ENGINEERING TRIPOS PART IB

Monday 3 June 2013 9 to 11

Paper 1

MECHANICS

Answer not more than **four** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

*Answer not more than **two** questions from this section.*

1 A uniform straight rigid rod AB of length L and mass m lies on a flat horizontal surface which may be assumed frictionless. The rod is subject to a horizontal force P applied at a distance of $2L/3$ from end A in a direction perpendicular to its length as illustrated in the plan view of Fig. 1. The rod is otherwise unrestrained. As a result of the application of P, the centre of the rod G has linear acceleration a and the rod has angular acceleration $\ddot{\theta}$.

(a) At the instant shown confirm that the rod is rotating about end A. [8]

(b) Obtain an expression for the instantaneous bending moment in the rod as a function of distance from end A. [6]

(c) Find the point within the rod where the instantaneous bending moment is a maximum and express its value in terms of P and L. [6]

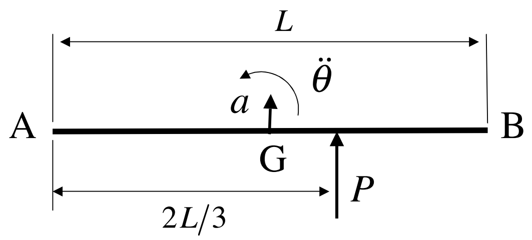

Fig. 1

2 Figure 2 shows a mechanism driven by the crank AB which rotates at steady speed ω. AB is attached to rod BD which passes through a slider pivot at C which is ℓ from A. The crank is also of length ℓ while the rod BD is $2\sqrt{2}\ell$. At the instant shown the angle at B is $45°$.

(a) By sketching a suitable velocity diagram, or otherwise, demonstrate that the velocities of points B and D are of equal magnitude but at right angles to one another. [8]

(b) Establish the value of the angular acceleration of rod BD and the magnitude and direction of the acceleration of the point D. [8]

(c) A mass m is attached at D while all other inertias are negligible. If the effect of gravity can be neglected find the instantaneous torque T required at A to maintain steady motion when the mechanism is in the configuration shown. [4]

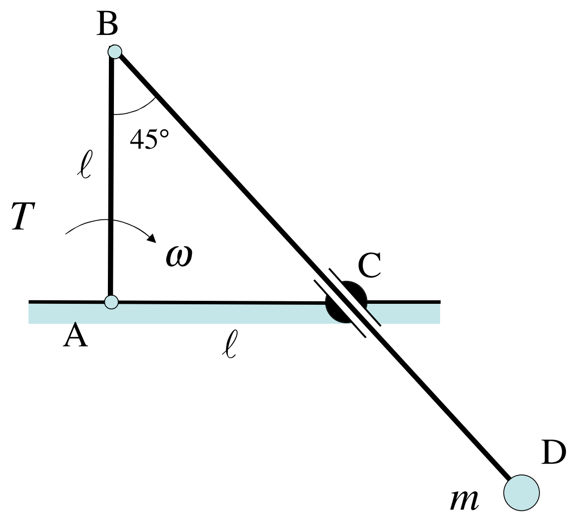

Fig. 2

3 A mass m is connected by a pulley, cable and spring to a fixed support as shown in Fig. 3. The inextensible cable AB is continuous and runs around the pulley which is of radius R. The mass of the pulley is m and its polar moment of inertia is I. The spring is of stiffness k.

It can be assumed that at all times the centre of gravity of the mass lies vertically below the axis of the pulley and that friction between the cable and the pulley is sufficient to prevent any slip.

(a) The system undergoes a small displacement from equilibrium so that the mass and the pulley descend a distance Δ and the pulley rotates through Δ/R. Show that the resultant *change* in the potential energy stored in the spring is given by

$$2(mg + k\Delta)\Delta \ . \qquad [6]$$

(b) (i) The mass and the pulley are now released. Determine the angular frequency with which they will oscillate in terms of k, m, I and R. [8]

(ii) A load cell is connected at A to measure the tension in the cable. If the pulley can be modelled as a simple disc, so that $I = mR^2/2$, what value of Δ will cause the output of the load cell to just reach zero? [6]

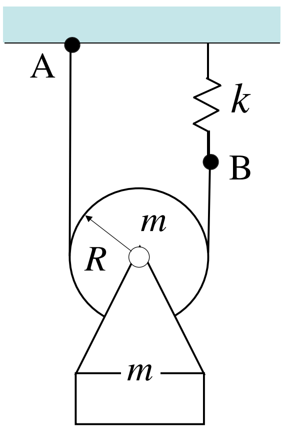

Fig. 3

SECTION B

*Answer not more than **two** questions from this section.*

4 Two thin uniform rods OQ and QR, each of mass m and length ℓ, are connected at Q by a frictionless pin as shown in Fig. 4. They are supported, without friction, in the horizontal plane and ϕ is the angle between OQ and QR. Unit vectors along OQ and QR are defined as $\underline{e}_1$ and $\underline{e}_2$ respectively. Rod OQ rotates about the fixed point O with steady angular velocity $\omega \underline{k}$ where $\underline{k}$ is a unit vector perpendicular to the plane containing the rods.

