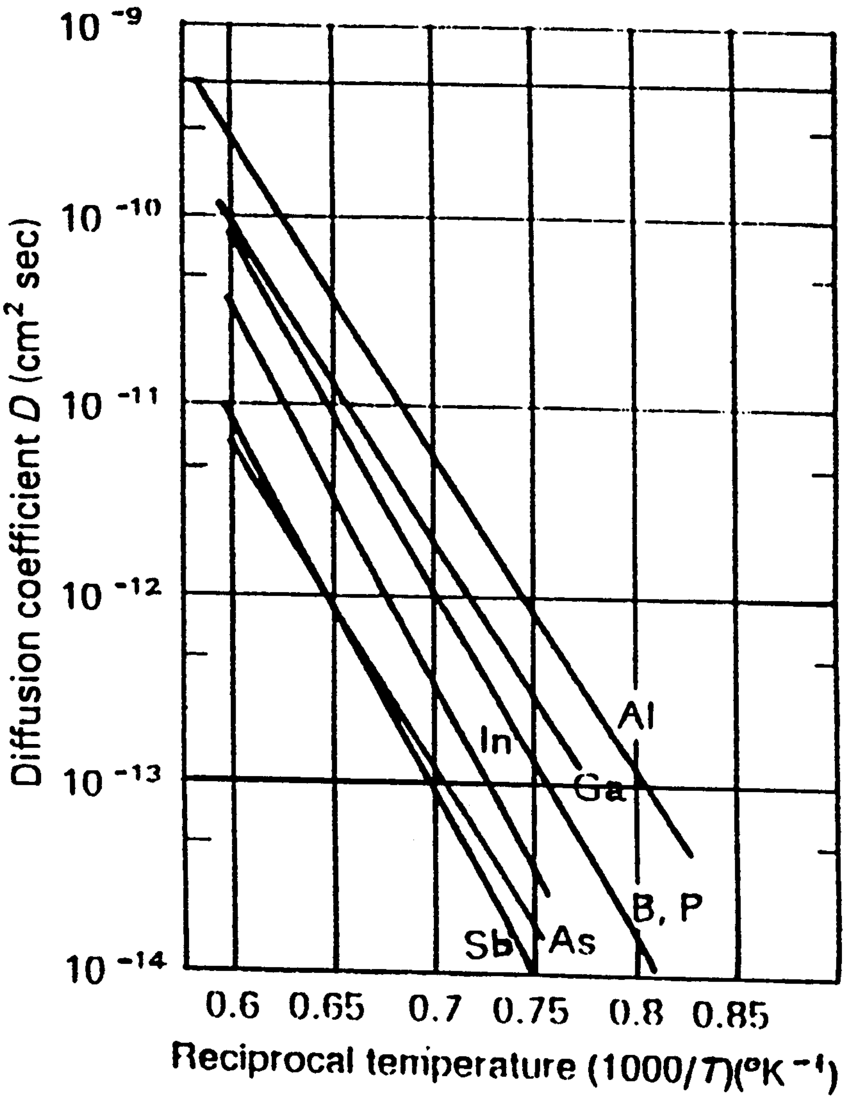
Schematic cross section of a resistance-heated oxidation furnace. The silicon wafer loading area is shown in a laminar hood.

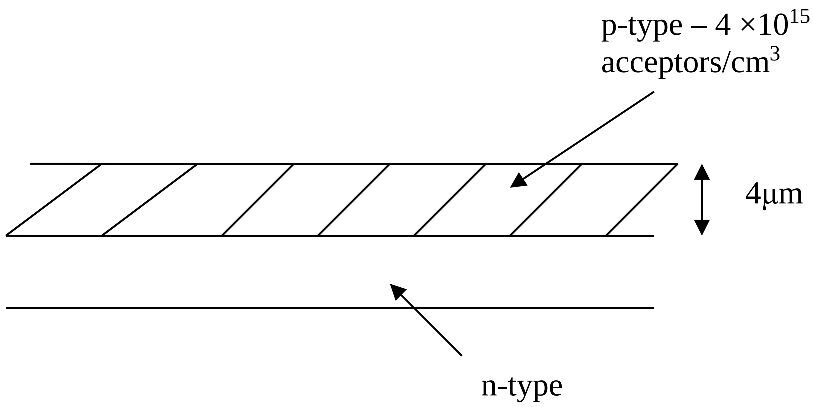
Wet oxidation $Si + 2\ H_2O \rightarrow SiO_2$ (solid) $+ 2H_2$

Dry oxidation $Si + O_2 \rightarrow SiO_2$ (solid)

e.g. Hafnia or Zirconia
HfO_2 ZrO_2

(c)


10⁻⁹
10⁻¹⁰
10⁻¹¹
10⁻¹²
10⁻¹³
10⁻¹⁴
Diffusion coefficient D (cm² sec)
Al
In
Ga
B, P
Sb
As
0.6 0.65 0.7 0.75 0.8 0.85
Reciprocal temperature (1000/T)(°K⁻¹)



T=1100ºC

Assumption : For a successful isolation, the density of the phosphorus impurities which get through the epi layer (4 μm thick) must be greater than the density of p-type impurities in the epi layer everywhere. We need to calculate the phosphorus concentration at 4 μm from the surface and determine whether or not it is > $4 \times 10^{15}/cm^3$.

at 1100°C Cs for P in Si is ~ $1.3 \times 10^{27}/m^3$ from Fig. 1

i.e ~ $1.3 \times 10^{21}/cm^3$

Diffusion coefficient for P in Si at 1100°C ~ 4×10^{13} cm^2/s from Fig.2.

For diffusion from an ∞ source use

$$C(x,t) = Cs\ erfc\left[\frac{x}{2(Dt)^{1/2}}\right]$$

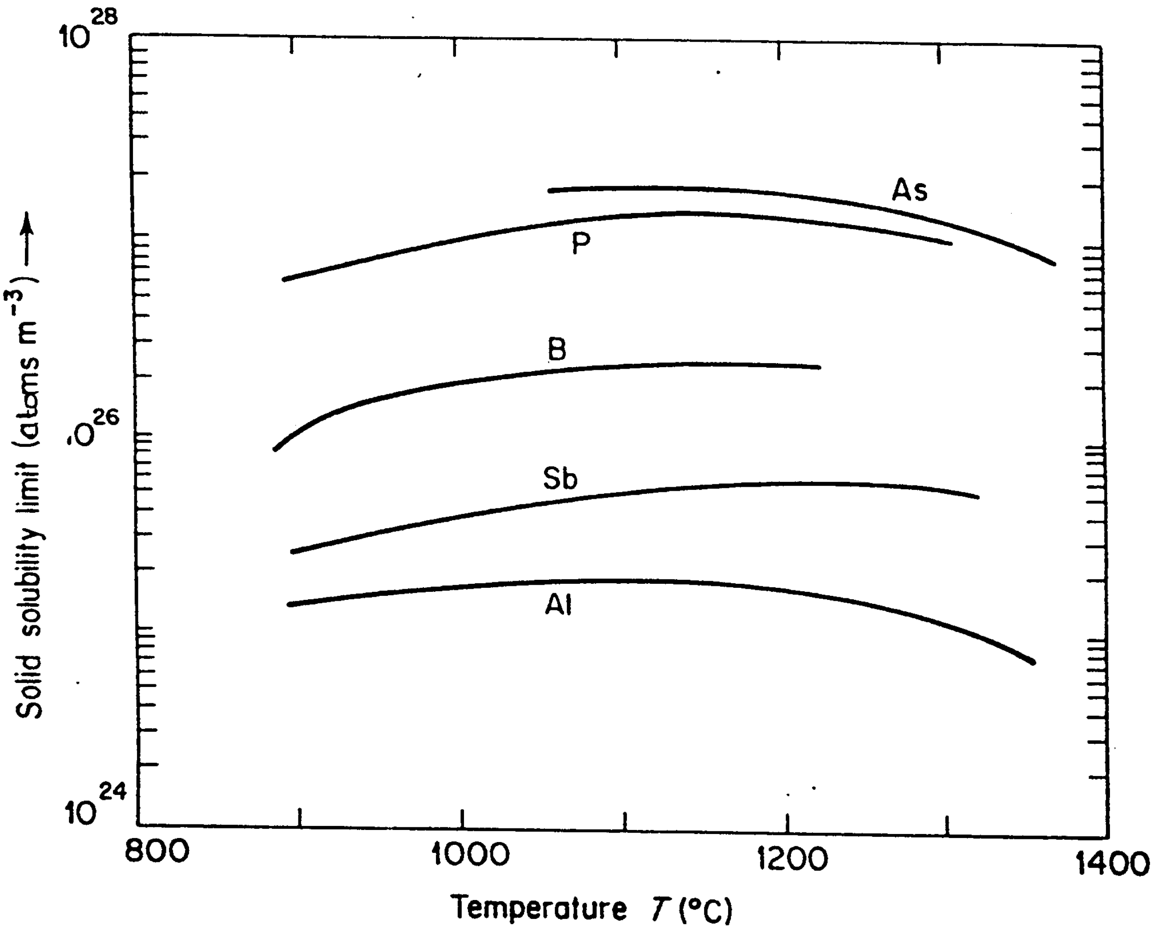
$$C(x,t) = 1.3\times10^{21} erfc\left[\frac{4\times10^{-4}}{2\left(4\times10^{-13}\times45\times60\right)^{1/2}}\right]$$

