ENGINEERING TRIPOS PART IB

Friday 7 June 2013 9 to 11.30
9 to 10.30 Foreign Language Option

Paper 8

SELECTED TOPICS

Answer ***one*** *question from Section A. In addition:*

> *If you are not taking the Foreign Language option, answer* ***four*** *questions, taken from only two of Section B – H. Not more than* ***two*** *questions from each section many be answered.*
> *If you are taking the Foreign Language option, answer* ***two*** *questions from* ***one*** *of Sections B – H.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Data Sheet for Section B (6 pages)
Data Sheet for Section E (2 pages)

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
CUED approved calculator allowed	

You may not start to read questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

V4

SECTION A *Introductory Business Economics*

Answer not more than **one** *question from this section.*

1 (a) Using an appropriate diagram or diagrams, explain how the profit maximising equilibrium levels of output and price differ in an industry which is a monopoly compared to one which is perfectly competitive. [5]

(b) Why are consumers considered to be worse off when supplied by a monopoly industry rather than by a perfectly competitive industry? [5]

(c) In what ways does the life cycle model of consumption differ from the Keynesian consumption function model? [10]

2 (a) Illustrate the Sweezy oligopoly model. [5]

(b) What is a perfectly competitive market? [6]

(c) What impact would the following have on the level of investment in the macroeconomy:

(i) a fall in the rate of interest; [3]

(ii) reduced expectations of future growth; [3]

(iii) a fall in business confidence? [3]

SECTION B *Civil and Structural Engineering.*

Answer not more than **two** *questions from this section.*

Note Data Sheets at end of paper.

3 A client is considering the construction of an underground railway tunnel in a congested city. The typical tunnel depth will be 20 m, but the ground conditions at this depth are highly variable, including soft clays with an undrained shear strength of 30 kPa, stiff clays with an undrained shear strength of 150 kPa and sands. The water table is close to the ground surface. Both soft and stiff clays can be assumed to have a unit weight of 16 kN m^{-3}.

(a) Define stability ratio as a means of determining the stability of a tunnel being constructed in clay, in terms of the depth of the tunnel axis below ground level, the support pressure applied to the face of the tunnel and the shear strength and unit weight of the clay. [5]

(b) What construction techniques would be suitable in the soft and stiff clay ground conditions? [3]

(c) What are the key engineering issues relating to tunnel construction in sands? What construction techniques might be appropriate in these ground conditions? [4]

(d) Explain the significance of ground movements caused by bored tunnelling. What kind of buildings are most susceptible and why? What can be done to minimise building damage due to tunnelling? [5]

(e) What instrumentation might be specified to control tunnel construction in urban areas? [3]

4 A 10 m deep excavation is to be constructed for a cut and cover tunnel supported by a stiff retaining wall that penetrates 5 m into the underlying stiff clay. During construction, the retaining wall is supported by an anchor attached to a 4 m deep wall, as shown in Fig. 1; this provides resistance from the passive pressure that it can generate. The ground conditions comprise 4 m of granular fill overlying a thick layer of stiff clay. The water table is at 20 m below the ground surface.

Both the granular fill and clay have a unit weight of 18 kN m^{-3}. The granular fill has a critical state friction angle of 30° whereas the stiff clay has an undrained shear strength of 100 kPa and a critical state friction angle of 25°.

(a) Assuming that the anchor does not move and using undrained conditions in the clay, calculate the factor of safety for the wall. [6]

(b) Assuming that the anchor does not move and using drained conditions in the clay, calculate the factor of safety for the wall. [6]

(c) Assuming the anchor-force to be 400 kN, is the anchor wall adequate? [4]

(d) Given the bending moment at the excavation level under drained conditions is 620 kNm m^{-1} and assuming a wall thickness of 400 mm, determine appropriate axial reinforcement for a singly-reinforced wall. Take f_y = 460 N mm^{-2} and f_{cu} = 30 N mm^{-2}. [4]

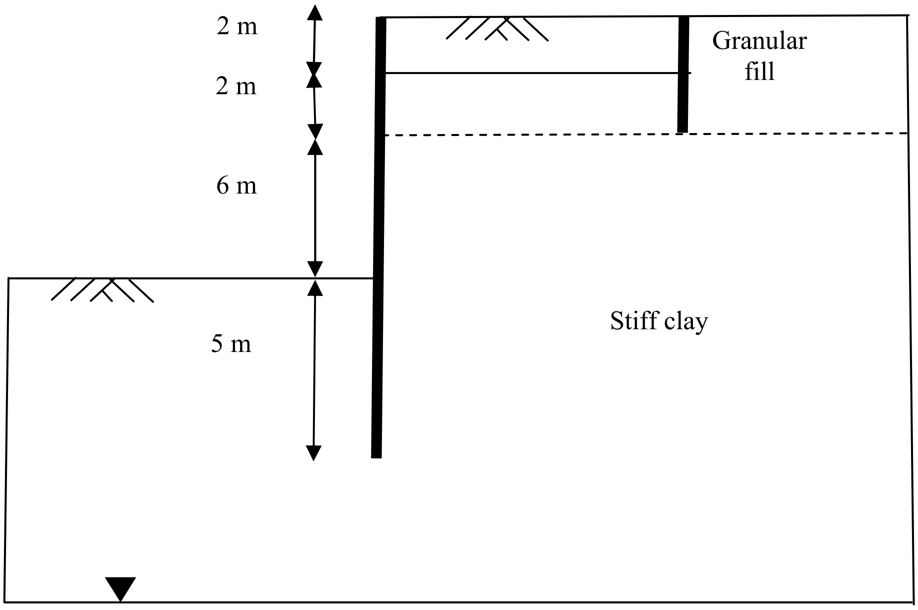


Fig. 1

5 A cut and cover road tunnel is to be constructed consisting of a reinforced concrete box, as shown in Fig. 2. The base slab of the tunnel is subjected to an upwards hydrostatic pressure with a design value of 100 kPa, taking into account appropriate partial factors on load and the slab self-weight, which is resisted by tension piles attached to the tunnel at A and B.

The degree of fixity at the corners of the tunnel cannot be guaranteed, so the designer must ensure that the base slab is adequate if there is full fixity at A and B or if it is simply supported at these points.

