ENGINEERING TRIPOS   PART IB

Monday 3 June 2013   2 to 4

Paper 2

STRUCTURES

Answer not more than ***four*** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: none.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

fc03

SECTION A

1 (a) Figure 1(a) shows a thin-walled cantilever box-beam having a square cross-section with width and height of 100 mm. The wall thickness is 2 mm everywhere. The cantilever is 2.5 m long and is subjected at its free end to an eccentric point load of 1.5 kN acting vertically at the corner A. The box-beam contains diaphragms that prevent changes to the cross-sectional shape.

(i) Calculate the second moment of area and the torsion constant for the cross-section. [4]

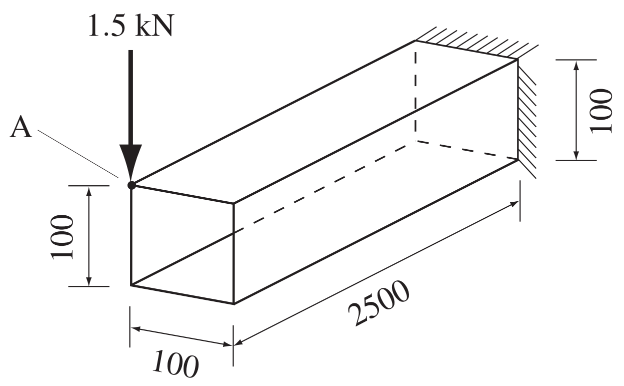
(ii) Calculate the maximum axial stress at the root of the cantilever. [2]

(iii) Calculate the maximum shear stress at the root of the cantilever. [6]

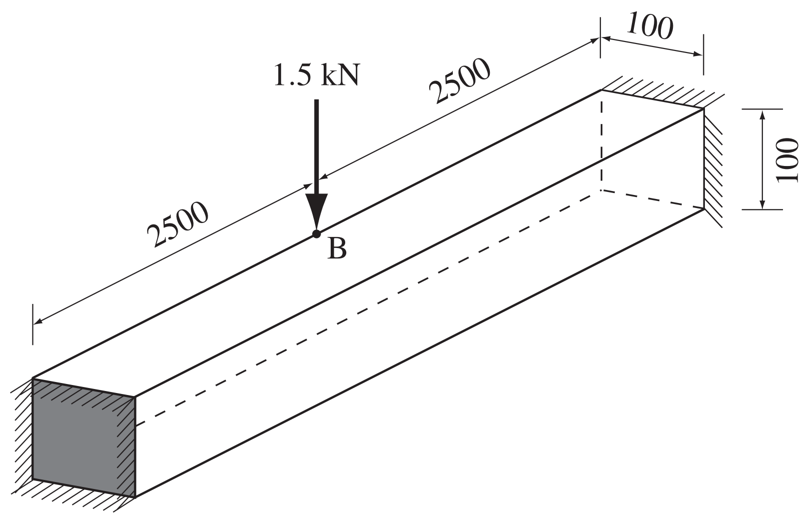
(b) Figure 1(b) shows a 5 m long thin-walled box-beam with both ends fixed to a foundation. The square cross-section has a width and height of 100 mm and the wall thickness is 2 mm everywhere. The beam is subjected at its mid-span to an eccentric point load of 1.5 kN acting vertically at the corner B. The Young's modulus and the shear modulus of the beam are E and G, respectively. The box-beam contains diaphragms that prevent changes to the cross-sectional shape.

(i) Calculate the rotation of the cross-section at the mid-span. [4]

(ii) Calculate the vector of displacement of the corner point B. [4]



(a)



(b)

(Dimensions in mm unless indicated otherwise. Not to scale.)

Fig. 1

2 (a) Figure 2(a) shows a weightless pin-jointed truss with three members. Each member has cross-sectional area A, Young's modulus E and all behaviour is elastic. The unloaded structure is free of stress.

