

ENGINEERING TRIPOS PART IIA

Module 3C6 Examination, 2003

Answers

1. (b) $u^{(n)} = \sin\left(\frac{n\pi x}{L}\right), \quad \omega_n = \frac{n\pi}{L} \sqrt{\frac{P}{m}};$

(c) $w(x,t) = \sum_n a_n \cos \omega_n t \sin\left(\frac{n\pi x}{L}\right),$ with $a_n = \begin{cases} a/2, & n=2 \\ -4a \sin\left(\frac{n\pi}{2}\right), & \text{otherwise} \\ \frac{a}{\pi(n^2-4)} \end{cases}$

2. (c) $\frac{L_1}{L_2} = \left(\frac{\omega_2}{\omega_1}\right)^{1/2} = 2^{1/24} = 1.029$

(d) $\omega \propto (\alpha L)^2, \quad 1:2.76:5.40$

3. (a) $V = \frac{k}{2L^2} \left[(y_3 + y_1 - 2y_2)^2 + (y_2 - 2y_1)^2 \right] + \frac{Sy_1^2}{2L^2}$

(c) $\mathbf{y} = [0 \quad 1 \quad 3]^T; \quad \omega = \frac{1}{L} \sqrt{\frac{2k}{5m}}$

(d) lower

4. (a) $\begin{cases} \mathbf{u}^{(1)} = [1 \quad 1 \quad 1]^T, \omega_1 = 0 \\ \mathbf{u}^{(2)} = [1 \quad 0 \quad -1]^T, \omega_2 = \sqrt{\frac{k}{J}} \\ \mathbf{u}^{(3)} = [1 \quad -2 \quad 1]^T, \omega_3 = \sqrt{\frac{3k}{J}} \end{cases}$

(b) $\theta_2(\sqrt{J/k}) = 0.143I\sqrt{Jk}$

(TURN OVER)