(a) Draw the bending moment diagrams for the base slab for conditions of both full-fixity at A and B and the simply supported case. Determine the design values of bending moment at A and at C (at midspan). [6]

(b) Determine the minimum effective depth of the slab if it is to be singly reinforced. Take $f_y = 460$ N mm^{-2} and $f_{cu} = 30$ N mm^{-2}. [4]

(c) On the assumption that the slab provided is 800 mm thick, design suitable reinforcement at A and C. [4]

(d) Suggest a reinforcement layout for the slab and for the junctions with the walls of the tunnel. [6]

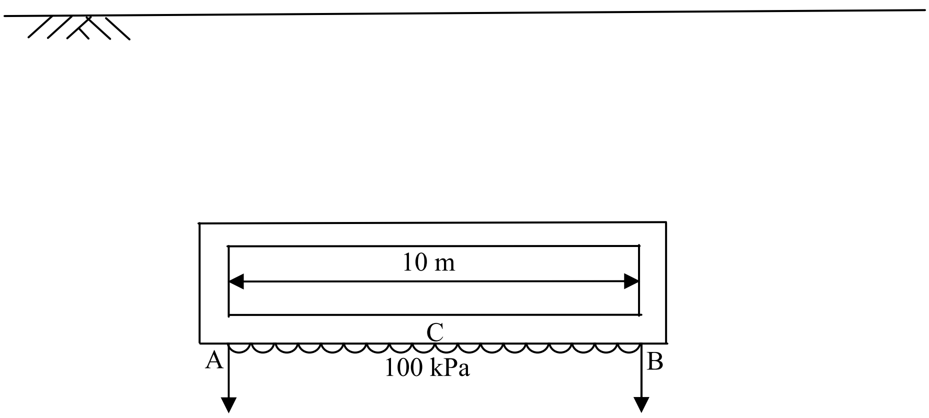


Fig. 2

SECTION C *Mechanics, Materials and Design*

Answer not more than ***two*** *questions from this section*

6 A horizontal-axis wind turbine of swept area A is operating in undisturbed air of density ρ moving at a uniform speed V. The Betz limit for such a turbine is 59%.

(a) Derive the Betz limit from first principles, including a clear diagram showing the ideal flow around the wind turbine. Identify on the diagram the control volume for which you consider continuity, energy and momentum flux. [10]

(b) Find an expression for the horizontal force acting on the tower when the turbine is operating at the Betz limit. You may assume that the speed of air passing through the turbine drops from V to $V/3$. [4]

(c) Define the terms “axial induction factor” and “angular induction factor” and explain with sketches why wake rotation must be included in a detailed analysis of a wind turbine. [6]

7. A wind turbine blade of length L is illustrated in Fig. 3. The spar depth $2d$ and cross-sectional area of the spar skins A vary linearly such that

$$\frac{d}{d_0} = \frac{A}{A_0} = \frac{x}{L}$$

where x is the distance from the blade tip and d_0 and A_0 are the values of d and A at the blade root. The thickness of the spar skins may be assumed to be much less than the depth $2d$.

(a) (i) Under the design load case, the transverse wind load per unit length w is also assumed to vary linearly with x, from zero at the blade tip. Show that the spar bending moment $M(x)$ is given by

$$M(x) = \frac{WL}{3}\left(\frac{x}{L}\right)^3$$

where W is the total wind load on the spar. [3]

(ii) Show that the second moment of area $I(x)$ of the spar section is

$$I(x) = A_0 d_0^2 \left(\frac{x}{L}\right)^3$$

Hence find the maximum stress in the outer surface of the skin. [3]

(iii) The spar is to be manufactured from a material of Young's modulus E, density ρ and failure stress σ_f. Derive an expression for the mass of the spar required to avoid failure, in terms of W, L, d_0 and the relevant material properties. [3]

(b) (i) By considering the differential equation governing beam bending, show that the spar tip deflection δ is given by

$$\delta = \frac{WL^3}{6EA_0 d_0^2}$$

[5]

(ii) The spar stiffness is governed by a maximum allowable tip deflection. Derive an expression for the mass of a spar of specified stiffness and hence find a criterion that identifies whether the design is stiffness or strength limited.

Consider this criterion for two identical blade designs, one manufactured from GFRP and the other from laminated bamboo, and comment on why one material might be favoured over the other. You may use the material properties given in Table 1. [6]

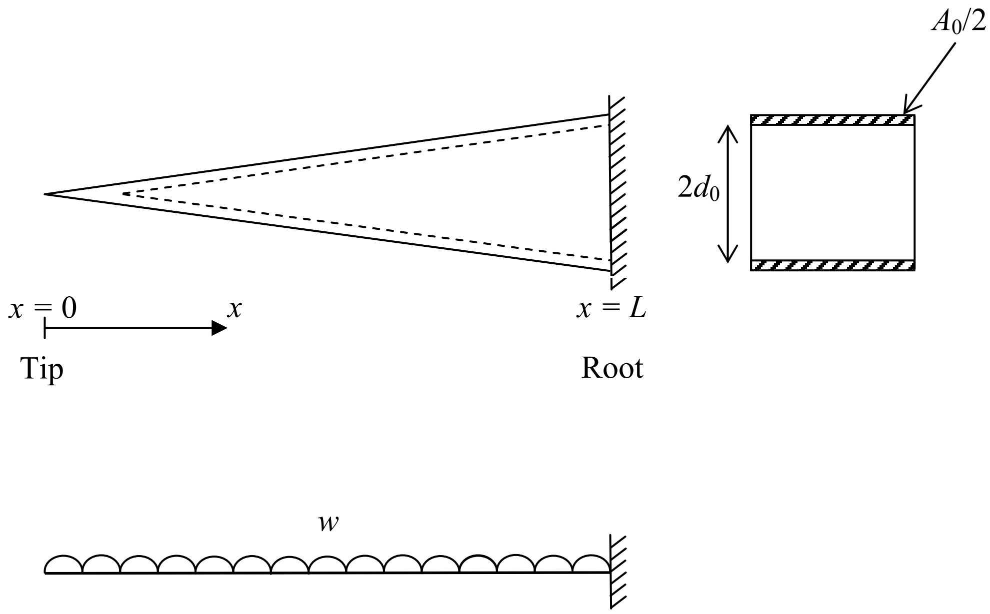


Fig. 3

	E [GPa]	σ_f [MPa]	ρ [kg m^{-3}]
GFRP	28	56	1800
Bamboo	18	36	700

Table 1

8 A 2.5 MW horizontal-axis wind turbine generator produces rated output at its rated wind speed of 12 m s^{-1}. At rated wind speed the system operates at its optimum tip-speed ratio of 8, at which the power coefficient is 0.38. The turbine drives a three-phase, star-connected, 12 pole cage rotor induction generator which has its stator windings connected directly to the 50 Hz, three-phase 6.6 kV grid via a gearbox. Take the density of air to be 1.23 kg m^{-3}.

