Version MJD/5

EGT1
ENGINEERING TRIPOS PART IB

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Monday 1 June 2015 2 to 4

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**Paper 2**

**STRUCTURES**

*Answer not more than **four** questions, which may be taken from either section.*

*All questions carry the same number of marks.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Answers to questions in each section should be tied together and handed in separately.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Engineering Data Book

**10 minutes reading time is allowed for this paper.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**SECTION A**

1 A thin-walled cantilever steel beam is shown in Fig. 1(a). A vertical point load of 1 kN is applied at point A and a torque of 2 kN m is applied at the mid-length of the beam, as shown. The hollow, rectangular cross-section of the beam is shown in Fig. 1(b), where the overall dimensions are to mid-thickness. All walls of the cross-section are of uniform thickness and points A, B, C and D all lie on lines of symmetry of the cross-section. The steel has a Young's modulus of 210 GPa, a shear modulus of 81 GPa and a uniaxial yield stress of 355 MPa.

(a) Find the vertical displacement at point B due to the applied loading. [7]

(b) Assuming the Tresca yield criterion applies, and considering only the stresses at points C and D, calculate the amount by which the torque can be increased before yield occurs. [18]

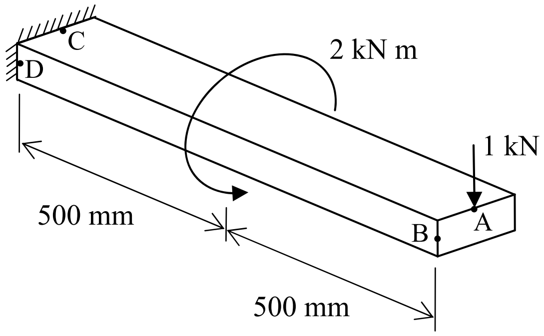


Fig. 1(a)

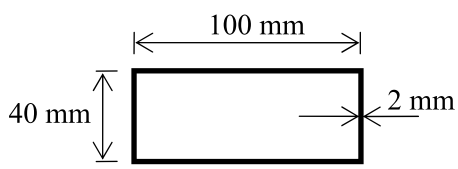


Fig. 1(b)

2 (a) A curved beam of constant radius $R$ is shown in Fig. 2(a). The beam is initially stress free, has a uniform bending stiffness, $EI$, and is built-in at point A. A moment $M$ is applied at point B as shown and all displacements are in-plane. Find the vertical displacement at B. [6]

(b) A roller support is now added at point B, as shown in Fig. 2(b). Find the support reaction and the angle of rotation at point B. [12]

$$\left( \text{Note:} \quad \int_0^{\pi/2} \sin^2\theta \, \mathrm{d}\theta = \frac{\pi}{4} \right)$$

(c) Explain how you would use your previous answers to find the vertical deflection at the location of the applied force $P$ in Fig. 2(c). Do not calculate the deflection. [7]

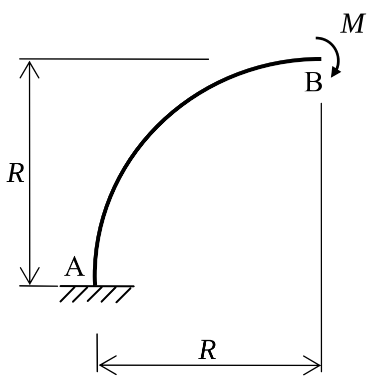


Fig. 2(a)

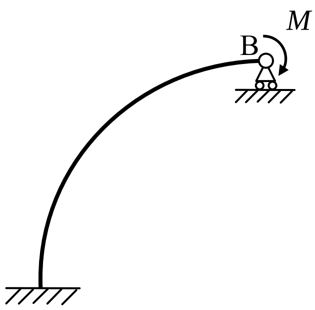


Fig. 2(b)

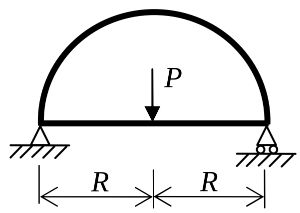


Fig. 2(c)

3 The frame structure shown in Fig. 3(a) is built-in at point A and is supported by a roller at point C. Two equal point loads of magnitude $P$ are applied as shown.

(a) Assume that the fully plastic moment of segment BC is $M_p$ and the fully plastic moment of segment AB is $2M_p$ . Using upper bound theory, find the collapse load $P$ in terms of $M_p$ . [6]

(b) Now instead assume that the capacity of the cross-sections of segments AB and BC are not yet defined.

(i) If segments AB and BC are uniform and identical in cross-section, use lower bound theory to determine the minimum required fully plastic moment. Assume the particular equilibrium solution and the state of self-stress shown in Fig. 3(b). [8]

(ii) If the beam segments AB and BC are uniform but are not identical, use lower bound theory to find a design that is more efficient. Provide your answer in terms of the fully plastic moment required for each segment. [8]

(iii) Including consideration of axial stresses, explain how you would select suitable cross-sections for each member of the frame. [3]

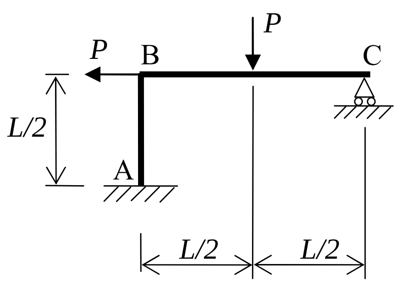


Fig. 3(a)

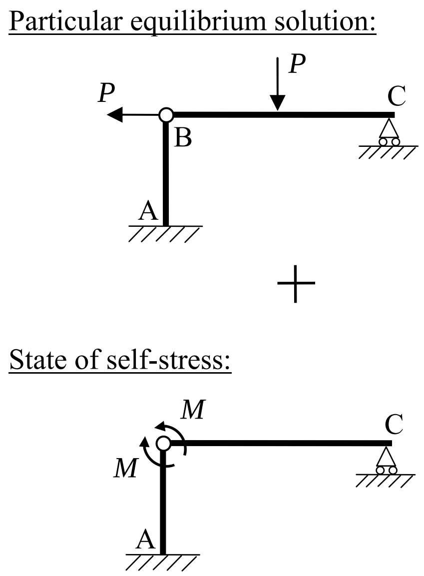


Fig. 3(b)

## SECTION B

4 The frame structure shown in Fig. 4(a) and Fig. 4(b) is built-in at point A and supported by rollers at points B and D. All members of the frame have bending stiffness *EI* and are axially rigid.