$$= 1.3\times10^{21} erfc[\sim 6]$$

from Fig.3

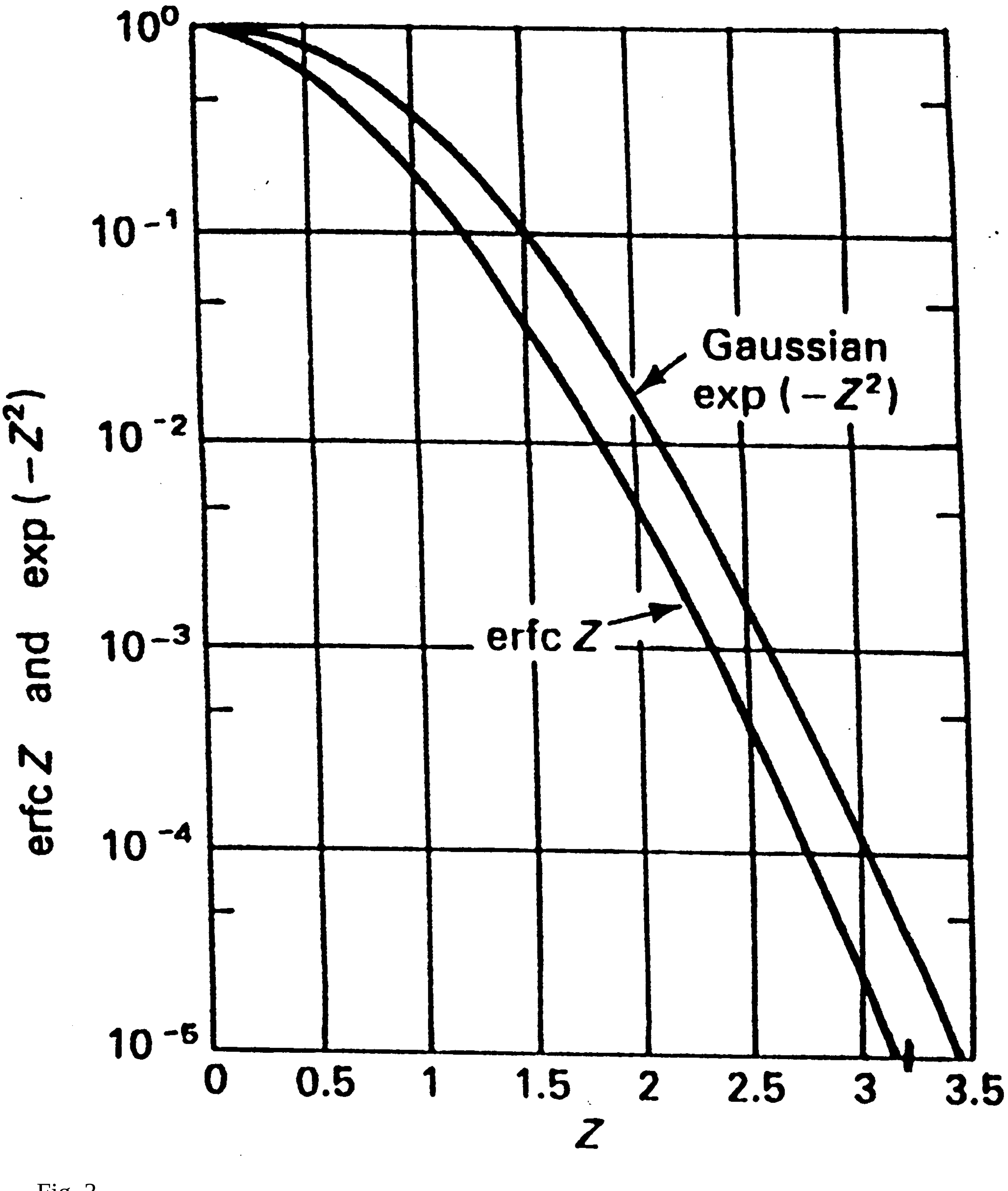
$$= 1.3\times10^{21}\times(\sim 0)$$

essentially no dopant at 4 μm $\therefore$ unsuccessful isolation



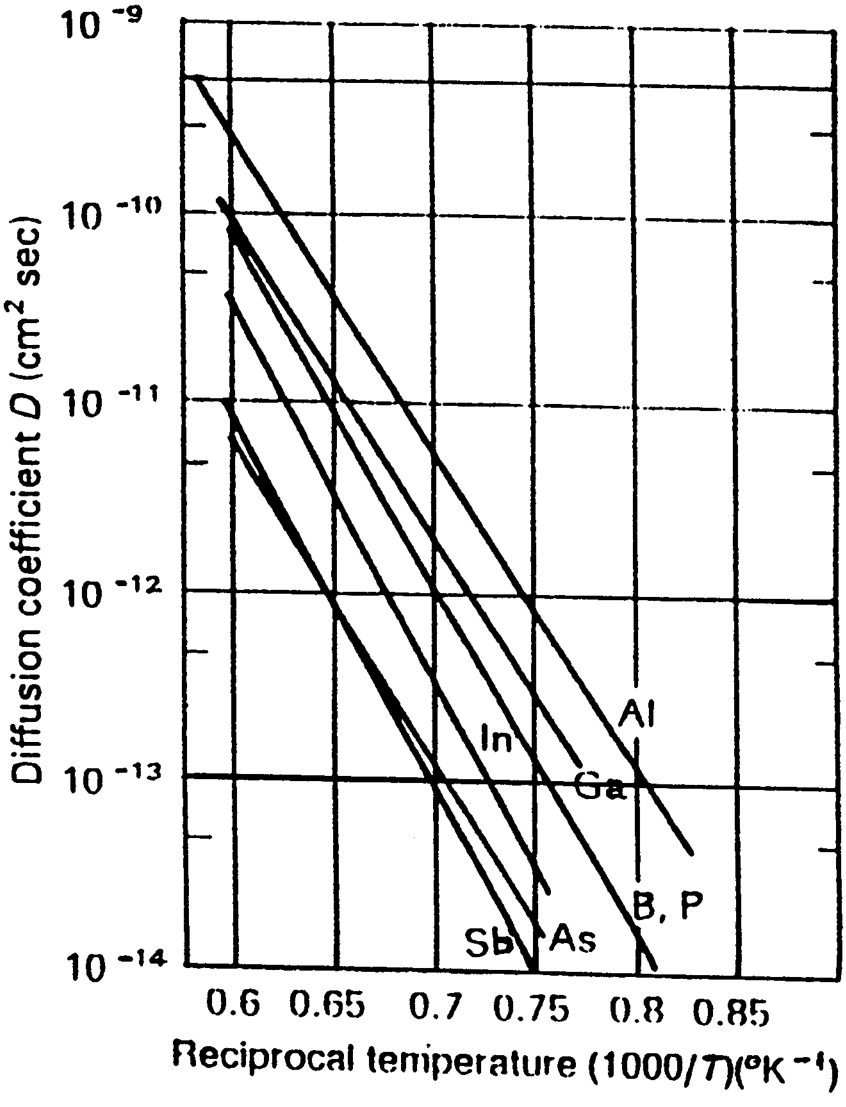
<u>The solid solubility versus temperature data for a range of atoms on silicon</u>