(a) Find the turbine diameter and its angular speed at rated wind speed. [2]

(b) Find the gearbox ratio such that the system operates at its optimum tip-speed ratio at the most probable wind speed of 6 m s^{-1}. Comment on the number of gearbox stages that would be needed to achieve this gear ratio. Also determine the turbine torque and the induction generator torque at this wind speed. [8]

(c) The cage rotor induction generator of part (b) has the following equivalent circuit parameters: $R_1 = R_2' = 1\ \Omega$; $X_1 = X_2' = 2\ \Omega$; X_m and R_0 are large enough to be ignored. Under the conditions of (b), find the slip at which the induction generator operates, and find its input current. [6]

(d) Explain the benefits of variable-speed operation of wind turbine-generator systems and outline the principles of the doubly-fed induction generator in allowing variable-speed operation. [4]

SECTION D *Aerothermal Engineering*

Answer not more than **two** *questions from this section.*

9 (a) Define the stage loading (or work) coefficient ψ and the flow coefficient ϕ for compressors and turbines. Explain why ψ may be as high as 2.0 for turbines but is usually limited to about 0.4 for compressors. What are the implications for an aircraft engine of exceeding these values in each case? [5]

(b) Give a qualitative explanation of how flow turning relates to the pressure rise or pressure fall in axial turbomachines. Give typical values of flow turning for compressor blades and for turbine blades and sketch typical blade shapes in each case. [5]

(c) A multistage axial air compressor is to be designed to give a stagnation pressure ratio of 10. The mean blade radius is constant at 0.45 m for all blade rows and the rotor speed is to be 6000 rpm for an inlet stagnation temperature of 288 K. Assuming that the work per stage is the same for all stages and that the overall isentropic efficiency is 90%, determine the number of stages required if ψ is not to exceed 0.4 . [5]

(d) If the rotor blade height (from hub to tip) for the first stage is 15 cm, *estimate* the stator blade height for the last stage, assuming the static density ratio is approximately equal to the density ratio based on stagnation conditions and that ϕ remains constant. [3]

(e) The compressor is to be driven by a single-stage turbine. Determine the mean radius of the turbine rotor blades if the stage loading coefficient is not to exceed 2.0. [2]

Take $\gamma = 1.4$ and $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ for air.

10 (a) Give the definitions of stagnation temperature and stagnation pressure for the compressible flow of air. Determine values for these quantities relative to an aircraft that is cruising at Mach 0.85 at an altitude of 10 km. Assume a standard atmosphere, as given in Table 2. [4]

(b) When the propelling nozzles of a turbo-fan engine are choked, as is normally the case, the total (bypass plus core) air mass flow rate through the engine may be expressed as,

$$\dot{m}_a = \mathrm{f}\left(p_{02}, T_{02}, T_{04}, c_p, A_N\right)$$

where p_{02} and T_{02} are the engine inlet stagnation pressure and stagnation temperature respectively, T_{04} is the post-combustion stagnation temperature, c_p is the isobaric specific heat capacity of air and A_N is the total cross-sectional area of the propelling nozzles. Find two dimensionless groups from these variables. [4]

(c) Stationary tests are conducted on a new turbo-fan engine at sea level. The total (bypass plus core) propelling nozzle area is 2.5 m^2. At the optimum dimensionless operating point (for which the nozzles are choked) the measured thrust is 250 kN and the air mass flow rate is 800 kg s^{-1}. Calculate the net thrust that would be developed at an altitude of 10 km and a flight Mach number of 0.85 if the engine is at the optimum operating point. You may assume that the dimensionless thrust, $\left(F_G + p_a A_N\right)/\left(p_{02} A_N\right)$, remains constant, where F_G is the gross thrust and p_a is the ambient pressure. [7]

(d) The thrust specific fuel consumption (SFC) measured during the ground tests is 0.3 kg h^{-1} kg^{-1}. By considering how the fuel mass flow rate relates to the variables in part (b), estimate the SFC at the flight condition of part (c). [5]

Altitude (km)	**Ambient pressure (bar)**	**Ambient Temperature (K)**
0 (sea level)	1.013	288.15
10	0.260	222.85

Table 2: Standard atmospheric pressure and temperature.

Take $\gamma = 1.4$ and $R = 287$ J kg^{-1} K^{-1} for air.

11 (a) Define the terms *thermal efficiency* and *propulsive efficiency*. Show that these efficiencies are related to the thrust specific fuel consumption, SFC, by

$$\mathrm{SFC} = \frac{V}{\eta_{th}\eta_p \mathrm{LCV}}$$

where V is the flight velocity, η_{th} and η_p are the thermal and propulsive efficiencies respectively and LCV is the lower calorific value of the fuel. [4]

(b) A four-engine plane is cruising at Mach 0.85 at an altitude of 9.5 km, where the pressure and temperature are 0.287 bar and 226.7 K. The engine jet velocity, V_j, is 400 ms^{-1}. Stating your assumptions, estimate the propulsive efficiency. [4]

(c) The plane suffers a complete failure of one of its engines. Explain why it is desirable to move to a lower altitude. [1]

(d) Following the engine failure, the plane descends to an altitude where the pressure and temperature are 0.400 bar and 241.5 K. Explain why it is desirable to keep the lift coefficient approximately constant and compute the new flight velocity on this basis. [3]

(e) At the new flight condition, the mass flow of air into each of the remaining engines is increased by 21.2% but the thermal efficiency of these engines is unchanged. Making appropriate assumptions, estimate the percentage reduction in the remaining range of the aircraft. You may use without proof the Breguet range equation,

$$s = \frac{V(L/D)}{g\,\mathrm{SFC}} \ln(W_1/W_2)$$

where s is the distance covered whilst the aircraft total weight reduces from W_1 to W_2, L/D is the aircraft lift-to-drag ratio and g is the acceleration due to gravity. [8]

Take $\gamma = 1.4$ and $R = 287$ J kg^{-1} K^{-1} for air.