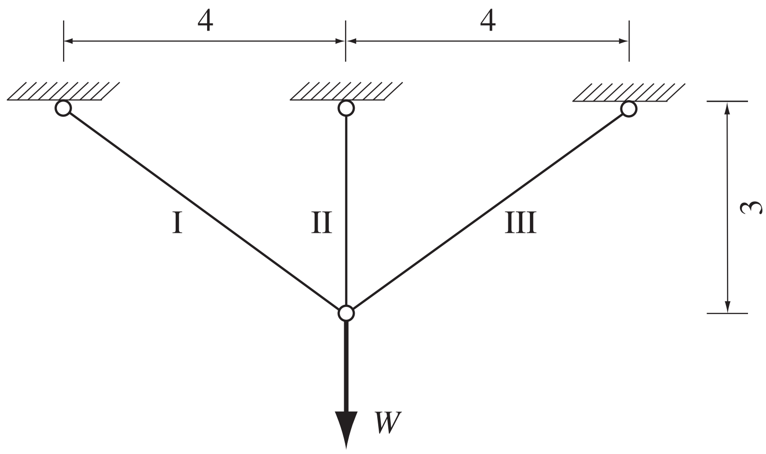
(i) Show that the structure has one redundancy and find a state of self-stress corresponding to a unit tension in bar II. [4]

(ii) Use the Force Method to determine the bar forces in the structure due to the applied load W. Specify bar II as the redundant member. [6]

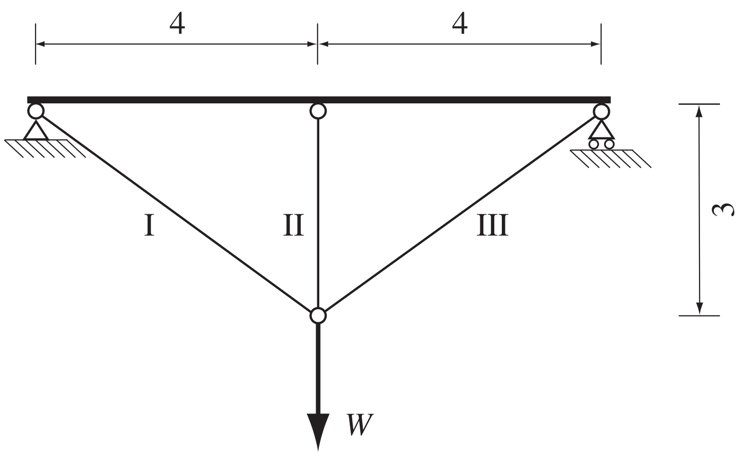
(b) Figure 2(b) shows a beam braced by a pin-jointed truss with three members. The flexural rigidity of the beam is EI_b and its axial rigidity EA_b is to be assumed infinite. Each member of the pin-jointed truss has axial rigidity EA_t. All behaviour is elastic and the unloaded structure is free of stress. The structure has one redundancy which is designated as a unit tension in bar II.

(i) Express the bending moment distribution in the beam due to the self-stress in the truss. [4]

(ii) By considering compatibility between the central deflection of the beam and the truss displacements, use the Force Method to find the maximum bending moment in the beam due to the applied load W. [6]



(a)



(b)

(Dimensions in m.)

Fig. 2

3 A thin-walled aluminium alloy circular cylinder of radius $R = 2$ m and uniform wall thickness $t = 15$ mm has closed ends. The Young's modulus of the aluminium alloy is $E = 70$ GPa and its Poisson's ratio is $\nu = 0.33$. In the following assume that the through-thickness component of stress is zero.

(a) The cylinder is subjected to an internal gauge pressure of $p = 1500$ kPa.

(i) Calculate the components of stress in the longitudinal and hoop directions. [4]

(ii) Calculate the components of strain in the longitudinal, hoop and through-thickness directions. [6]

(b) In addition to the internal gauge pressure the cylinder is subjected to a torque T. The aluminium alloy obeys Tresca's yield criterion and has a yield stress in tension of 260 MPa. Calculate the maximum torque that can be applied before inelastic behaviour occurs for:

(i) an internal gauge pressure of $p = 1500$ kPa; [6]

(ii) an internal gauge pressure of $p = 750$ kPa. [4]

SECTION B

4 Figure 3 shows a portal frame carrying a vertical load V and a horizontal load H. The uniform fully plastic moment for the frame is M_p, except in the vertical column, between points D and E, where it is only $M_p/2$.

