ENGINEERING TRIPOS PART IB

Thursday 6 June 2013 2 to 4

Paper 6

INFORMATION ENGINEERING

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig.1

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

Answer not more than ***two*** *questions from this section.*

1 (a) Briefly describe a procedure to measure experimentally the Bode diagram of a physical system. What conditions should the system satisfy if this procedure is to be carried out? [5]

(b) Figure 1 shows the Bode diagram of a stable linear system with transfer function of the form

$$G(s) = \frac{as(1+bs)}{(1+cs)(1+0.4(s/\omega_n)+(s/\omega_n)^2)}.$$

(i) Estimate the values of a, b, c and ω_n. [7]

(ii) The system is to be controlled in a unity gain negative feedback system with pre-compensator

$$K(s) = k\,\frac{s+1}{0.1s+1}.$$

For $k = 1$ sketch the Bode diagram of the compensated system $G(j\omega)K(j\omega)$ on the attached figure. [5]

(iii) Use your sketch to estimate the value of k for which the compensated system in part (b)(ii) has a phase margin of 45^{o}. [3]

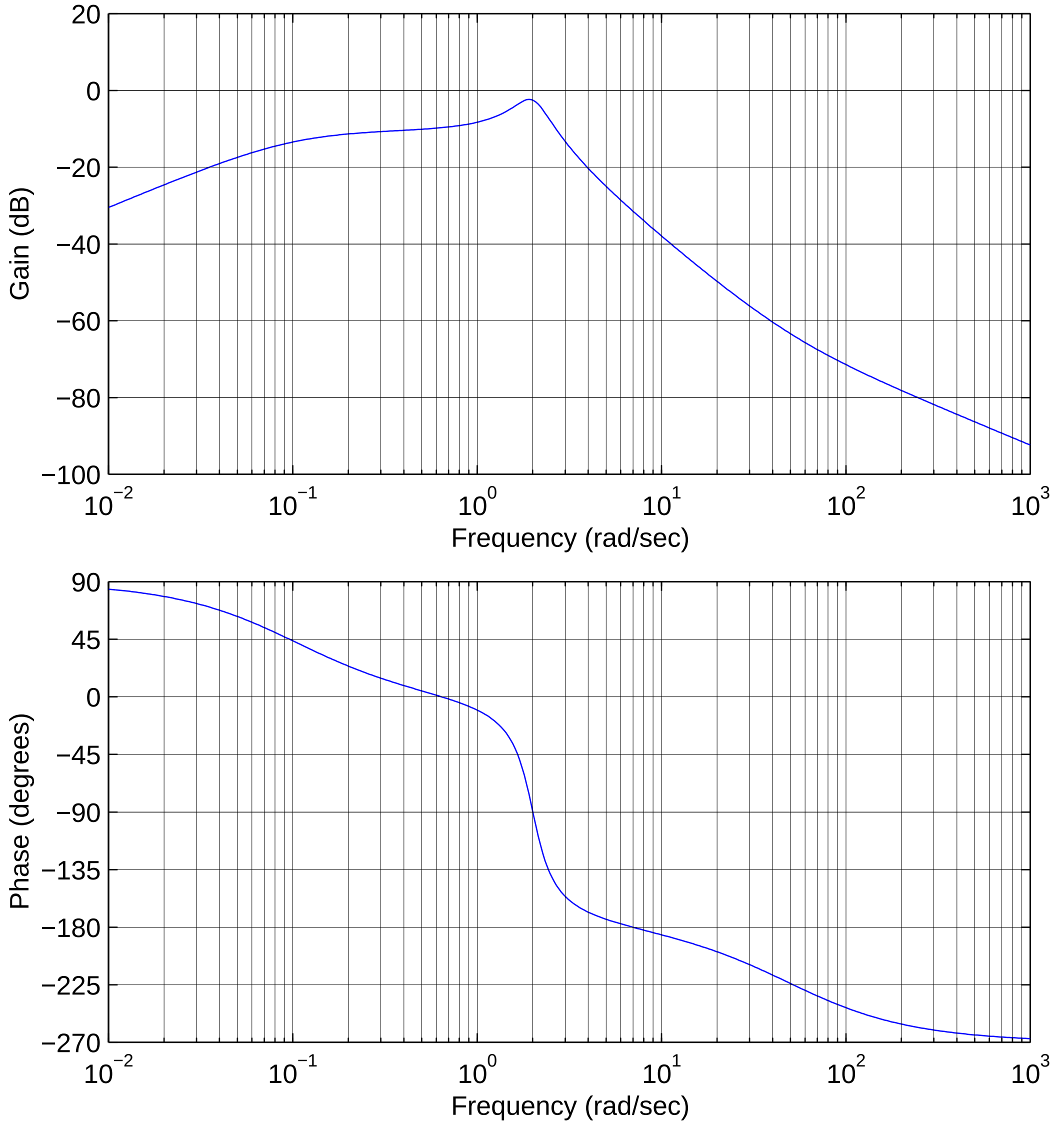


Fig. 1

Note: an additional copy of Fig. 1 is attached at the end of this paper. This should be annotated with your constructions and handed in with your answer to this question.

2 (a) State the Nyquist stability criterion. [3]

(b) (i) Show that the Laplace transfer function relating v_o to v_i for the operational amplifier circuit shown in Fig. 2 is given by

$$\frac{-K}{Ts+1}$$

where $K = R_1/R_2$ and $T = R_1C_1$. It may be assumed that all circuit elements behave in an ideal manner. [5]