SECTION E *Electrical Engineering*

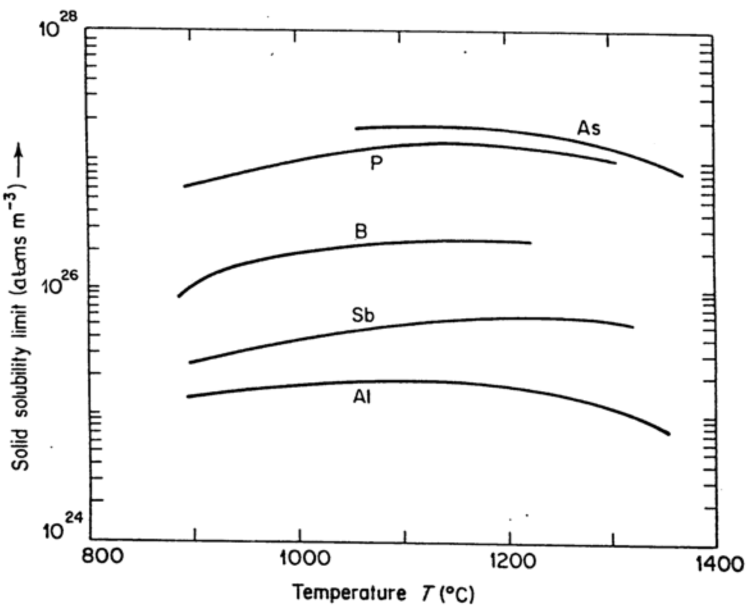
Answer not more than **two** *questions from this section.*

Note the Data Sheet at end of paper.

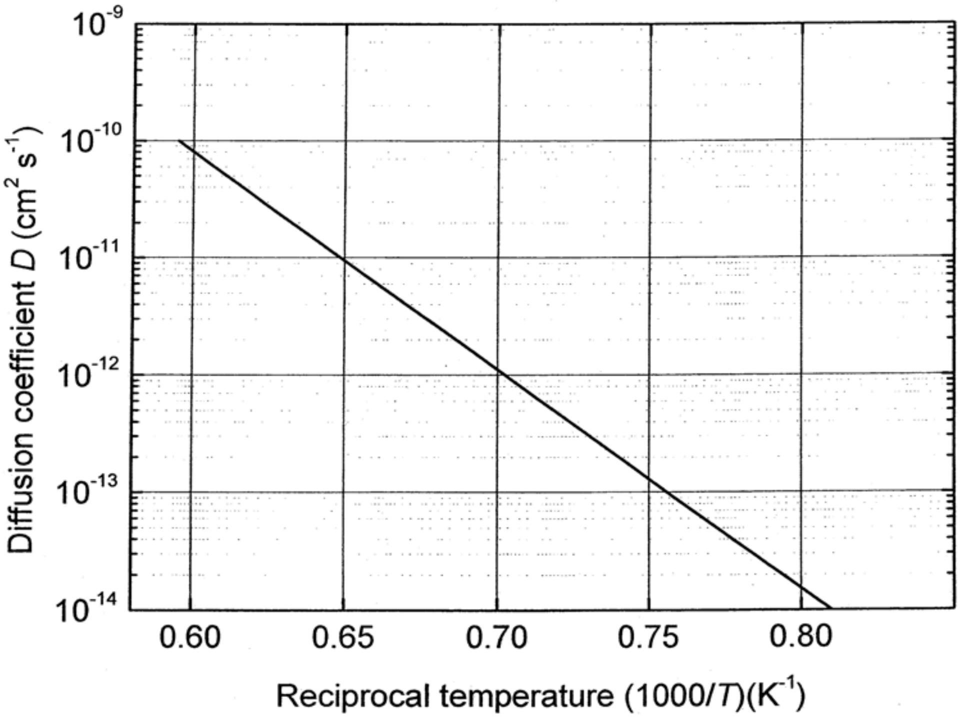
12 (a) When a silicon dioxide layer of thickness d is grown by thermal oxidation of a silicon wafer, what is the thickness of the silicon which is consumed? Assume that the molecular weight of silicon is 29 g mol^{-1}, the density is 2.33 g cm^{-3} and the molecular weight of SiO_2 is 60.00 g mol^{-1} and its density is 2.20 g cm^{-3}. [5]

(b) Compare and contrast the "wet" and the "dry" methods of silicon dioxide growth and suggest two alternative oxides that can be used in the manufacture of silicon- based devices as dimensions continue to shrink. [6]

(c) With the aid of Figs. 4 and 5 provided, determine whether a successful p-type pocket isolation diffusion is obtained by a phosphorus diffusion from an infinite source through a 4 μm thick epitaxial p-type silicon layer. The p-type layer contains 4×10^{15} cm^{-3} acceptor impurities and is grown onto a n-type substrate. The diffusion takes place at 1100°C for 45 minutes. State any assumptions made. [9]



The solid solubility versus temperature data for a range of atoms in silicon
Fig. 4



Diffusion coefficient for phosphorus in Si vs reciprocal temperature

Fig. 5

13 A MOSFET with a transit time of 20 ps is to be designed using a 0.3 μm thick layer of n-type semiconductor on an insulating substrate. The supply voltage will be 1 V. The semiconductor has a conductivity of 32 $\Omega^{-1}m^{-1}$, and mobility of 0.5 $m^2V^{-1}s^{-1}$.