Fig 1



erfc Z and exp (−Z²)
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
0
0.5
1
1.5
2
2.5
3
3.5
Z
Gaussian
exp (−Z²)
erfc Z

Fig. 3



10 -9
10 -10
10 -11
10 -12
10 -13
10 -14
Diffusion coefficient D (cm2 sec)
Al
In
Ga
B, P
Sb
As
0.6
0.65
0.7
0.75
0.8
0.85
Reciprocal temperature (1000/T)(°K -1)

Fig. 2

Q14

a) $N = 4.1 /(3.61\ 10^{-10})^3 = 8.5\ .10^{28}\ /m^3$

$\sigma = Ne\mu$

$\mu = 6\ 10^7 / (8.5.10^{28} \times 1.6\ 10^{-19}) =$ **$0.0044\ m^2\ /V.s$**

σ (semiconductor) = $N_{semi} / N_{copper} .\sigma_{copper} = 4.10^{21} / 8.5\times10^{28} \times 6.10^7 = 2.82$ ohm-1 m-1.

b) book work.

energy = $\frac{1}{2} mv^2 /e = 9.1\times10^{-31} (5.10^4)^2/2 \times 1.6.10^{-19} =$ **0.0071 eV**

c) $v = \mu E$

thus, $E = 5.10^4 / 0.0044 = 1.14\ 10^7$ V/m.

V =2 V, thus $L = 2/ 1.14\ 10^7 =$ **$1.76\ 10^{-7}$ m.**

$t = L/v = 1.76\ 10^{-7} /5.\ 10^4 =$ **$3.4\ 10^{-12}$ s**

Q13

a) V = 1V, σ= 32 /ohm.cm , $v_s = 10^5$ m/s, μ = 0.5 m2/V.s

$\sigma = Ne\mu$, therefore $N = 32/ (1.6.10^{-19}\ 0.5) =$ **$4.\ 10^{20}\ m^{-3}$**.

b) $t = L/v$, $v = \mu E$, $E = V/L$,

so $t = L^2 / \mu V$ and $L = (t\mu V)^{1/2}$ or $L = (20.10^{-12}.\ 0.5\ .\ 1)^{1/2} =$ **$L = 3.1\times10^{-6}$ m**.

$E = V/L = 1 /3.1.10^{-6} = 3.1\times$**$10^5$ V/m**.

c) W/L =20, so $W = 6.2\times.10^{-5}$ m

$I = dW.\ E\ \sigma = (3\times10^{-7}\ 6.2.10^{-5}\ 3.1\times10^5\ 32) = 1.8\times10^{-4}$ **A**

d) $V = Ned^2/ 2\varepsilon$ d = channel thickness, 3.10^{-7} m.

$V_{gate} = 4.10^{20}\ 1.6.10^{-19}\ .\ 9.10^{-14} / (2\times15\times\ 8.85\ 10^{-12}) = 0.022$ V

Part 1B paper 8 2013 Solutions

January 28, 2013

Z9 Q17. Q15.

(a) The most probable image is the image with all white pixels. Since the probability of a black pixel is 0.4, the probability of a white pixel is 0.6, so the all white image has probability $(0.6)^{100}$.

(b) To get the ML estimate for data set D we first write the likelihood:

$$P(D|\theta) \propto [\theta^{60}(1-\theta)^{40}][\theta^{70}(1-\theta)^{30}]$$

We then maximise the (log) likelihood with respect to θ:

$$\log P(D|\theta) = 130 \log \theta + 70 \log(1-\theta) + c.$$

Taking derivatives and setting to zero:

$$\frac{\partial \log P(D|\theta)}{\partial \theta} = \frac{130}{\theta} - \frac{70}{1-\theta} = 0$$

Solving:

$$\begin{aligned} 130 - 130\theta &= 70\theta \\ \theta &= \frac{130}{200} = 0.65 \end{aligned}$$

Note that this is a very intuitive result.

(c) This requires the use of Bayes rule and some integration over θ for M_2. (We can ignore the combinatorial term $c = \binom{100}{60}\binom{100}{70}$ since it is the same for both models.)

$$\begin{aligned} P(D|M_1) &= c(0.5)^{60}(0.5)^{40}(0.5)^{70}(0.5)^{30} = c(0.5)^{200} \\ &\approx c\exp\{-138.6\} \\ P(D|M_2) &= c\int_0^1 (\theta)^{60}(1-\theta)^{40}(\theta)^{70}(1-\theta)^{30}d\theta \\ &= c\,\frac{\Gamma(131)\,\Gamma(71)}{\Gamma(202)} \\ &= c\frac{130!\;70!}{201!} \approx c\exp\{-132.0\} \end{aligned}$$

$$
\begin{aligned}
P(M_2|D) &\approx \frac{0.01c\exp\{-132.0\}}{0.01c\exp\{-132.0\} + 0.99c\exp\{-138.6\}} \\
&= \frac{0.01}{0.01 + 0.99\exp\{-138.6 + 132.0\}} \\
&\approx 0.8813
\end{aligned}
$$

Q 16 NGK

(a) To correct the image the following operations would be needed:

(i) To enlarge the image by $\frac{1}{0.75}$ requires each pixel in the ~~new~~ output image to be taken from a different pixel position in the input image, using interpolation (probably bi-linear) to deal with non-integer locations in the input image. The centre of the output image needs to correspond to a pixel 80 ~~to~~ samples to the left of centre in the input image in order to correct for the offset.

(ii) The filament light source will cause the red intensities to be ~~to~~ scaled up by 1.5, relative to a pure white source, and it will cause the blue intensities to be scaled down by 0.5. Hence we will need to multiply the red component of each output pixel by $\frac{1}{1.5} = 0.67$ & multiply the blue component by $\frac{1}{0.5} = 2$, in order to ~~go~~ produce an image corresponding to white illumination.

(iii) The blurring of the image causes higher frequencies to be attenuated compared with lower frequencies. To correct for this we must apply a highpass filter to the image, such that the gain at lower frequencies is unity, and at $\frac{1}{4}$ of the sampling frequency it increases to $\frac{1}{0.7}$. De-focus Blurring is usually an isotropic filtering process, so we need an isotropic highpass filter (equal frequency response in all directions) in order to compensate for this. This can be achieved by subtracting a lowpass filtered version of the input from the input image, with appropriately chosen gains. Gaussian 1-D lowpass filters can be used to achieve isotropic 2-D lowpass filters, by separable 1-D filtering in the horizontal & vertical directions, which is more efficient than implementing a 2-D filter directly.