(a) Show that $\underline{v}_P$ the velocity of the point P, which is the centre of mass of rod QR, can be written as

$$\underline{v}_P = \ell\omega\underline{e}_1^* + \frac{\ell}{2}\omega\underline{e}_2^* + \frac{\ell}{2}\dot{\phi}\underline{e}_2^*$$

$$\text{where } \underline{e}_1^* = \underline{k}\times\underline{e}_1 \text{ and } \underline{e}_2^* = \underline{k}\times\underline{e}_2 .$$

Obtain the corresponding expression for $\underline{a}_P$ the acceleration of P. [10]

(b) Rod QR is initially held so that $\phi = 90°$ and then released.

(i) Show that in the subsequent motion $\dot{\phi}$ satisfies the equation

$$\dot{\phi}^2 = 3\omega^2\cos\phi \quad .$$ [5]

(ii) Obtain an expression for the tension in the rod OQ when $\phi = 0$ in terms of m, ω and ℓ. [5]

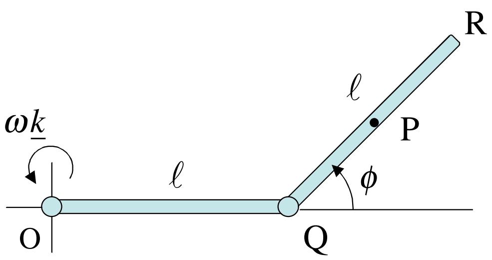

Fig. 4

5 (a) Under what circumstances are (i) moment of momentum, and (ii) total mechanical energy conserved? [4]

(b) A uniform disc of radius a rolls slowly at speed V over a horizontal surface when it encounters a defect AB of width $\sqrt{2}a$, as shown in Fig. 5. The disc does not make contact with the base of the defect.

(i) Confirm that as the disc rotates about the point A its centre descends a distance Δ such that

$$\Delta = \left(1 - \sqrt{2}/2\right)a .$$

Hence show that the angular velocity ω of the disc just before it makes contact with corner B satisfies the equation

$$(a\omega)^2 = V^2 + \frac{4g\Delta}{3} .$$ [5]

(ii) In a particular case $V = 2\sqrt{ag}$. Assuming that there is no rebound, find the angular velocity of the disc after it makes contact with corner B and the proportion of its initial kinetic energy which has been lost in the impact. [6]

(iii) To what value can V be reduced before it is impossible for the disc to regain the horizontal surface to the right of B? [5]

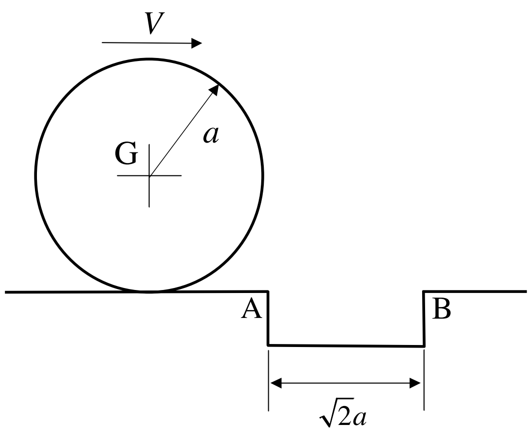

Fig. 5

6 A rectangular rigid plate is symmetrically welded to a vertical shaft and supported by two bearings A and B, separated by distance L, as shown in Fig. 6. The plate and shaft have moment of inertia J about the axis AB and are initially stationary.

The plate is hit by a projectile of mass m at point P. The coordinates of P in the system shown, which has its origin at the centre of the plate, are $(d, h, 0)$. The speed of the particle before impact is V and is in a direction perpendicular to the plate. After the impact the particle remains attached to the plate. The effects of gravity can be neglected.

(a) Show that the angular velocity ω_0 of the plate immediately after the impact is given by the expression

$$\omega_0 = \frac{mVd}{J + md^2}$$

and hence determine the value of d which maximises its magnitude. [8]

(b) A constant frictional torque Q acts at the bearings and causes the rate of rotation of the plate to decrease. If ω_0 takes the maximum value found in part (a), find:

(i) the time after impact for the plate to come to rest; [5]

(ii) the magnitudes of the components of the forces acting at each bearing just after impact in terms of m, d, h, L, V and Q. [7]

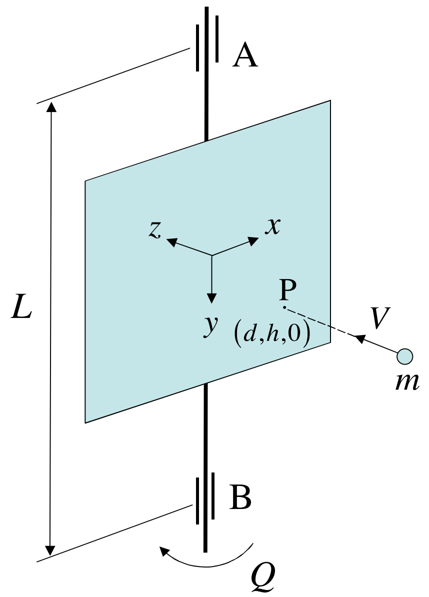

Fig. 6

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