(a) Determine the donor density in the semiconductor. [2]

(b) Determine the electric field and source-drain separation for the above transit time. [5]

(c) The ratio of the width to length of the MOSFET channel is 20. Determine the source current which flows for a gate-source voltage of 0 V. [6]

(d) Determine the gate voltage required to turn off the transistor, assuming the dielectric constant of the semiconductor is 15. [7]

14 (a) Copper has a face-centred cubic structure lattice, with a lattice constant of 3.61 Angstroms and 4 atoms per unit cell. It has effectively 1 valence electron per atom and its conductivity is 6×10^{7} $\Omega^{-1}m^{-1}$. Calculate the valence electron density and mobility. If a semiconductor has the same mobility as copper, what is its conductivity if it is doped to a donor concentration of 4×10^{21} m^{-3}. [6]

(b) Sketch the velocity-field diagrams of electrons in Si and GaAs. Explain the meaning and cause of the scattering limited velocity. If the scattering limited velocity of the doped semiconductor in part (a) is 5×10^{4} ms^{-1}, what is the relevant phonon energy in electron volts if the free electron mass is 9.1×10^{-31} kg ? [6]

(c) If the doped semiconductor in part (a) is used to make an FET operating at 2 V, at what length does the conduction become limited by the scattering limited velocity? What electron transit time does this correspond to? [8]

SECTION F *Information Engineering*

Answer not more than **two** *questions from this section.*

15 Consider a model of a binary (black and white) 10×10 pixel images where each pixel is black with probability θ and all pixels are independent.

(a) Assuming $\theta = 0.4$, what is the most probable image under this model and what is its probability? [4]

(b) Derive the maximum likelihood estimate for θ given a data set D consisting of two images with 60 and 70 black pixels, respectively. Show your derivation. [8]

(c) You wish to compare two models, M_1 and M_2, where under model M_1 the value of $\theta = 0.5$, and under M_2 the value of θ is unknown but given a uniform prior on the interval [0,1]. For the data set D in (b) above, and the priors $P(M_1) = 0.99$ and $P(M_2) = 0.01$, compute the posterior probability $p(M_2 \mid D)$. You may use the fact that $\int_0^1 x^{a-1}(1-x)^{b-1}\,dx = \Gamma(a)\,\Gamma(b)/\Gamma(a+b)$, and that $\Gamma(a) = (a-1)!$.

[8]

16 It is desired to use a photo-editor to correct the defects in a digital photograph of size 1024 by 768 pixels with the following defects:

- The main subject (a person) is too small, by a factor of 0.75, and is offset 80 pixels to the left of centre.

- The illumination of the image is from a conventional filament light-bulb whose red, green and blue intensities are scaled by 1.5, 1.0 and 0.5 respectively, relative to a pure white light source.

- The image is blurred by poor focussing such that frequency components at one quarter of the pixel sampling frequency, in either the horizontal or vertical directions, have been attenuated (reduced in gain) by a factor of 0.7, while very low frequency components are unaffected by the blur.

(a) Briefly describe what operations would need to be performed on the image in order to create a corrected image, of the same overall size (in pixels) as the input image and with all three types of defect eliminated as far as possible. [10]

(b) For each of the three correction processes for the above image, give mathematical and algorithmic descriptions of the techniques used, and discuss any design tradeoffs that might be needed. [10]

You should assume that any image filters are either Gaussian in form or are simple linear combinations of Gaussian filters, and that the Fourier transform for a Gaussian is given in the data book.

$$\frac{1}{b\sqrt{2\pi}}\exp\left(\frac{-t^2}{2b^2}\right) \Leftrightarrow \exp\left(\frac{-\omega^2 b^2}{2}\right)$$

17 Consider an algorithm to detect interest points (features of interest) in a 2-D image for use in matching.

(a) The image is first smoothed with a low-pass filter before image gradients are computed. What smoothing filter is used in practice? Give an expression for computing the intensity of a smoothed pixel in terms of two discrete 1-D convolutions. [2]

(b) Show how different resolutions of the image can be represented efficiently in an *image pyramid*. Your answer should include details of the implementation of smoothing within an octave and subsampling of the image between octaves. [4]

(c) Show how image features such as *blob-like* shapes can be localized in both position and scale by computing the Laplacian of the smoothed images over multiple scales. [3]

(d) Explain why this can be considered to be *band-pass* filtering. How can *band-pass* filtering at different scales be implemented efficiently using the image pyramid? [3]

(e) How can the orientation of the image feature be assigned from image gradients in the neighbourhood of the feature? [2]

(f) Hence show how the neighbourhood of each image feature can be normalised to a 16×16 patch of pixels and give details of a suitable descriptor for use with matching image features in different images. [3]

(g) How are these descriptors used to find correspondences over different images? [3]

SECTION G *Engineering for Life Sciences*

Answer not more than **two** *questions from this section.*

18. One third of the total focussing power of the eye comes from the crystalline lens.

(a) Describe the composition and structure of the crystalline lens. [4]

(b) Explain the process of lens accommodation and how it changes with ageing. [5]

(c) A young lens with a small-strain shear modulus G = 60 Pa is indented to a depth $d = 100\mu m$ with an indenter of radius R = 4 mm. Assuming incompressibility, the load is given by $P = 8RdG$. Recall that indentation strain $\varepsilon = 0.2 \times (d/R)^{0.5}$.

(i) Calculate the peak force for this indentation.

(ii) Calculate the peak force for an indentation to depth $d = 200\mu m$.

(iii) Calculate the peak force, with the original indentation to a depth $d = 100\mu m$, for an aged human lens. [6]

(d) Describe cataracts and discuss typical cataract treatment. [5]

19 (a) Scanning laser ophthalmoscopy and time-domain optical coherence tomography can both be used for *depth-sectioning* of the eye. What is *depth-sectioning*, and how is this achieved for both of these imaging modalities? [6]

(b) A 2D cross-sectional image of the eye is created with the horizontal image axis x laterally across the eye and the vertical image axis y representing depth into the eye. Explain what determines the image resolution in both the x and y directions, giving typical values for these resolutions, for each of the following imaging modalities:

(i) scanning laser ophthalmoscopy; [2]

(ii) time-domain optical coherence tomography; [2]

(iii) ultrasound imaging. [2]

(c) An ultrasound imaging system employs a simple acoustic pulse with duration $2t_p$ and shape $P(t)$ given by:

$$P(t) = \begin{cases} +1 & -t_p < t < 0 \\ -1 & 0 < t < t_p \\ 0 & \text{otherwise} \end{cases}$$

(i) Calculate and sketch the frequency spectrum of *P(t)* for $t_p = 0.05\mu s$. [6]

(ii) How would this pulse spectrum be modified after the pulse has passed through several centimetres of tissue at the back of the eye? [2]

20 (a) Describe the receptive field properties of retinal ganglion cells and how the receptive fields are constructed from the photoreceptors. Why might such receptive field properties be computationally useful? [3]

(b) The black text on a newspaper in bright sunlight reflects as much light as the white paper inside a normal room, yet they look quite different shades. What mechanisms are responsible for this perceptual difference? [2]

(c) If you stare at a green apple and yellow banana for some time and then stare at a white wall, what colours might you see in the afterimage and why? [2]

(d) Sketch a contrast sensitivity function. Use the contrast sensitivity function to explain how you would produce a poster with text that could only be read from far away but not when a person was close to it. [3]

(e) The eyes of an ancient animal are reconstructed by palaeontologists. The reconstruction suggests a simple eye with a convex lens whose two symmetrical surfaces are spherical, and the diameter of the lens is 1 cm when viewed from the front. We also know that the retina was 1 cm away from the midline of the lens.