(b) The algorithms, described above, can be implemented in Matlab (or equivalent) as follows:

(i) To enlarge and shift the image, we first need ~~to crop the image~~ to compute where each output pixel will come from in the original image. If the output location is (s,t), then the input location (p,q) will be given by: $p = (a s + b)$

$q = (c\,t + d)$

where $a = c = 0.75$ in order to achieve the enlargement and b & d must be chosen to centre the new image at a point 80 pixels to the left of the centre of the old image. If $(p, q) = (512-80, 384)$ when $(s, t) = (512, 384)$, then

$b = 512 - 80 - 0.75(512) = 48$

$d = 384 - 0.75(384) = 96$

Note (p, q) will be non-integer so interpolation (bilinear) of the input image will be needed. To use the interp2 function in Matlab to achieve this efficiently, we would create a 1024-vector of p values and a 768-vector of q values, corresponding to the row & column addresses of the output image as $s = 1{:}1024$ & $t = 1{:}768$.

Or we could do this by cropping the image first & then resizing it.

(ii) To adjust the lighting intensities it is simply necessary to multiply each vector of RGB pixel values by the diagonal matrix

$$\begin{bmatrix} 0.667 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

This can be done efficiently in Matlab using the .* operator (element-wise product). If any blue values exceed the maximum display intensity then they should be clipped at this max value (typ 255) using min(x, 255).

(iii) Let the de-blurring highpass filter be formed from

$$H(\omega) = \alpha - \beta\, G(\omega)$$

where $G(\omega)$ is an isotropic lowpass filter with gain = 1 at low frequencies and gain = 0 at high frequencies.

For correct reproduction of the unaffected low frequency component of the blurred image, we need $H(\omega)$ to have unit gain at low frequencies. Hence $\alpha - \beta = 1$.

At one quarter of the pixel sampling frequency, the gain of $H(\omega)$ needs to be $\frac{1}{0.7} = \sqrt{2}$, in order to correct for the blur. If we assume this is the $\frac{1}{\sqrt{2}}$ gain point of the lowpass filter $G(\omega)$,

then $\alpha - \frac{\beta}{\sqrt{2}} = \sqrt{2}$

Solving for α + β gives:

$$\alpha = 1 + \beta$$

$$\therefore 1 + \beta - \frac{\beta}{\sqrt{2}} = \sqrt{2}$$

$$\therefore \beta = \frac{\sqrt{2} - 1}{1 - \frac{1}{\sqrt{2}}} = \underline{\underline{\sqrt{2}}}$$

$$\therefore \alpha = \underline{1 + \sqrt{2}}.$$

If $G(\omega) = \exp\left(\frac{-\omega^2 b^2}{2}\right)$, then at the freq ω_c where $|G(\omega_c)| = \frac{1}{\sqrt{2}}$, $\frac{-\omega_c^2 b^2}{2} = \ln\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}\ln(2)$

$$\therefore \omega_c b = \sqrt{\ln(2)}$$

(iii) If the filter breadth b is measured in pixel intervals, then at $\frac{1}{4}$ of the pixel sampling frequency, $\omega_c = 2\pi . f_c = 2\pi . \frac{1}{4}$

$$\therefore \quad 2\pi . \tfrac{1}{4} . b = \sqrt{\ln(2)}$$

$$\& \quad b = \tfrac{2}{\pi} \sqrt{\ln(2)} = \underline{0.530}$$

The impulse response of this filter is then given by

$$h(t) = \frac{1}{b\sqrt{2\pi}} . \exp\left(-\frac{t^2}{2b^2}\right).$$

In practice we would use the sampled version of this at integer values of t, & convolve this (using conv2(·)) both horizontally and vertically with the image. We would then multiply the result by β & subtract it from α times the input image to get the highpass-filtered (de-blurred) result.

RC 1

Q17
~~Q16~~ Feature detection

(a) (i) - Smoothing — reduce high frequency noise before differentiation (latter amplifies noise)

- Use a low-pass filter; Gaussian Kernel

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

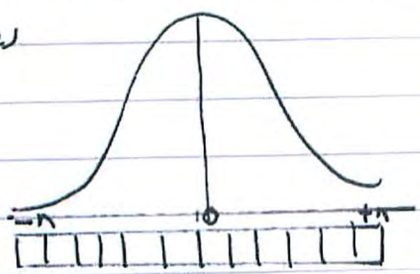
$$= g_\sigma(x)\, g_\sigma(y)$$

- Increasing σ increases amount of blurr, i.e. suppresses high frequencies + cut-off frequency is reduced and only ~~very~~ larger spatial structures survive.

(a)(ii).
$$S(x,y) = \sum_{-n}^{n}\sum_{-n}^{n} I(x-u, y-v)\, G_\sigma(u,v)$$

$$= \sum_{-n}^{n} g_\sigma(u) \sum_{-n}^{n} g_\sigma(v)\, I(x-u, x-v)$$

i.e. implement as 2 1D discrete convolutions where $g_\sigma(x)$ is 1D gaussian samples



2n+1 samples of kernel

(2)

P2 3

Q16(b) Image pyramids and blob-detection (scale-space)

(i). Need to generate a discrete set of images with difference amount of blur. We sample $S(x, y, \sigma^2)$, logarithmically spaced;

$$\sigma_i = 2^{\frac{i}{s}} \sigma_0 \quad , \quad \sigma_{i+1} = 2^{\frac{1}{s}} \sigma_i \qquad (1)$$

with s images per octave (i.e. after s images, σ has doubled).

— apply incremental blur (gaussian σ_K) between images in octave to get images with increasing amount of blur.

$$G(\sigma_{i+1}) = G(\sigma_i) * G(\sigma_K) \qquad \sigma_{i+1}^2 = \sigma_i^2 + \sigma_K^2 \text{ and } \sigma_{i+1} = 2^{\frac{1}{s}}\sigma_i$$