(ii) Four identical circuits of the form shown in Fig. 2 are connected in series. It may be assumed that this cascade connection continues to behave in an ideal manner with transfer function equal to

$$\frac{K^4}{(Ts+1)^4}.$$

Sketch the Nyquist diagram of this transfer function. Calculate the intersection points of the Nyquist diagram with the real axis. [7]

(iii) The cascade circuit of part (b)(ii) is connected in a unity gain negative feedback loop. Show that the loop is stable providing

$$0 < K < \sqrt{2}.$$

Find the corresponding condition for a unity gain positive feedback loop. [5]

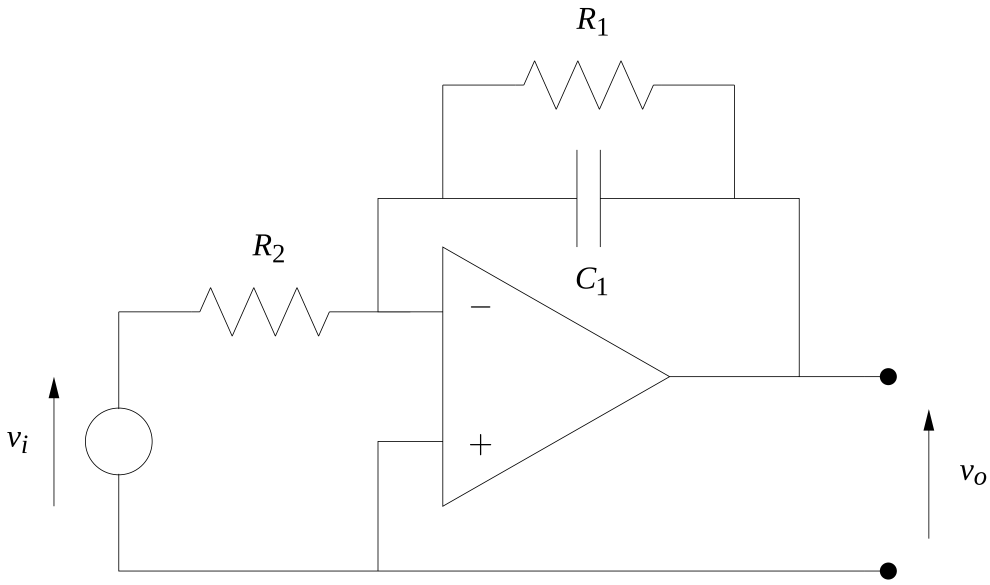


Fig. 2

3 A stirred tank with heater has inflow and outflow pipes with equal flow rates. The temperature of the liquid in the tank and outflow pipe is T and the temperature in the inflow pipe is T_i. The rate of supply of heat from the heater is denoted by Q_{in}. An energy balance per unit time for the system results in the equation

$$V\frac{dT}{dt} = F\left(T_i - T\right) + \frac{1}{\rho c_p} Q_{\text{in}}$$

where $V = 5\ \text{m}^3$ is the volume of liquid in the tank, $F = 0.1\ \text{m}^3\,\text{s}^{-1}$ is the flow rate, c_p is the specific heat capacity of the liquid and ρ is its density. A sensor measures the temperature at a point in the outflow pipe to provide the signal $y(t) = T(t-\tau)$ where $\tau = 2\,\text{s}$ is the "transport" delay. The heat supply is set by the feedback law

$$\frac{1}{\rho c_p} Q_{\text{in}}(t) = k_p\left(T_0(t) - y(t)\right)$$

where T_0 is the desired temperature and k_p is a proportional gain parameter.

