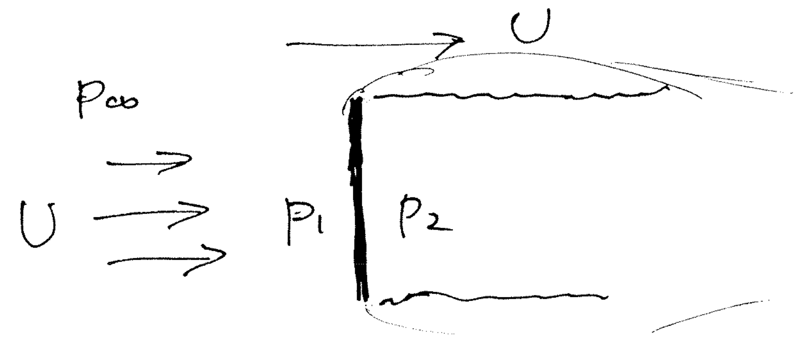
3A1 EXAM CRIBS 2002/3

(1) (a)



Pressure on front of plate is

$\simeq$ stagnation pressure $= p_\infty + \frac{1}{2}\rho U^2$

Pressure on rear $\simeq$ static pressure $= p_\infty$
(since streamlines are not strongly curved)

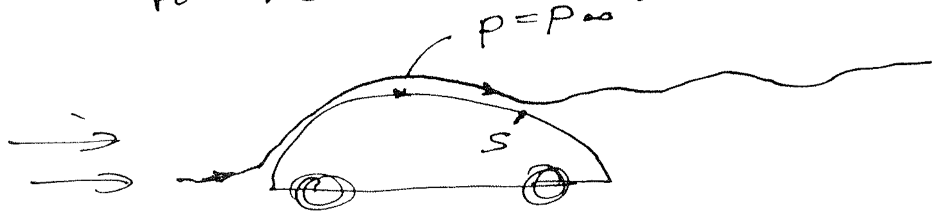
Drag Force $= (p_1 - p_2) A = \frac{1}{2}\rho U^2 A$

$$C_D = \frac{\text{Drag force}}{\frac{1}{2}\rho U^2 A} = 1$$

(b) Power = Force $\times$ Velocity

$$= \frac{1}{2}\rho U_c^2 A \times U_c = \frac{1}{2}\rho U_c^3 A$$

(c) The car has some streamlining which means the pressure has a chance to recover before it separates



Pressure in wake is = Pressure at separation

Pressure at S $> p_\infty$ since velocity is less than free stream. $\rightarrow$ lower drag

D(d)

Apply momentum from inlet to exit

flow out = flow in

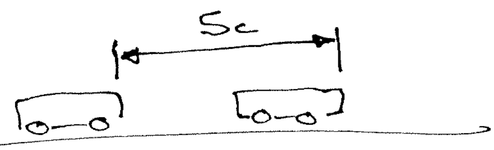
Area out = Area in

So no change in momentum.

→ hence no pressure change.

$$\sum \text{forces} = 0$$

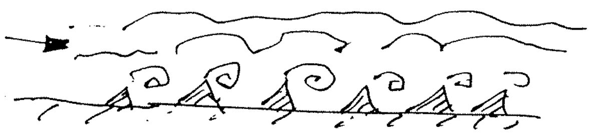
$$\underbrace{\tfrac{1}{2}\rho U_T^2 C_f P_T L}_{\text{skin friction}} = \tfrac{1}{2}\rho (U_c - U_T)^2 C_D A_c \times \frac{L}{\underbrace{S_c}_{\text{length of car + space.}}}$$

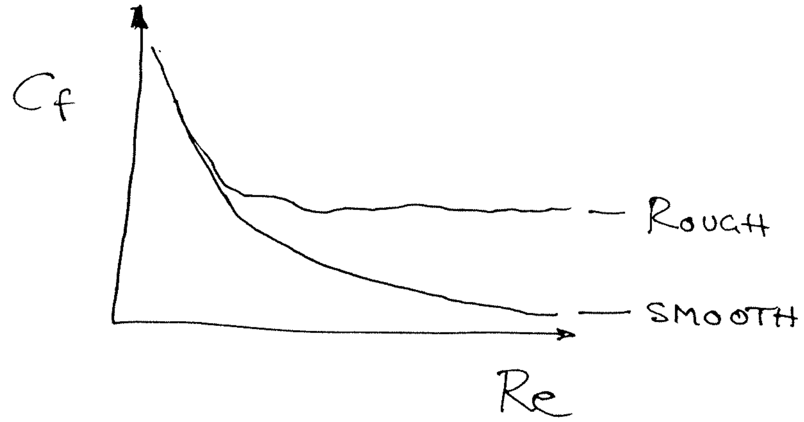


$$\left(\frac{U_c - U_T}{U_T}\right)^2 = \frac{C_f P_T S_c}{C_D A_c}$$

$$\frac{U_T}{U_c} = \frac{1}{1 + \left(\frac{C_f P_T S_c}{C_D A_c}\right)^{1/2}}$$