$$\sigma_K = \sigma_i \sqrt{2^{\frac{2}{s}} - 1} \qquad (2)$$

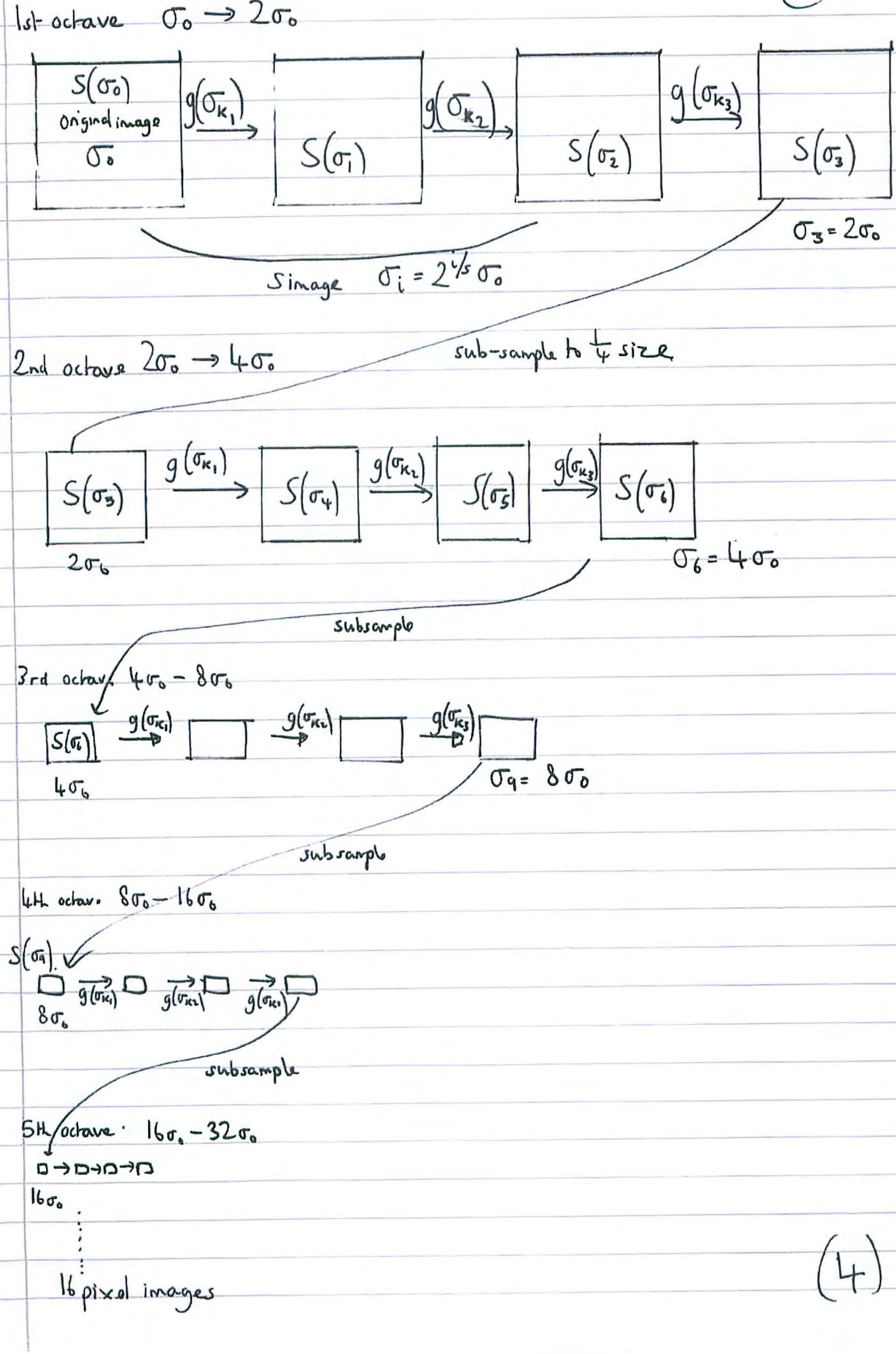
— Each blur is performed as 2 1D convolutions (see (a) ii).

— After scale has doubled (i.e. s blurs) resize image by subsampling by 2 (i.e. ¼ size images). We can represent blurred images with fewer pixels without loss of information (Nyquist). [biggest saving] (3)

— same (small) incremental blur kernels used in each octave. $\sigma_{K_1}, \sigma_{K_2}, \sigma_{K_3}$ etc. on sub-sampled images, but really represent filtering with larger kernels, $2\sigma_{K_1}$ ..... $4\sigma_{K_1}$ ... $8\sigma_{K_1}$.

R 3

16b(i)

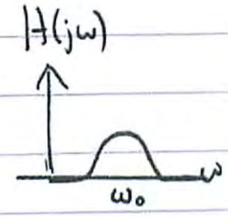

1st octave $\sigma_0 \rightarrow 2\sigma_0$
$S(\sigma_0)$
original image
σ_0
$g(\sigma_{k_1})$
$S(\sigma_1)$
$g(\sigma_{k_2})$
$S(\sigma_2)$
$g(\sigma_{k_3})$
$S(\sigma_3)$
$\sigma_3 = 2\sigma_0$
S image $\sigma_i = 2^{i/s}\sigma_0$
2nd octave $2\sigma_0 \rightarrow 4\sigma_0$
sub-sample to $\frac{1}{4}$ size
$S(\sigma_3)$
$2\sigma_0$
$g(\sigma_{k_1})$
$S(\sigma_4)$
$g(\sigma_{k_2})$
$S(\sigma_5)$
$g(\sigma_{k_3})$
$S(\sigma_6)$
$\sigma_6 = 4\sigma_0$
subsample
3rd octave $4\sigma_0 - 8\sigma_0$
$S(\sigma_6)$
$4\sigma_0$
$g(\sigma_{k_1})$
$g(\sigma_{k_2})$
$g(\sigma_{k_3})$
$\sigma_9 = 8\sigma_0$
subsample
4th octave $8\sigma_0 - 16\sigma_0$
$S(\sigma_9)$
$8\sigma_0$
$g(\sigma_{k_1})$
$g(\sigma_{k_2})$
$g(\sigma_{k_3})$
subsample
5th octave $16\sigma_0 - 32\sigma_0$
$16\sigma_0$
16 pixel images

(4)

P 4

16(c)

16 b(ii) Band-pass filtering



— tuned to a small band of spatial frequencies (difference of high pass and low pass filtered o/p)

— $\nabla^2 G_{\sigma_i} * I = \nabla^2 S(\sigma_i^2) \simeq S(\sigma_{i+1}^2) - S(\sigma_i^2)$

i.e. subtract neighbouring images in same octave.

— generate a pyramid of DOG images by subtraction in same octave (2)

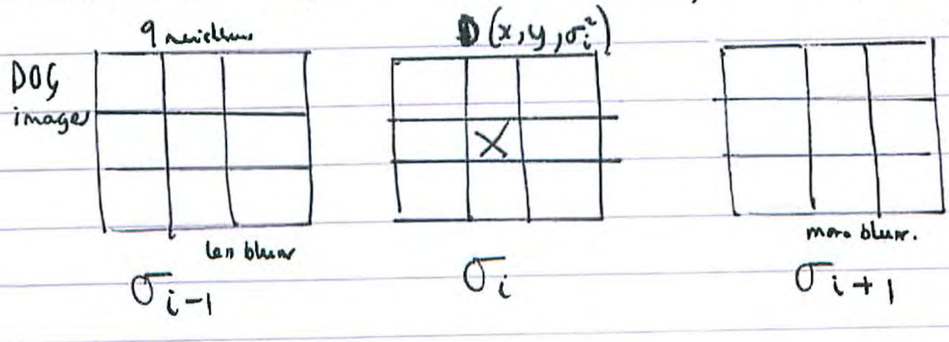
b(iii) Blobs are localised at max/min of $\nabla^2 (G_\sigma * I)$ response. [3]
Need to search over $\nabla^2 S(x, y, \sigma_i^2)$ for (max/min) local.

16(d) — More efficient to search over difference of gaussian (DOG) pyramid

$$\nabla^2 S \approx D(x,y,\sigma_i^2) = S(x, y, \sigma_{i+1}^2) - S(x, y, \sigma_i^2)$$

for max/min in x, y and σ.

— Evaluate 26 neighbours of $D(x, y, \sigma_i^2)$ to see if local max/min



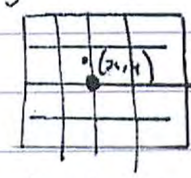
— Local max/min is blob location (x_i, y_i); scale (size of feature) is σ_i [3]

— SIFT descriptor looks at 16x16 pixels sampled from $S(x, y, \sigma_i^2)$ image in correct octave.

R | 1

Q17 SIFT, matching and classification

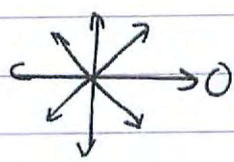
a (i) – For each interest point at location (x, y) and scale σ estimate the dominant orientation, θ, by looking at histogram of edge gradients from $\nabla S(x, y, \sigma^2)$.
Bins 10° apart + smoothed by gaussian.



– Sample 16×16 gradients from (x,y) and aligned with θ at image of scale, σ

(f) – Smooth these gradients with $g(1.5\sigma)$ to emphasise gradients at interest point.

– produce orientation histogram (HOG) for each 4×4 block (cell)
Each bin records grad. magnitude interpolated over 8 dir's (quadrants) only.



(ii) – concatenate 16 histograms of gradients (HOG's) to vector of 128D

– normalize to unit length; truncate to 0.2 to avoid illumination effects and normalize to 1.