(a) Sketch a block diagram of the system. [5]

(b) Calculate the closed-loop transfer-functions $H_1(s)$ and $H_2(s)$ for which

$$\overline{T}(s) = H_1(s)\overline{T}_0(s) + H_2(s)\overline{T}_i(s).$$

[5]

(c) Calculate and sketch the response of $T(t)$ to a step change of $1\,\text{K}$ in T_i, assuming that $k_p = 0$, i.e. the heater is switched off. [5]

(d) It is necessary to select k_p so that the closed-loop system is stable. It may be assumed that $k_p = 0.05$ is such a choice. If T_i is a sine wave of amplitude $1\,\text{K}$ at a frequency of $1\ \text{rad}\,\text{s}^{-1}$, what is the steady-state amplitude of oscillation in T? [5]

SECTION B

*Answer not more than **two** questions from this section.*

4 (a) The Fourier transforms of $f(t)$ and $g(t)$ are $F(\omega)$ and $G(\omega)$ respectively.

(i) If $h(t)$ is the convolution of $f(t)$ and $g(t)$, i.e. $h(t) = f(t) * g(t)$, prove that the Fourier transform of $h(t)$, which we write as $H(\omega)$, is given by

$$H(\omega) = F(\omega)G(\omega)$$

[3]

(ii) If the *cross-correlation*, $R_{fg}(t)$, of two real functions $f(t)$ and $g(t)$ is defined by

$$R_{fg}(t) = \int_{-\infty}^{\infty} f(\tau)g(\tau+t)d\tau$$

give an expression for the Fourier transform of $R_{fg}(t)$ in terms of $F(\omega)$ and $G(\omega)$. [2]

(iii) Describe how $f * g$ is related to $g * f$, and how R_{fg} is related to R_{gf}. [2]

(b) The function $p(t)$ has Fourier transform $q(\omega)$.

(i) Show that $q(t)$ has a Fourier transform given by $2\pi p(-\omega)$. This is the *duality* property. [4]

(ii) Using duality or otherwise, find the Fourier transform of $\operatorname{sinc}(t)$. [4]

(iii) Using Parseval's Theorem and the result in part (b)(ii), show that

$$\int_{-\infty}^{\infty} \operatorname{sinc}^2 t \, dt = \pi$$

[5]

5 (a) If a continuous time signal $y(t)$ is sampled at a frequency f_s, which is greater than twice the maximum frequency component, f_{max}, in $y(t)$, describe how we can recover the original signal from the sampled signal, $y_s(t)$, via a *reconstruction filter* with impulse response, $h_r(t)$. The explicit form of $h_r(t)$ should be given. [6]

(b) The signal

$$y(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

is sampled at 0.25s intervals starting at $t = 0$. If we take 8 samples, y_n, for $n = 0,..,7$, we can form 8 discrete Fourier transform (DFT) coefficients, Y_k, for $k = 0,..,7$. Find the first 3 DFT coefficients, verifying that $Y_2 = 0$. By considering the continuous Fourier transform of $y(t)$, comment on whether you expect Y_2 to be zero. [8]

(c) A digitiser has two analogue input channels each able to receive input signals of bandwidth up to 20 kHz. After sampling and quantising the input signals, the digitiser combines them into a joint data stream which is transmitted at a rate of 1.5 $\text{Mbit}\,\text{s}^{-1}$. Estimate how many bits per sample are available if the input signals are sampled so as to avoid aliasing. If the input signals can be approximated by sinusoidal components, determine the maximum signal to quantisation noise power ratio that can be achieved on each channel, stating any assumptions that are made. [6]

6 (a) Modulation techniques which can accommodate multiple users in a communications channel are termed *Multiple Access* methods. Describe and compare the three main multiple access techniques. [5]

(b) A channel with additive, white Gaussian noise (with a power spectral density given by N_0) is to have total bandwidth B and a capacity of C. What is the total transmitted power P? [3]

(c) Assuming no overlap nor guard bands, give expressions for the channel capacity per user for the FDMA and TDMA multiple access techniques in terms of total bandwidth, noise characteristics, total power transmitted and number of users. [3]

(d) A wireless channel extends from 50 MHz to 250 MHz and we wish to use FDMA to transmit binary data from M users at a rate of 50 kbit s^{-1} per user.