1 e) C_f is likely to be indept of Re since the walls will be aerodynamically rough at this high Reynolds number – flow near wall is dominated by inertia forces and indept of viscosity.





f) S.F.E.E.

From continuity

$$\cancel{\dot{Q}} - \dot{W}_x = \dot{m}\,\Delta\left(h + \cancel{\tfrac{1}{2}U^2} + \cancel{gz}\right)$$

ADIABATIC WALLS AND NO HEAT ADDED IF $\eta_c = 1$

HORIZ

$$W_x = \frac{1}{2}\rho\,(U_c - U_T)^2\,U_c\,C_D\,A_c\,\frac{L}{S_c}$$

$$= \rho\,A_T\,U_T\,C_p\,\Delta T$$

$$\Delta T = \frac{\frac{1}{2}C_D A_c L\,(U_c - U_T)^2\,U_c}{A_T\,C_p\,U_T\,S_c}$$

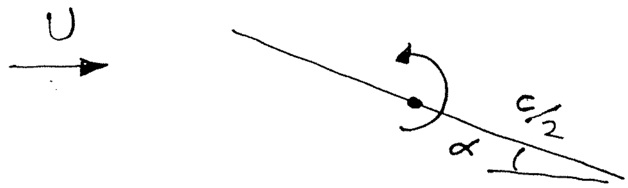
1 (g) Now we have heat added due to the inefficiency of the engines which produce

$$\dot{Q} = (1-\eta_c)\,\dot{W}_x \quad \text{waste heat/unit time}$$

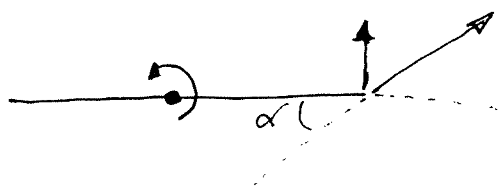
Substuting this in (to S.F.E.E.) leads to

$$\Delta T = \frac{\Delta T(\eta_c = 1)}{\quad} \times (2-\eta_c)$$

② a)



Consider velocities at trailing edge.
To simplify diagram rotate aerofoil



We want zero velocity component normal to the plate (Γ must be negative)

$$\frac{\Gamma}{2\pi\left(\frac{c}{2}\right)} + U\sin\alpha = 0$$

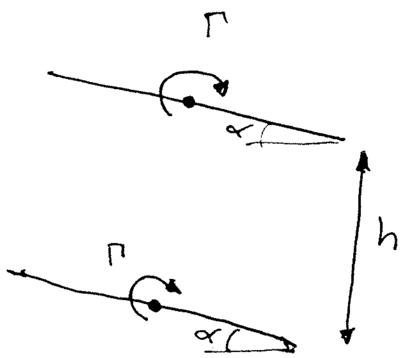
$$\Gamma = -\pi c U \sin\alpha$$

$$C_L = \text{Lift coefficient} = \frac{\text{Lift force / unit length}}{\frac{1}{2}\rho U^2 c}$$

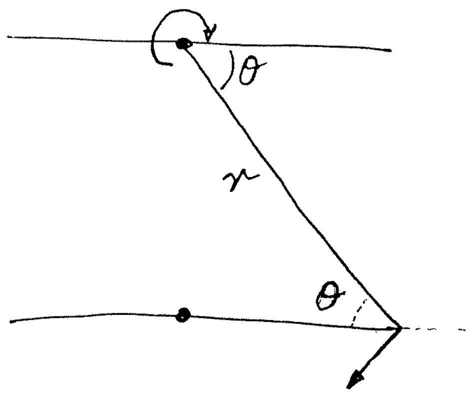
$$= \frac{-\rho U \Gamma}{\frac{1}{2}\rho U^2 c}$$