(i) Following simple geometric arguments, calculate the minimal radius of curvature of the lens. [1]

(ii) Assume that the material of the lens was homogeneous, and its refractive index (r_{lens}) could not be higher than 1.5. Explain, with reasons, whether it is likely that the animal was aquatic, terrestrial or either. Support your answer with calculations regarding the minimal object distance the eyes could bring into focus. Use the following physical constants in your calculations: $r_{\text{air}} = 1$, $r_{\text{water}} = 1.3$. [6]

(iii) How does your answer to the previous question change if it is possible that the lens was inhomogeneous? [2]

(iv) Besides broadening the range of possible focal lengths, what other advantage does an inhomogeneous lens confer? [1]

SECTION H *Manufacturing, Management and Design*

Answer not more than **two** *questions from this section.*

21 (a) Describe the advantages and disadvantages of each of the following types of business models for generating commercial value from an invention:

(i) selling the invention;

(ii) licensing the invention;

(iii) partnering to commercialise the invention;

(iv) doing everything yourself. [8]

(b) An entrepreneur is seeking funding for a new business that will manufacture, install and manage control systems for off-shore wind turbines.

(i) Describe the information that a Venture Capitalist (VC) would expect to see in a business plan for this venture.

(ii) Discuss the key factors that would influence the VC's decision to invest in this venture. [12]

22 Cambridge Display Technology (CDT) Ltd is seeking to commercialise intellectual property (IP) relating to Polymer Organic Light Emitting Diodes (P-OLEDs). CDT owns some of the most important patents relating to P-OLED technologies, and its core patent was filed in 1989. P-OLEDs are now just beginning to reach the market within products manufactured by large display and lighting companies. CDT is a small company and its business strategy is currently described as "*Scaling up of manufacturing processes; Licensing to leverage IP and manufacturing know-how; Establishing close partnerships with major display and lighting manufacturers*."

(a) What is the usual maximum life of a UK patent? [1]

(b) Discuss the intellectual property (IP) issues that CDT's management team are likely to have confronted when implementing their business strategy. [9]

(c) Discuss the challenges that CDT may have faced in setting up and managing "*close partnerships with major display and lighting manufacturers*." [10]

23 (a) In the context of the design process for a new product:

(i) describe four roles of prototyping; [4]

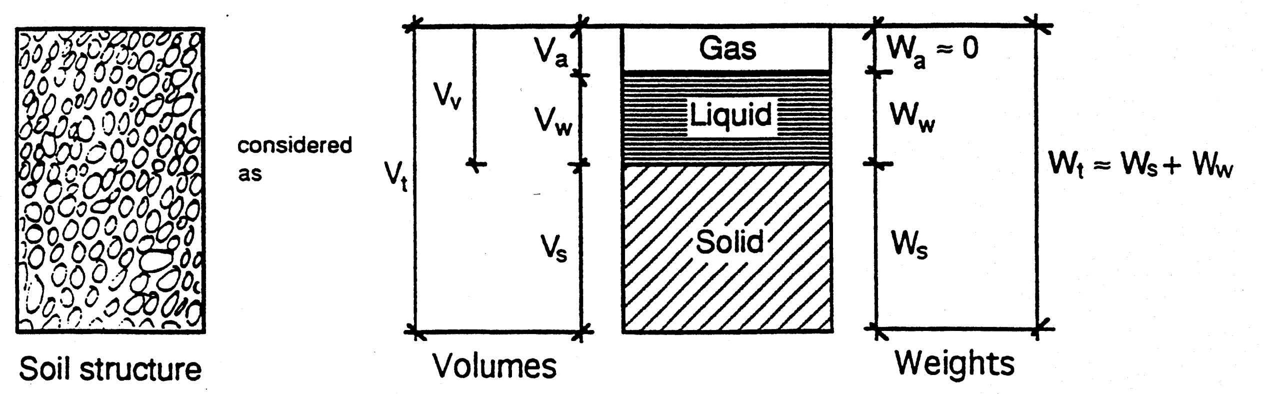
(ii) discuss potential pitfalls of prototyping. [4]

(b) Discuss the relationship between product specification and design of the manufacturing process. [12]

END OF PAPER

Data sheet – Soil Mechanics

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\, G_s / e)$
Unit weight of water	$\gamma_w = 9.81$ kN/m^3 (although we assume 10 kN/m^3)
Unit weight of soil	$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e} \right) \gamma_w$
Buoyant (effective or submerged) unit weight	$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w$ (soil saturated)
Unit weight of dry soil	$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e} \right) \gamma_w$
Relative density	$I_d = \frac{(e_{max} - e)}{(e_{max} - e_{min})}$

where e_{max} is the maximum voids ratio achievable in the quick tilt test (for sands), and
e_{min} is the minimum voids ratio achievable by vibratory compaction (for sands).