(a) Sketch *three* reasonable collapse mechanisms where plastic hinges occur at some subset of the five points marked A–E. For each mechanism, show clearly where plastic hinges have formed. [6]

(b) Find an upper-bound estimate of the collapse load for each of the three mechanisms sketched in (a). [8]

(c) Combine all your results obtained in (b) into a single interaction diagram. For a given horizontal load $H = 2\lambda M_p/L$ and vertical load $V = \lambda M_p/L$ use the interaction diagram to find the load factor λ at collapse. [6]

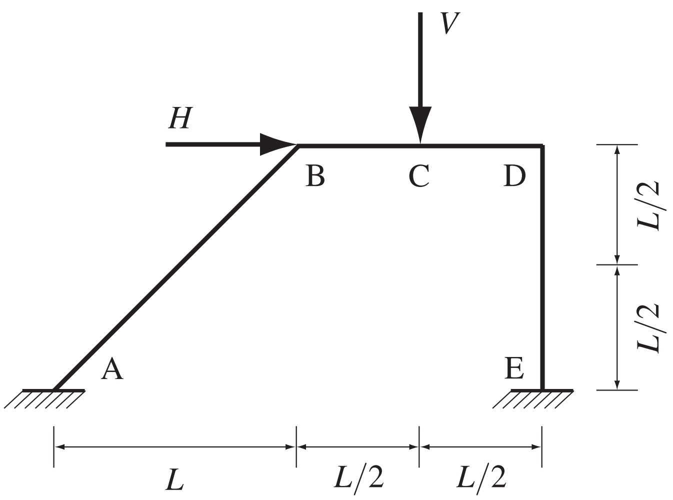


Fig. 3

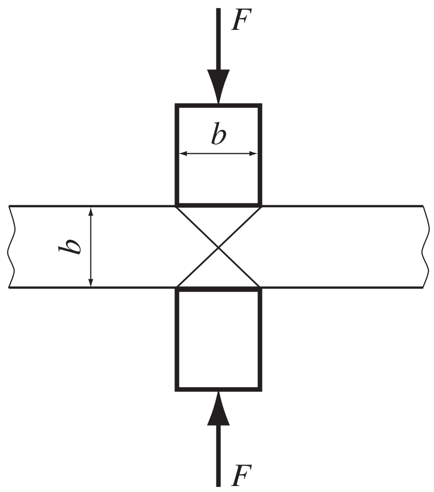
5 (a) To generate an upper bound for a collapse load, which of the three basic principles of structural analysis, namely equilibrium, compatibility and material law, are required? [3]

(b) In Fig. 4 the cross-sections of three symmetric metal forming problems are shown. In each of the three problems the material behaves as rigid-perfectly plastic and has a yield stress in shear of k. The thin lines in the cross-sections indicate the slip planes to be assumed in the calculations. All the contacting walls are smooth.

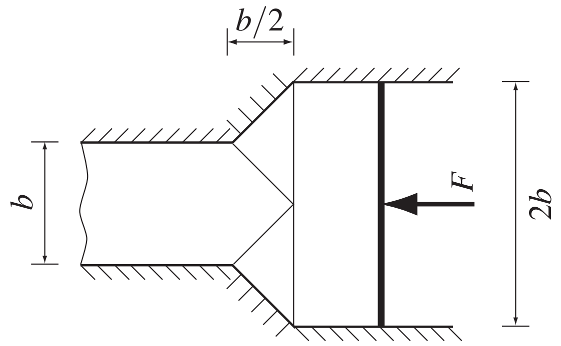
(i) Calculate the force per unit depth into the page F required for the indentation problem shown in Fig. 4(a). [5]

(ii) Calculate the force per unit depth into the page F required for the extrusion problem shown in Fig. 4(b). [6]

(iii) Calculate the force per unit depth into the page F as a function of the length L for the extrusion problem shown in Fig. 4(c). Find an optimal upper bound for the force F. [6]