$$= 2\pi \sin\alpha$$

b) Consider now the bi-plane

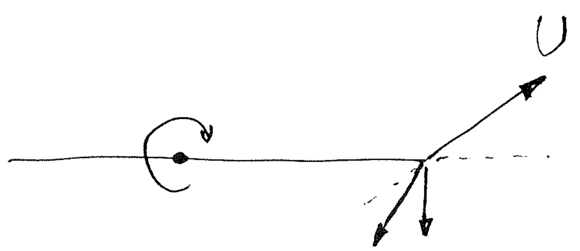


Taking α as small we now have another vortex nearby on the other wing



α small

$$r = \sqrt{\left(\frac{c}{2}\right)^2 + h^2} \qquad \text{(approx)}$$



zero normal component

$$U\sin\alpha - \frac{\Gamma}{2\pi\left(\frac{c}{2}\right)} - \frac{\Gamma}{2\pi r}\cos\theta = 0$$

$$r = \sqrt{\left(\frac{c}{2}\right)^2 + h^2}$$

$$\cos\theta = \frac{\frac{c}{2}}{r} =$$

$$\Rightarrow U\sin\alpha - \frac{\Gamma}{2\pi}\left[\frac{2}{c} + \frac{\frac{c}{2}}{\left(\frac{c}{2}\right)^2 + h^2}\right] = 0$$

$$\frac{\Gamma}{2\pi}\left[\frac{2\left(\frac{c^2}{4} + h^2\right) + \frac{c^2}{2}}{c\left(\frac{c^2}{4} + h^2\right)}\right] = U\sin\alpha.$$

$$\frac{\Gamma}{2\pi}\left[\frac{2h^2 + c^2}{c\left(\frac{c^2}{4} + h^2\right)}\right] = U\sin\alpha.$$

$$\Gamma = 2\pi U\sin\alpha\, c\,\frac{(c^2 + 4h^2)}{8h^2 + 4c^2}$$

$$= 2\pi U\sin\alpha . c\left(\frac{1 + 4\left(\frac{h}{c}\right)^2}{8\left(\frac{h}{c}\right)^2 + 4}\right)$$

$$\text{Lift} = -\rho U\Gamma = -2\pi\rho U^2\sin\alpha\, c\left(\frac{1 + 4\left(\frac{h}{c}\right)^2}{4 + 8\left(\frac{h}{c}\right)^2}\right)$$

as $\frac{h}{c} \to \infty$

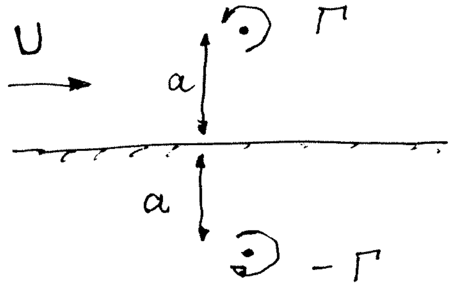
$$= -\pi\rho U^2\sin\alpha\, c$$

But its actually +ve since we drew the original diagram with Γ -ve. (i.e. CW).

③ Flow past a 90° corner

In the z-plane

a) $$F(z) = Uz + \frac{-i\Gamma}{2\pi}\ln(z-ia) + \underbrace{\frac{i\Gamma}{2\pi}\ln(z+ia)}_{\text{Image.}}$$



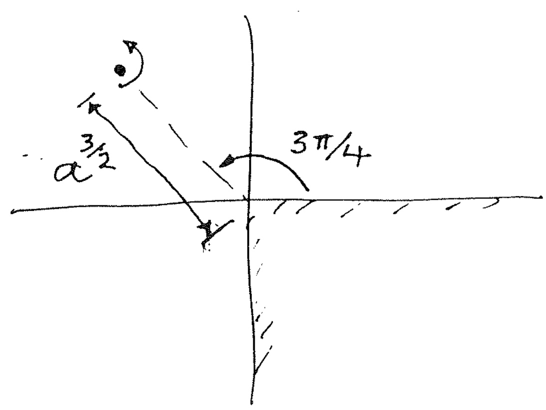
b) $$z = \zeta^{2/3} \qquad z = re^{i\theta} \qquad \zeta = Re^{i\phi}$$

Position of vortex in z-plane is

$$z = ia$$
$$= ae^{i\frac{\pi}{2}} = R^{\frac{2}{3}}e^{i\frac{2\phi}{3}}$$

$$R = a^{3/2}, \quad \phi = \frac{3\pi}{4}$$

So in the ζ-plane



c) We want $\dfrac{dF}{d\zeta} = 0$ at $\zeta = 0$

$$\frac{dF}{d\zeta} = \frac{dF}{dz}\frac{dz}{d\zeta} = 0$$

Now $\dfrac{dz}{d\zeta} = \dfrac{2}{3}\zeta^{-\frac{1}{3}}$ → which is singular at $\zeta = 0$

but if $\dfrac{dF}{dz} = 0$ at same point then we are OK.