– SIFT encodes 2D shape – invariant to lighting by using edges + normalization step + to exact 2D position by histogram / bin effects [pooling]
Edges taken around a blob in centre.

(4)

17g(ii) Each descriptor $\underline{d}_q$ is a 128D vector. Search for **nearest-neighbour** (first nearest and second nearest) $\underline{d}_1$ $\underline{d}_2$ in all descriptors seen.
Done efficiently by arranging descriptors in a **kd tree** (binary search tree data structure).

Accept match if $\frac{\cos^{-1}(\underline{d}_1 \cdot \underline{d}_q)}{\cos^{-1}(\underline{d}_2 \cdot \underline{d}_q)} < r$ (threshold $0.6 \to 0.7$)

This is used to distinguish reliable matches from ambiguous matches (LOWE '99)
Alternatively measure euclidean distance between descriptors $\underline{d}_q$ and $\underline{d}_1$ and accept if close.

(iii). Each feature (SIFT descriptor) points to a target image.
Each match will vote for a target. (See inverted file index)

Can use total votes to hypothesise targets. Verify by estimating pose of target and checking agreement [geometric verification]

SECTION G *Engineering for the Life Sciences*

Answer not more than two questions from this section

18 One third of the total focussing power of the eye comes from the crystalline lens.

(a) Describe the composition and structure of the crystalline lens. [4]

Answer: The crystalline lens sits behind the iris and contributes 1/3 of the total focussing power of the eye. The lens is about a cm across and half a cm thick. The transparent, biconvex lens structure changes shape to change focus. There is an exterior capsule that contains the lens, which is in two parts, the nucleus and the cortex. The nucleus is older lens fibers and the cortex is the newer lens fibers; the capsule is the source of new lens fiber cells.

The lens is made up of "lens fibers" which are specialised elongated epithelial cells surrounded by unusual proteins called crystallines (30% by mass). The overall structure of the lens is complex and "onion-like" in terms of being in layers. There are no blood vessels in the lens.

(b) Explain the process of lens accommodation and how it changes with ageing. [5]

Answer: Lens accommodation is the process of lens shape change that allows the eye to adjust for focus on objects nearer or further away. Lens curvature is controlled by ciliary muscles, and by changing curvature, one can focus the eye on objects at different distances. ÒAmplitude of accommodationÓ is the max amount that the lens can accommodate in diopters (D), equal to the reciprocal of the focal length measured in metres.

The lens continually grows throughout life, laying new cells over the old cells, which results in stiffening of the lens as well as growth of the lens size. It then becomes more difficult for the lens to change shape under the action of the ciliary muscles, and thus the lens gradually loses accommodation ability with age. This is called Presbyopia and it is part of the natural ageing process and happens to nearly everyone. The nucleus stiffens more than the cortex with ageing.

The near point is the closest object that can be brought into focus naturally. This ranges from a few cm in children to an armÕs length in advancing middle age to beyond an armÕs length in old age.

(c) A young lens with a small-strain shear modulus $G = 60\,\text{Pa}$ is indented to a depth $d = 100\,\mu\text{m}$ with an indenter of radius $R = 4\,\text{mm}$, where load $P = 8RdG$, assuming incompressibility. Recall that indentation strain $\varepsilon = 0.2 * (d/R)^{1.5}$.

(i) Calculate the peak force for this indentation.

Answer: P = 8 * 4e-3 m * 100e-6 m * 60 = 0.19 mN

(ii) Calculate the peak force for an indentation to depth $d = 200\,\mu\text{m}$.

Answer: P = 8 * 4e-3 m * 200e-6 m * 60 = 0.38 mN

HOWEVER, the elastic modulus of biological materials is not constant, since the stress-strain response is more like quadratic instead of linear. Therefore this is a lower bound estimate and the load is larger than this. We can estimate how much greater if, assuming quadratic stress-strain, we assume that the small strain modulus

$G = k * \varepsilon$

and calculating a new effective modulus for the new strain (assuming k is constant). So

ε_1 = 0.2 (100e-6/4e-3)$^{3/2}$ = 0.032

and

ε_2 = 0.2 (200e-6/4e-3)$^{3/2}$ = 0.045.

From that,

k = 60 Pa/0.032 = 1897 Pa

and

G_2 = 1897* 0.045 = 85 Pa

and

P = 8 * 4e-3 m * 200e-6 m * 85 = 0.54 mN

(iii) Calculate the peak force, with the original indentation to a depth $d = 100\,\mu\text{m}$, for an aged human lens.

Answer: P = 8 * 4e-3 m * 100e-6 m * 15,000 = 48 mN

NB the value of 15 kPa is not given; the idea of this part of the question is to see if people recall that the stiffness goes up by about three orders of magnitude. Any value for G here in the range of tens of kPa will work.

[6]

(TURN OVER for continuation of Question 18

(d) Describe cataracts and discuss typical cataract treatment. [5]

Answer: A cataract is a cloudy or opaque lens, and is the leading cause of blindness in the developing world, and a major problem with senior citizens in the developed world. In treating cataracts, the lens is removed completely and replaced with an artificial intraocular lens (IOL). The artificial lenses were originally very stiff, glassy polymers with elastic moduli in the range of GPa. However, more modern lenses are flexible and rubber-like, with moduli in the MPa-range. This enhances the minimally invasive nature of the surgery, as the flexible rubbery lenses can be folded up for deployment, and thus the incisions needed for surgery are smaller. IOLs are fixed focus, so the 1/3 of total eye focussing power that was once associated with lens accommodation is gone. Multifocal and accommodating IOLs are under development.

19 (a) Scanning laser ophthalmoscopy and time-domain optical coherence tomography can both be used for *depth-sectioning* of the eye. What is *depth-sectioning*, and how is this achieved for both of these imaging modalities? [6]

Answer: *Depth-sectioning* refers to the ability of these imaging modalities to return image data from a specific depth into the tissue, rather than just returning data from the first visible surface (like a photograph) or summed over a wide focal region (like a conventional microscope).

For the scanning laser ophthalmoscope, depth-sectioning is achieved by confocal optics. The reflected light is focused at a specific depth into the tissue, and this is directed towards a very small aperture (called the confocal aperture). This aperture is positioned so that light reflected from this depth will be precisely focused at the location of the aperture and will hence all pass through to the single detector which is positioned after the aperture. Any light from shallower or deeper regions is not focused at the confocal aperture, hence only a small fraction is detected. Moving the confocal aperture therefore changes the depth at which the detector is most sensitive to reflected light.

In time-domain coherence tomography, a laser pulse is used rather than a continuous laser. Part of this pulse is sent to the eye, and part is sent to a reference mirror. The reflections from the eye and the reference mirror are then re-combined before being sent to a photo-detector. When the optical path lengths to the mirror and to the back of the eye are nearly identical, this causes interference fringes. These fringes show up as oscillations in the time-domain light signal, and are picked up by looking for high frequency content in this signal. Moving the reference mirror changes the depth at which interference will occur, and hence also the depth at which the detector is sensitive to reflected light.

(b) A 2D cross-sectional image of the eye is created with the horizontal image axis x laterally across the eye and the vertical image axis y representing depth into the eye. Explain what determines the image resolution in both the x and y directions, giving typical values for these resolutions, for each of the following imaging modalities:

(i) scanning laser ophthalmoscopy; [2]

Answer: The lateral (x) resolution is determined by the spot size of the laser as it hits the back of the eye, which itself is determined by the optics of the instrument and the lens of the eye being imaged. Typically achievable values are 15 μm for a normal system, 2 μm if adaptive optics are used. The depth (y) resolution is determined by the confocal system, the most important aspect being the width of the confocal aperture, though the numerical aperture of the imaging lenses will have some impact on this too. Typical values are in the $100\,\mu$m to $400\,\mu$m range.

