# Engineering Tripos Part IIA
## 2003
## Paper 3F1: Signals and Systems

## Answers

1. (a)(i) $\dfrac{1}{1 - \beta z^{-1}}$

   (a)(ii) $\dfrac{z - \beta}{z - \alpha}$

   (a)(iii)

$$y_{k+1} - \alpha y_k = u_{k+1} - \beta u_k$$

   (b)

$$\text{pulse response: } h_k = \begin{cases} 1 & k = 0 \\ \alpha^k - \beta\alpha^{k-1} & k \geq 1 \end{cases}$$

$$\text{step response} = \begin{cases} 1 & k = 0 \\ \frac{1}{1-\alpha}((\beta - \alpha)\alpha^k + 1 - \beta) & k \geq 1 \end{cases}$$

   (c) $\alpha = 0.969$, $\beta = 0.689$.

2. (b) $k = 1$ gives the most probable position.

3. (a) $r_{XY}(\tau) = h(\tau) \star r_{XX}(\tau)$
   (b) $S_Y(\omega) = S_X(\omega)|H(\omega)|^2$
   (c) $h(t) = \frac{1}{2}\delta(t - T) + \delta(t - 2T) + \frac{1}{2}\delta(t - 3T)$
   (d) Formula in (b) doesn't tell us the phase of $H(\omega)$.

4. (b)(i) 0.8113
   (b)(ii) 0.5721
   (c)(i) 0.2392
   (c)(ii) 0.0859
   (d) 0.4188