So

$$\frac{dF}{dz} = 0 = U + \frac{i\Gamma}{2\pi}\left[\frac{1}{z+ia} - \frac{1}{z-ia}\right]$$

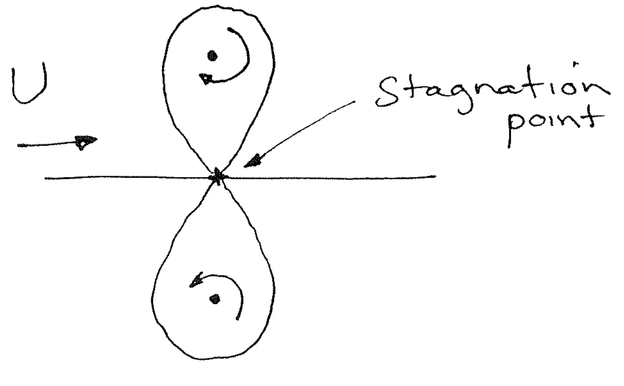
at $\zeta = 0$ which is the same as $z = 0$

$$0 = U + \frac{i\Gamma}{2\pi}\left[\frac{1}{ia} + \frac{1}{ia}\right]$$

$$= U + \frac{\Gamma}{\pi a}$$

$$\Rightarrow \Gamma = -\pi a U$$

So in z-plane.



d) We want $\dfrac{dF}{d\zeta}$ at $\zeta = a^{\frac{3}{2}} e^{i\frac{3\pi}{4}}$

$$\frac{dF}{d\zeta} = \frac{dF}{dz}\frac{dz}{d\zeta} \qquad z = \zeta^{\frac{2}{3}}, \; \zeta = z^{\frac{3}{2}}$$

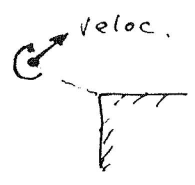
$$\frac{dz}{d\zeta} = \frac{2}{3}\zeta^{-\frac{1}{3}}$$
$$= \frac{2}{3}\left(a^{\frac{3}{2}} e^{i\frac{3\pi}{4}}\right)^{-\frac{1}{3}}$$
$$= \frac{2}{3} a^{-\frac{1}{2}} e^{-i\frac{\pi}{4}}$$

Now $$\frac{dF}{dz} = U + \frac{i\Gamma}{2\pi}\left[\frac{1}{z+ia} - \frac{1}{z-ia}\right]$$

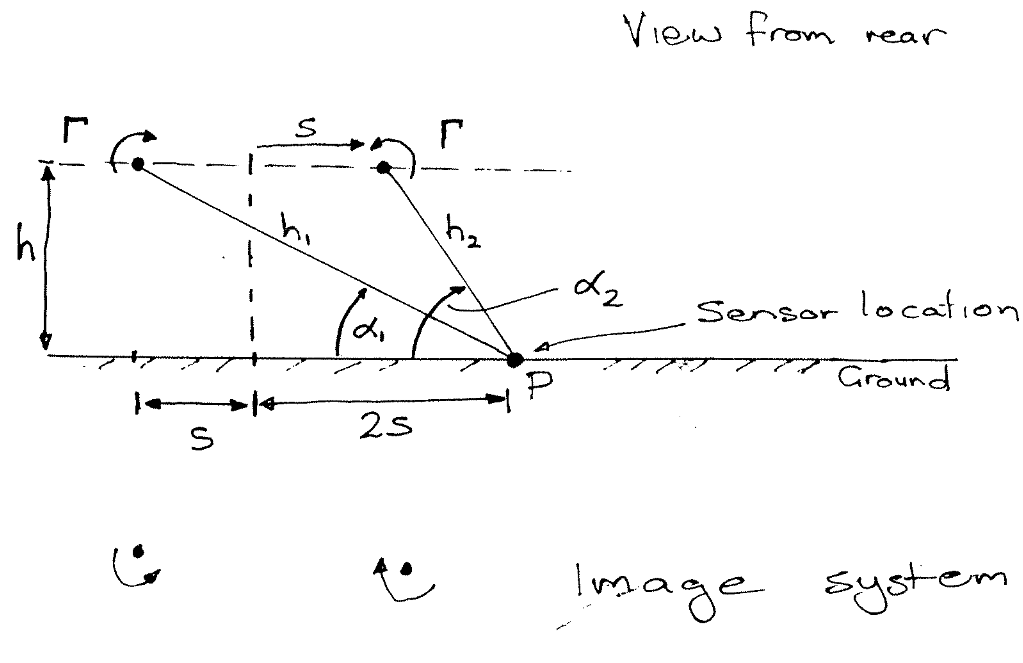
And the position of the vortex in the ζ-plane corresponds to $z = ia$. Drop the second term since it is velocity induced by vortex on itself and is singular