(ii) time-domain optical coherence tomography; [2]

Answer: As in scanning laser ophthalmoscopy, the lateral (x) resolution is determined by the size of the laser spot and has similar value to (i). However, in this case the depth (y) resolution is determined by the length of the laser pulse, or more specifically the correlation length of the laser pulse. This will tend to be better for lasers with a higher centre frequency and a wider bandwidth. Achievable values are about $5\,\mu$m, depending on the laser properties.

(iii) ultrasound imaging. [2]

Answer: Lateral (x) resolution is dependent on the lateral (electronic) focusing, which is a function of how many elements are used and the width of the transmission and reception aperture. The value will also depend on the acoustic centre frequency. It is very variable with depth, never better than about 0.5 mm, and often much worse than this. Depth (y) resolution, as with optical coherence tomography, is largely a function of the properties of the acoustic pulse, though it is also affected by the electronic focusing. Typical values for ultrasound are in the range 0.1 mm to 1 mm.

(c) An ultrasound imaging system employs a simple acoustic pulse with duration $2t_p$ and shape $P(t)$ given by:

$$P(t) = \begin{cases} +1 & -t_p < t < 0 \\ -1 & 0 < t < t_p \\ 0 & \text{otherwise} \end{cases}$$

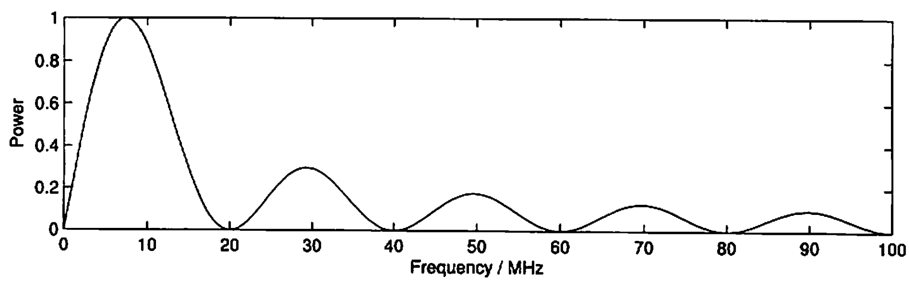
(i) Calculate and sketch the frequency spectrum of $P(t)$ for $t_p = 0.05\,\mu$s. [6]

Answer: To calculate the spectrum we need to take the complex Fourier

Transform of the power P:

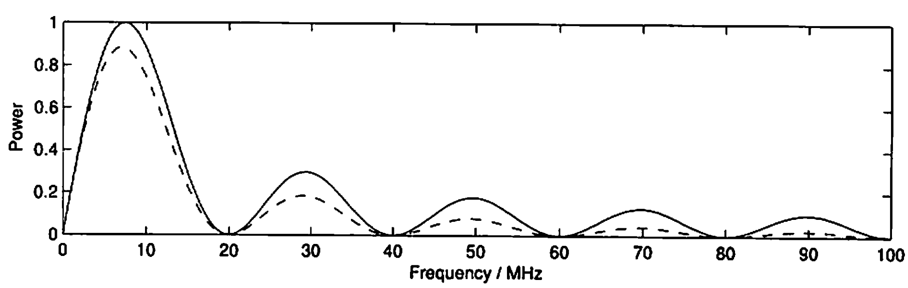
$$
\begin{aligned}
\mathscr{F}(P) &= \int_{-t_p}^{0} +1e^{-j\omega t}\,\mathrm{d}t + \int_{0}^{t_p} -1e^{-j\omega t}\,\mathrm{d}t \\
&= \left[\frac{1}{-j\omega}e^{-j\omega t}\right]_{-t_p}^{0} + \left[\frac{-1}{-j\omega}e^{-j\omega t}\right]_{0}^{t_p} \\
&= \frac{1}{j\omega}\left(e^{j\omega t_p}-1\right) + \frac{1}{j\omega}\left(e^{-j\omega t_p}-1\right) \\
&= \frac{2}{j\omega}\left(\frac{1}{2}\left(e^{j\omega t_p}+e^{-j\omega t_p}\right)-1\right) \\
&= \frac{2}{j\omega}\left(\cos\omega t_p - 1\right)
\end{aligned}
$$

The magnitude of this function is plotted below:



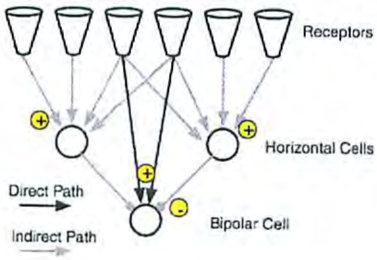
(ii) How would this pulse spectrum be modified after the pulse has passed through several centimetres of tissue at the back of the eye? [2]

Answer: The acoustic pulse will be attenuated as it travels through the eye, with the higher frequencies being attenuated more than the lower frequencies. Hence the spectrum will look like the dashed line in the graph below:



20 (a) Describe the receptive field properties of retinal ganglion cells and how the receptive fields are constructed from the photoreceptors. Why might such receptive field properties be computationally useful? [3]

Answer: Ganglion cells have roughly circular receptive fields from a few minutes of arc at fovea, to few degrees at periphery. On-centre ganglion cells have an excitatory receptive field centre and inhibitory surround thereby respond optimally to differential illumination. Off-centre ganglion cells have an inhibitory receptive field centre and excitatory surround. Two pathways are used to construct the receptive field. A central direct excitatataory and in an indirect inhibitory pathway through horizontal cells onto bipolar cells. Bipolar cells excite ganglion cells and have similar receptive fields.



A. WIRING DIAGRAM

The receptive field makes the cells sensitive to contrast differences rather than absolute luminance and this is important as lighting can change and these cells can respond better to enable edge detection.

(b) The black text on a newspaper in bright sunlight reflects as much light as the white paper inside a normal room, yet they look quite different shades. What mechanisms are responsible for this perceptual difference? [2]

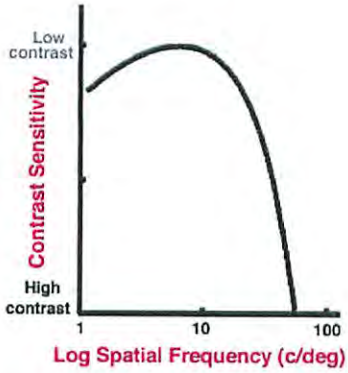
Answer: At least three features can contribute. Adaptation of the sensitivity of the retina due to dark adaptation and the pupillary reflex, the receptive field properties of ganglion cells which tend to respond to contrast gradients rather than illumination and higher level procession that tend to discount the illuminant.

(c) If you stare at a green apple and yellow banana for some time and then stare at a white wall what colours might you see in the afterimage and why? [2]

Answer: The green apple would turn to red and the yellow banana to blue and these are the colour opponency axes which come from the principal components of colour space for natural objects and are the way the brain codes colour. Fatiguing the visual system leads to an afterimage that is in the opposite direction in colour opponency space.

(d) Sketch a contrast sensitivity function. Use the contrast sensitivity function to explain how you would produce a poster with text that could only be read from far away but not when a person was close to it. [3]

Answer:



Produce the text by low pass filtering the normal text so that only low frequencies remain which are below the contrast sensitivity when near but high enough to be read when far away. Include high spatial frequency mask that can only be seen when near but is invisible when far away - this is similar to the Monroe/Einstein illusion.

(e) The eyes of an ancient animal are reconstructed by palaeontologists. The reconstruction suggests a simple eye with a convex lens whose two symmetrical surfaces are spherical, and the diameter of the lens is 1 cm when viewed from the front. We also know that the retina was 1 cm away from the midline of the lens.