Classification of particle sizes

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D — equivalent diameter of soil particle

D_{10}, D_{60} etc — particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

Stress components

Principle of effective stress (saturated soil):

total stress	σ	=	effective stress	$\sigma' +$	pore water pressure	u
	τ	=		$\tau' +$		0

and

σ_v = vertical stress

σ_h = horizontal stresss

τ = shear stress

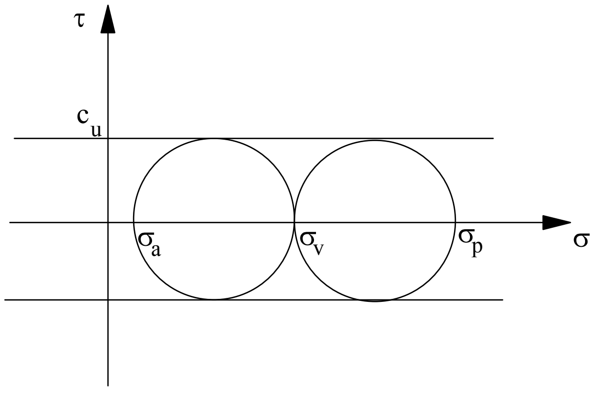
Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the *undrained shear strength* c_u which is mobilized when the shear stress $\tau = c_u$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, σ_a and σ_p respectively, are given by

$$\sigma_a = \sigma_v - 2\,c_u$$

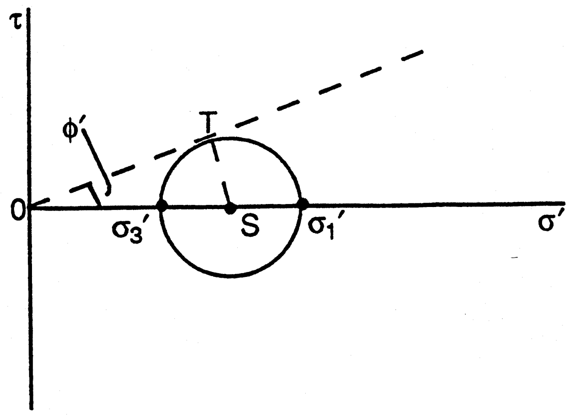
$$\sigma_p = \sigma_v + 2\,c_u$$

where σ_v is the total vertical stress.



Strength of sands

Mobilised angle of shearing ϕ'
where $\tau = \sigma' \tan \phi'$



$$\sin \phi' = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

$$\therefore \phi' = \sin^{-1}\left[\frac{\left(\sigma_1'/\sigma_3'\right) - 1}{\left(\sigma_1'/\sigma_3'\right) + 1}\right]$$

Earth pressure coefficient K: $\sigma_h' = K\sigma_v'$

Active pressure: $\sigma_v' > \sigma_h'$ $\therefore$ $\sigma_1' = \sigma_v'$

$\sigma_3' = \sigma_h'$

$K_a = (1 - \sin \phi'/1 + \sin \phi')$

Passive pressure: $\sigma_h' > \sigma_v'$ $\therefore$ $\sigma_1' = \sigma_h'$

[We assume principal stresses $\sigma_3' = \sigma_v'$

are horizontal and vertical] $K_p = (1 + \sin \phi')/(1 - \sin \phi') = \dfrac{1}{K_a}$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left(\sigma_1'/\sigma_3'\right)_{max}$

at critical state ϕ'_{crit} after large strains.

Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi' = \phi'_{max}$ and

$$\phi'_{max} = \phi'_{crit} + \phi'_{dilatancy}$$

where ϕ'_{crit} is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero $(\phi'_{dilatancy} -> 0)$ as $\phi'_{max} -> \phi'_{crit}$.

ϕ'_{crit} is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values (±2°):

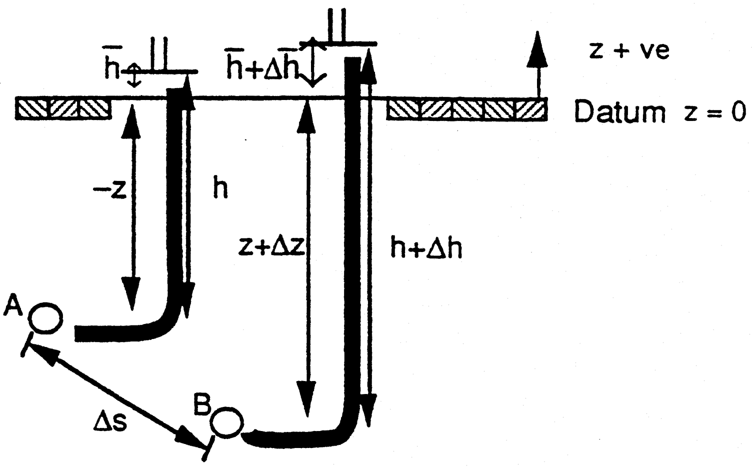
	ϕ'_{crit}	ϕ'_{max}	
feldspar	40°		
quartz	33°	53°	($I_d = 1$, and mean effective stress OS < 150 kPa)
mica	25°		

Seepage

Excess pore water pressure

Head $h = u/\gamma_w$

Potential $\bar{h} = h + z$



Total pore water pressure head at A: $u = \gamma_w h = \gamma_w \left(\bar{h} - (-z)\right)$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w \left(\bar{h} + z + \Delta\bar{h} + \Delta z\right)$

[Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w \left(\bar{h} + \Delta\bar{h}\right)$]

Hydraulic gradient A–B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s} = -\dfrac{\Delta\bar{u}}{\gamma_w \Delta s}$

Darcy's law $\quad v = ki$

v = average or superficial seepage velocity

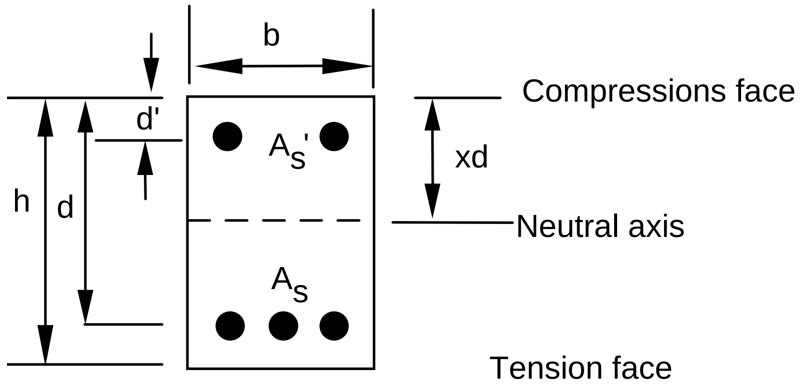
k = coefficient of permeability

Typical permeabilities

$D_{10} > 10\text{mm}$	:	non-laminar flow
$10\text{ mm} > D_{10} > 1\mu\text{m}$	:	$k \cong 0.01(D_{10}\text{in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



Design Stresses

Cube strength for concrete f_{cu}. At failure in bending, stress in concrete $= 0.4f_{cu}$ over whole area of concrete in compression.