(a)



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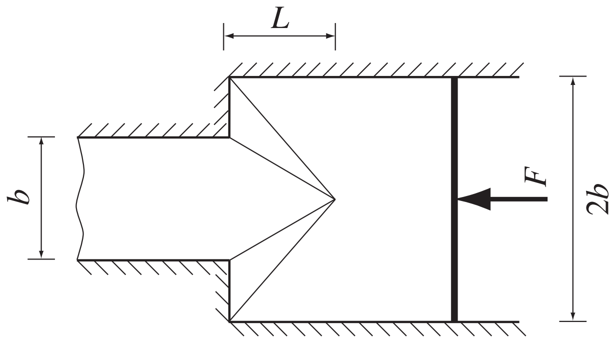


(c)

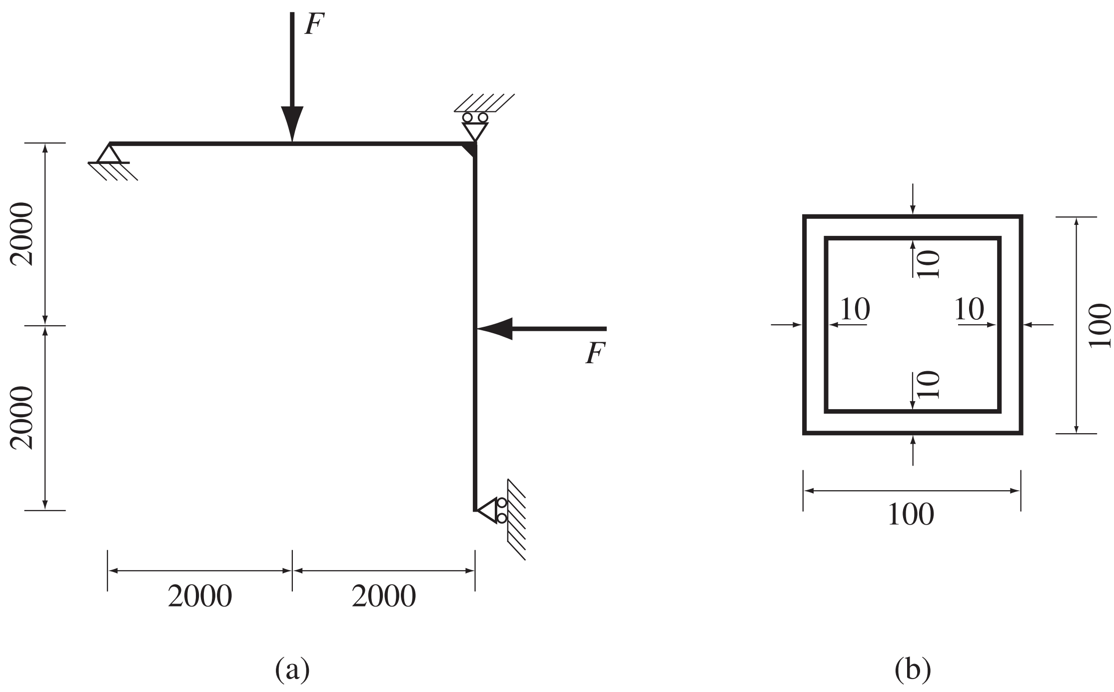
Fig. 4

6 Figure 5(a) shows a frame consisting of two beams that are rigidly connected to each other at the corner support. The two beams are each of 4 m length and their cross-section is shown in Fig. 5(b). The yield stress of the material can be assumed to be 355 MPa. As indicated in Fig. 5(a) the frame carries a horizontal and a vertical point load of the same magnitude. The frame is initially unstressed, and self-weight may be ignored.

(a) Calculate the elastic and the plastic section modulus of the cross-section shown in Fig. 5(b). [7]

(b) Perform an elastic analysis to find the magnitude of the point loads F when first yield occurs in the structure. [7]

(c) Perform a lower bound analysis to find the magnitude of the point loads F when collapse will occur. [6]



(Dimensions in mm. Not to scale)

Fig. 5

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