(i) Following simple geometric arguments, calculate the minimal radius of curvature of the lens. [1]

Answer: The minimal value for the curvature radius of the lens, R, is its frontal radius (which is half of its frontal diameter) – this is attained when the lens is a sphere. Therefore $R > 5$ mm.

(ii) Assume that the material of the lens was homogeneous, and its refractive index (r_{lens}) could not be higher than 1.5. Explain, with reasons, whether it is possible that the animal was aquatic or terrestrial. Support your answer with calculations regarding the minimal object distance the eyes could bring into focus. Use the following physical constants in your calculations:

$r_{air} = 1$, $r_{water} = 1.3$. [6]

Answer: Let us first assume that the animal was aquatic. Using the lensmaker's formula for thin (homogeneous) lenses, the minimal Matthiessen's ratio for this lens is

$$\frac{f}{R} = \frac{1}{2}\frac{r_{water}}{r_{lens} - r_{water}} > \frac{1}{2}\frac{1.3}{1.5 - 1.3} = 3.25$$

We know from the previous question that $R > 5$ mm, so the minimal focal length of the lens is

$$f > 3.25 \cdot 5 \text{ mm} = 16.25 \text{ mm}$$

The image must be formed on the retina, so the distance of the image from the midline of the lens is $S_2 = 1$ cm $= 10$ mm. Using the formula relating focal length to object distance, S_1, and image distance, we can drive the following:

$$\begin{aligned} \frac{1}{16.25}\text{ mm}^{-1} &> \frac{1}{f} = \frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{S_1} + \frac{1}{10}\text{ mm}^{-1} \\ \frac{1}{S_1} &< \frac{1}{16.25}\text{ mm}^{-1} - \frac{1}{10}\text{ mm}^{-1} < 0\text{ mm}^{-1} \end{aligned}$$

This is a contradiction because the object distance must be positive and hence its reciprocal must also be positive. Therefore the animal could not have been aquatic.

Let us now assume that the animal was terrestrial. Following the same logic as above we obtain the following results:

$$\begin{aligned} \frac{f}{R} &= \frac{1}{2}\frac{r_{air}}{r_{lens} - r_{air}} > \frac{1}{2}\frac{1}{1.5 - 1} = 1 \\ f &> 1 \cdot 5\text{ mm} = 5\text{ mm} \\ \frac{1}{5}\text{ mm}^{-1} &> \frac{1}{f} = \frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{S_1} + \frac{1}{10}\text{ mm}^{-1} \\ \frac{1}{S_1} &< \frac{1}{5}\text{ mm}^{-1} - \frac{1}{10}\text{ mm}^{-1} = 0.1\text{ mm}^{-1} \\ S_1 &> 10\text{ mm} \end{aligned}$$

Therefore it is possible that the animal was terrestrial and in this case the smallest object distance that it could have possibly focussed on was 10 mm.

(iii) How does your answer to the previous question change if it is possible that the lens was inhomogeneous? [2]

Answer: If the lens is inhomogeneous then its Matthiessen's ratio can be smaller than that calculated by the lensmaker's formula. Therefore its focal length can also be smaller than those values determined above, which means that we could obtain positive object distances even for an aquatic animal. Thus, in this case, the animal could have been aquatic as well as terrestrial.

(iv) Besides broadening the range of possible focal lengths, what other advantage does an inhomogeneous lens confer? [1]

Answer: An inhomogeneous lens can also correct for spherical aberration which is the phenomenon by which a lens with spherical surfaces does not quite collect all beams parallel to its axis into a single point thus leading to blur in the image. Inhomogeneities in the material of the lens can correct for this thus making the image less blurry.

END OF PAPER

21. (a).

(i) Pros
- Can get a profit from invention immediately.
- do not need to take a risk for production or marketing.
- do not need to raise a fund for production or marketing

Cons
- Cannot maxmise a benefit from invention
- should find someone to buy it.

(ii) Pros
- Can realise a profit without being involved production (as long as their intellectual properties are legally valid, in case of patents or IPs)
- little risk / responsibility for production process
- do not need to raise a fund ~~for~~ production

Cons
- can only enjoy small portion of potential profit.
- shuld find someone to license it.
- should care about IP management.

(iii)

pros
- can make use of partners' complementary asset.
- can reduce/share risk,
- can ~~share~~ reduce financial investment.

cons
- Have to share a profit
- Have to endure any inefficiency involved in collaboration (due to different culture / interests / communication methods)

(iv) Pros
- Can enjoy maximum benefits
- can control every detail

Cons
- Have to endure all risks
- Have to raise a fund for production & marketing.

21. (b)

(i) Venture Capitalist (VC) may expect specified information in the business plan. For example, what is missing in the market (market), what this entrepreneur can make or provide to address this gap (product or service, such as wind turbine control system), with whom he/she can initiate and to what extent they have their expertise in wind turbine (management team), how he/she can do it (business operation), how and when he can make a profit (financial projections), how he can make customers buy his product and service (marketing strategy), what resources are required to start his business (resource required), and how he can make a profit for investors (exit oppertunities).

(ii) VC will concern about the following factors: management (whether this team is newly established or ~~no~~ has worked together before), technology feasibility, market (whether there is a actual demand), access to markets, regulation effects, funding (how much my investment will be diluted), and exit (whether I can realise my investment for exit).

However, by adding some elements, this enterpreneur may be able to persuade investors and make its company look more attractive. As this ~~the~~ firm can provide alternative business model by providing services related to installation and control system management. He can accentuate the 'soft' start-up model of his company.

22. (a) 20 years

(b) When licensing patents (IP), CDT should...

- find trustworthy licensee who will offer appropriate royalty payments.
- decide the extent of patents they will licence
- decide the duration of licensing.

The latter two factors can be important, as CDT is currently scaling up its manufacturing process. (If necessary, some core patents should not be licensed)

When manufacturing know-how is leveraged, some vital information can be revealed to partners. CDT management team should consider 'Confidential Disclosure Agreement' (CDA). CDA should be signed before they initiate any kind of discussion with potential partners in order to prevent information leakage.

(c) • Due to size difference, CDT may have had difficulties in finding proper ~~par~~ counterpartners in large established firms. In addition, if the partner company is not located proximately, communication might lead more difficult problems.

- Partnership is established when the strategic need of CDT is accord with that of the large partner firm, but the dynamic nature of strategy and business model can influence/change partnership.

- If CDT and its partner have differnt clock-speeds, CDT might encounter serious problems in its cash-flow.
- Due to its limited resources, CDT might spend too much resources in building partnership, consequently paying less attention to other business activities.
- Due to its short experience, CDT might have insufficient partnering capability.

23. (a)

(i) • A prototype enables us to test an unknown dimension By capturing essential ideas, visual models can provide detailed information and play an important role as a communication tool.

- A prototype also enables us to reduce market risks by testing new features/concepts with customers.
- It also enables us to reduce technical risks by testing whether it really works as intended.
- It ~~can~~ also enables us to find any potential problem ~~in production and to test performance and life~~ which might appear in the production stage. By doing performance and life testing, We can increase product quality.

(ii) Manufacturers should be carful about pitfalls of prototyping.

- They should clearly identify the purpose by recognising what is being assessed.
- They should decide who will evaluate a protype. Evaluation can vary according to people who participate in (design team or customers).
- Whether a proper evaluation plan including test equipment is establised is also important.
- They should know (or make a plan) for the results. Prototyping is important itself, but how it can be used as a feedback can be more important.

23. (b) Product specification refers to defining what should be designed and what should be constrained by providing precise details about the product. Although usually only ~~core~~ the technology aspects of core product are dealt with, ~~whold~~ whole design mix should be considered together. It is because of prime importance lies in delivering benefits to customers rather than in just making products. Thus, like a 'elevator pitch', specification should cover entire value proposition, and provide detail at system, sub-system, and individual component level. It also consider dynamic characteristics of design as a living document.