(i) If the data from each user is to be modulated using BPSK with user i having a carrier frequency of f_c^i, determine and sketch the BPSK power spectrum. [5]

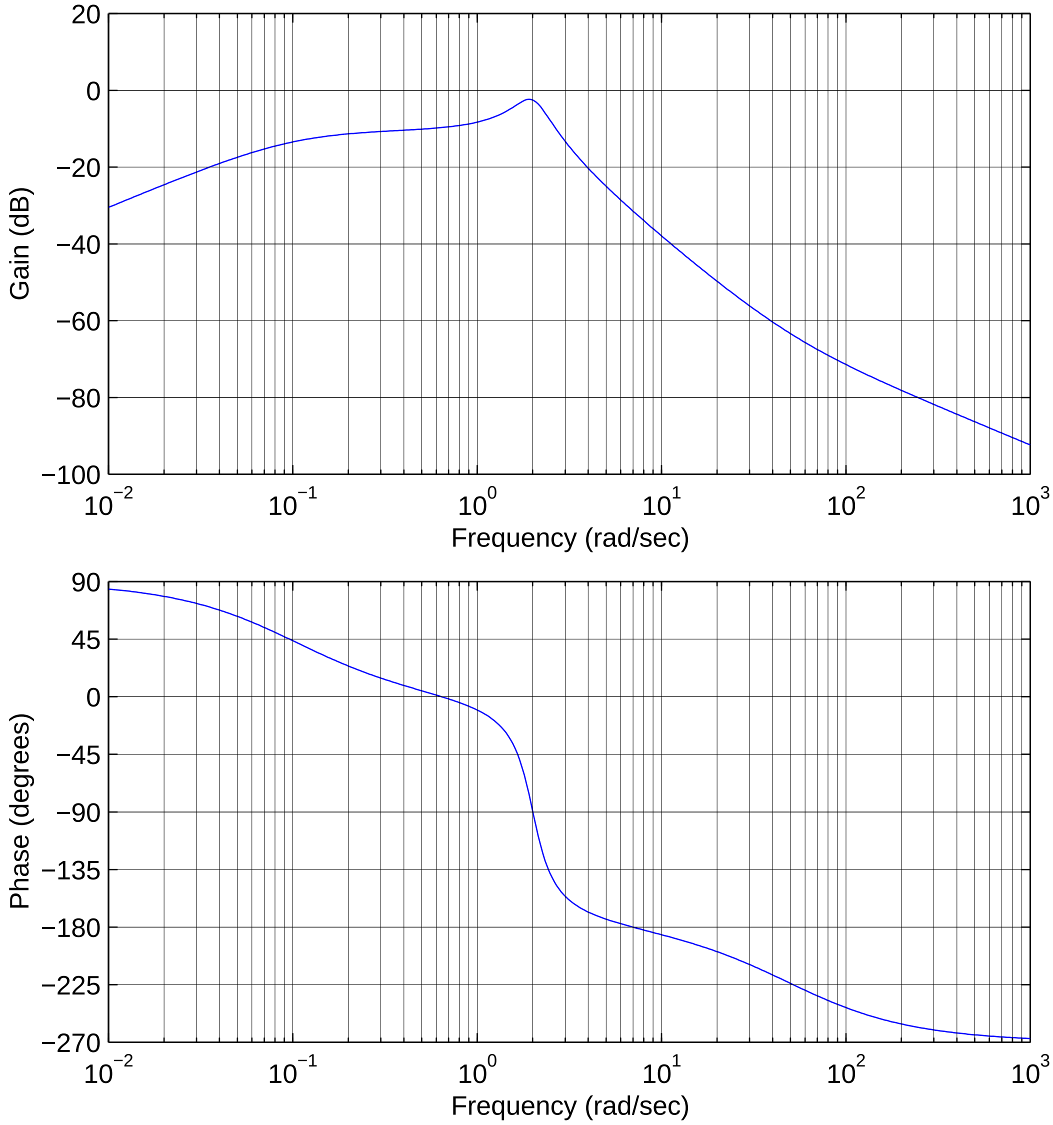
(ii) Using the result in part (d)(i), find the maximum value of M assuming that any overlaps between the spectra of adjacent users are to occur beyond the first sidelobe. State clearly any assumptions made. [4]

END OF PAPER

Candidate Number:

ENGINEERING TRIPOS PART IB

Thursday 6 June 2013, Paper 6, Question 1.



Copy of Fig. 1. This should be annotated with your constructions and handed in with your answer to question 1.