Tensile yield stress of steel f_y. At failure in bending, stress in bars in tension $=$ $0.87f_y$, stress in bars in compression $= 0.75f_y$.

Design Equations

Moment capacity of singly reinforced beam

$$M \le 0.15\, f_{cu} bd^2$$
$$M = 0.87 f_y A_s d(1 - x/2)$$
$$x = 2.175(f_y/f_{cu}).(A_s/bd) \quad (\le 0.5 \text{ to avoid over reinforcement})$$

Moment capacity of doubly reinforced beam

$$M = 0.15 f_{cu} bd^2 + 0.75 f_y A'_s (d - d')$$
$$0.87 f_y A_s = 0.75 f_y A'_s + 0.2 f_{cu} bd$$

Shear capacity of all beams

Total shear capacity $V = (v_c + v_s)bd$

Where, $v_c = 0.68(100A_s/bd)^{0.33}.(400/d)^{0.25}$ $\left(N/mm^2\right)$

and $v_s = 0.87 f_y A_{sq}/(bs)$

in which s = shear link spacing, A_{sq} is total area of all shear bars in a link and A_s is the total area of effective longitudinal *tension* steel at the section.

Standard bar sizes

Diameter (mm)	6	8	10	12	16	20	25	32	40	50
Area (mm^2)	28	50	78	113	201	314	491	804	1256	1963

Available steel types

Deformed high yield steel $f_y = 460 \quad N/mm^2$

Plain mild steel $f_y = 250 \quad N/mm^2$

Lap and anchorage lengths 40 bar diameters

Density of reinforced concrete: $24\ kN/m^3$

Reinforcement areas per metre width

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540
Areas calculated to 3 significant figures according to B.S.I recommendations										

April 2010

Part IB Data Sheet: Electrical Engineering Elective

Transistor Design Summary Sheet

Gauss's Theorem

$\varepsilon_o \varepsilon_r E_1 - \varepsilon_o \varepsilon_r E_2 =$ charge per unit area enclosed between upper surface 1 and lower surface 2.

FET Design Summary

- $\tau_t \rightarrow$ switching time as 1st approx. (scattering limited transit time).
- $\tau_{eff} = \tau_t + R_{load} C_{eff(output)} \rightarrow$ switching time as 2nd approx.
- $L = v_s \tau_t$ (source-drain spacing).
- $I_{sat} = e N v_s W d_s = e N W L d_s / \tau_t$
- Aspect ratio W/L (technology?).
- $(1/2) eN(d_s)^2 / \varepsilon_o \varepsilon_r =$ (Max Gate Voltage)
- $E'_{peak} = eN d_s / \varepsilon_o \varepsilon_r < E_{breakdown}$
- Minimum Drain Source Voltage $\sim E_s L$ (E_s is the field required to reach limiting velocities).

Mutual Conductance

$g_{mo} \sim I_{sat} / V_{gate(max)}$

Mutual conductance reduces with frequency as $g_m(\omega) \approx g_{mo} / (1 + j\omega \tau_t)$;

$$v_{out} = g_m(\omega)\, R\left(1 + j\omega RC_{eff(out)}\right) \approx g_{mo} R / [1 + j\omega(\tau_t + RC_{eff(out)})]$$
$$= g_{mo} R / [1 + j\omega \tau_{eff}]$$

Capacitances for FET

Parallel Plate Capacitance: $\varepsilon_o \varepsilon_r$ Area/spacing

Used for rough estimates of parasitic capacitance.

Effective Capacitances for FET

$C_{eff(out)} \rightarrow C_{gate/drain} + C_{drain/source} + C_{load;}$

$C_{eff(in)} \rightarrow MC_{gate/drain} + C_{gate/source\,(proximity)} + C_{gate/source\,(electronic);}$

$C_{electronic} = g_{mo} \tau_t$; $M = (1 + g_{mo} R_{load})$.

Time Constants for FET

$\mu = e\tau / m^*$ relates mean free time τ and mobility.

Transit time τ_t over distance L and scattering limited velocity v_s are related by $\tau_t = L / v_s$.

$v_{out} \approx g_{mo} R / [1 + j\omega \tau_{eff}] = g_{mo} R / [1 + j\omega / (2\pi f_t)]$

$1/(2\pi f_\tau) = \tau_t + RC_{eff(out)} = \tau_{eff}$ The transition frequency is f_t.

10% to 90% rise time is $T = 2.2\, \tau_{eff} = (2.2/2\pi)(1/f_t) = 0.35/f_t$.

Electrical Engineering Elective:
Tunnel Barrier Design Summary Sheet

Schrodinger's Equation

Complex Wave $\Psi = A\ \exp\left(-\ j2\pi f\ t +\ j2\pi\, x/\lambda\right) = A\exp\left(-\ j\omega\, t\right)\exp\left(jkx\right);$

<momentum> $\Psi = p\Psi = \left(h/\lambda\right)\Psi = -j\left(h/2\pi\right)\partial\Psi/\partial x;$

<Total energy> $\Psi = E\Psi = hf\Psi = j\left(h/2\pi\right)\partial\Psi/\partial t$

$\left(h/2\pi\right) \rightarrow \hbar;\quad h = 6.625\times 10^{-34}\ \text{J/s}.$

Schrodinger's equation:-

$$E\Psi = \left(1/2m\right)\left[-\ j\hbar\partial/\partial x\right]^2\Psi + e\phi\Psi$$

Tunneling (Rectangular barrier $\varepsilon\phi$)

Propagating waves outside barrier with incident kinetic energy $U_{incident} = \left(\hbar k_i\right)^2/2m^*$

Evanescent waves inside barrier: $-\left(\hbar k\right)^2/2m^* = \left(\hbar k_i\right)^2/2m^* - e\phi$

Technology Design Summary

Diffusion

Constant Surface Concentration:

$$C\left(x,t\right) = C_s\ \operatorname{erfc}\left[\frac{x}{2\left(Dt\right)^{1/2}}\right]$$

Constant Total Dopant:

$$C\left(x,t\right) = \frac{S}{\left(\pi Dt\right)^{1/2}}\ \exp\left[\frac{-x^2}{4Dt}\right]$$

W I Milne
May 2011