(a) Member BD is loaded with a uniformly distributed load of *w* per unit length, as shown in Fig. 4(a). Find the support reaction at point D. [6]

(b) Member CD is loaded with a uniformly distributed load of 2*w* per unit length, as shown in Fig. 4(b).

(i) Find the support reaction at point D. [14]

(ii) Find the horizontal displacement at point D. [5]

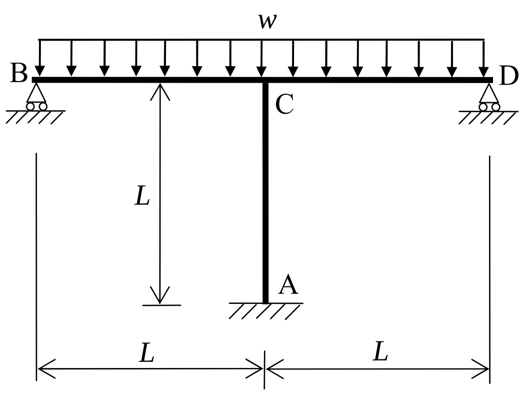


Fig. 4(a)

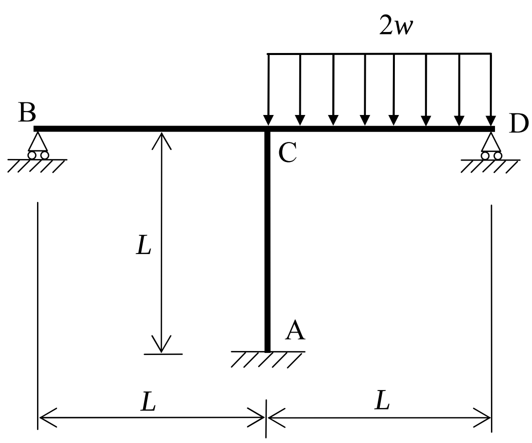


Fig. 4(b)

5 The ductile plate shown in plan view in Fig. 5(a) is simply supported at corners A and B, and is fully clamped along edge CD. The rest of the perimeter is free. The plate is subjected to a uniform pressure $p$ acting on the shaded area shown in Fig. 5(a). The fully plastic moment per unit length is $m$ for all yield lines.

(a) Figure 5(b) shows a collapse mechanism where the dashed line indicates hogging and the solid lines indicate sagging. Find the collapse load. [12]

(b) Propose a different compatible mechanism and calculate the collapse load. [13]

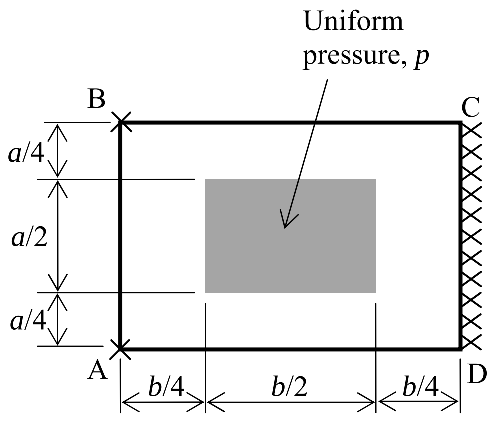


Fig. 5(a)

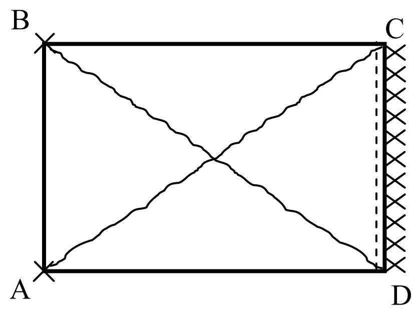


Fig. 5(b)

6 At a certain location on the surface of a thin steel plate the strains are found to be:

$$\varepsilon_{xx} = 100\times10^{-6}, \quad \varepsilon_{yy} = -60\times10^{-6}, \quad \gamma_{xy} = -120\times10^{-6}$$

The through thickness ($z$-axis) stress is zero and no temperature change has occurred. Assume material properties $E$ = 210 GPa, $\nu$ = 0.3, and $\alpha = 11\times10^{-6}$ K$^{-1}$.

(a) Using a square of unit side length in the $x$- and $y$- directions to represent the undeformed element, sketch the deformed element at this location. Exaggerate the deformation and label the sketch. [3]

(b) Draw Mohr's circle of strain and find the in-plane principal strains and their orientations with respect to the $x$-axis. [4]

(c) Find the third principal strain. [7]

(d) The temperature is then increased by 10 $^{o}$C, while the plate is constrained so that $\varepsilon_{xx}$ , $\varepsilon_{yy}$ , and $\gamma_{xy}$ remain unchanged and the through thickness ($z$-axis) stress remains zero. Find the principal strain in the $z$-direction. [11]

**END OF PAPER**

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