

ENGINEERING TRIPOS PART IIA 2003

MODULE 3C5: ANSWERS

1. (a)  $ma^2/3, ma^2/3, 2ma^2/3$   
 (b) Missing terms are zero, except the 3-1 entry is  $3\sqrt{2}$ .  
 (c)  $11ma^2/3, 11ma^2/3, 2ma^2/3$
  
2. (b)  $\dot{v}_2 = Fa^2/A, A = 7ma^2/5$   
 (c) No
  
3. (b)  $2mga(A + ma^2)/C^2\omega^2$   
 (c)  $mga(A + ma^2)/C^2\omega^2$
  
4. (a)  $T = (m/2)\{\dot{x}^2 + \dot{y}^2 + (L+x)^2\dot{\theta}^2 + 2\dot{y}\dot{\theta}(L+x)\cos\theta + 2\dot{x}\dot{y}\sin\theta\}$   
 $V = -mg(L+x)\cos\theta + kx^2/2$   
 (b)  $m\ddot{x} - m(L+x)\dot{\theta}^2 + m\ddot{y}\sin\theta - mg\cos\theta + kx = 0$   
 $m(L+x)^2\ddot{\theta} + 2m\dot{x}\dot{\theta}(L+x) + m\ddot{y}(L+x)\cos\theta + mg(L+x)\sin\theta = 0$   
 (d)  $\theta = \alpha \cos(\omega_n t + \phi) + A \sin \omega t / [L(\omega_n^2 - \omega^2)]$
  
5. (b) 
$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{pmatrix} + (g/L) \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = (F/mL) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
  
 (c)  $\omega_1^2 = (g/L)(2 - \sqrt{2}), \quad \mathbf{u}_1 = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix}; \quad \omega_2^2 = (g/L)(2 + \sqrt{2}), \quad \mathbf{u}_2 = \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix}$