$$\left.\frac{dF}{dz}\right|_{z=ia} = U + \frac{\Gamma}{4\pi a} = \frac{3}{4}U \quad \left(\begin{array}{l}\text{subst.} \\ \text{value of} \\ \Gamma \text{ from (c)}\end{array}\right)$$

Hence $$\left.\frac{dF}{d\zeta}\right|_{\zeta = a^{\frac{3}{2}} e^{i\frac{3\pi}{4}}} = u - iv = \frac{1}{2} U a^{-\frac{1}{2}} e^{-i\frac{\pi}{4}}$$



(4)



a) There are FOUR individual contributions to the velocity at P, two from the image system

The bound vortices do not contribute since P is in line with the wing.

Each tip vortex is semi-infinite

so $v = \frac{\Gamma}{4\pi h}\left[\cos 90^\circ + \cos 0^\circ\right] = \frac{\Gamma}{4\pi h}$

The horizontal contribution of the tip vortex and its image are the same and the vertical component from a vortex and its image cancel out at the ground

Hence only need to consider horizontal components.

This means we can just consider the two tip vortices and their horisontal components then double the result to account for the images

$$V_p = 2\left[-\underbrace{\frac{\Gamma}{4\pi h_1}\sin\alpha_1}_{\text{HORIZ}} + \underbrace{\frac{\Gamma}{4\pi h_2}\sin\alpha_2}_{\text{HORIZ}} \right]$$

where

$$h_1 = \sqrt{(3s)^2 + h^2} \qquad h_2 = \sqrt{s^2 + h^2}$$

$$\sin\alpha_1 = \frac{h}{\sqrt{(3s)^2 + h^2}} \qquad \sin\alpha_2 = \frac{h}{\sqrt{s^2 + h^2}}$$

Substitute and simplify

$$\boxed{V_p = \frac{4s^2\Gamma}{\pi} \; \frac{h}{9s^4 + 10s^2h^2 + h^4}}$$

(b) Max interference $\frac{dV_p}{dh} = 0$

$$\frac{4s^2\Gamma}{\pi} \quad \frac{9s^4 + 10s^2h^2 + h^4 - h(20s^2h + 4h^3)}{(9s^4 + 10s^2h^2 + h^4)^2} = 0$$

$$9s^4 - 10s^2h^2 - 3h^4 = 0$$

$$h^2 = -\frac{5}{3}s^2 \pm \sqrt{\frac{25}{9}s^4 + \frac{27}{9}s^4}$$

ONLY SOLN >0 IS PHYSICAL

$$h^2 = \frac{\sqrt{52} - 5}{3}s^2 \qquad h = 0.86s$$

(c) According to the horseshoe vortex model, the tip vortex induced velocity would be double if the aircraft was ∞ distance ahead of the sensor (the vortex would be infinite, rather than semi-infinite)

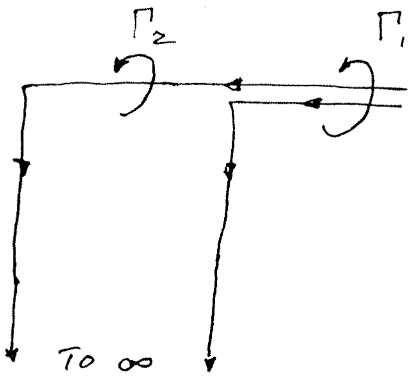
In reality the vortices would decay.

In this case there is also an additional contribution from the bound vortices, but this is small.

In summary the largest effect is likely to be when the aircraft is a little ahead of the sensor and it is likely to be larger than that calculated in (a)

(5) P.14

a) Vortex lines can't end in the fluid so any change in circulation along the wing causes a line vortex in the trailing vortex sheet



$\Gamma_1 > \Gamma_2$

$$\underbrace{d\Gamma}_{\text{Strength of incremental vortex}} = \underbrace{\frac{d\Gamma}{dz}}_{\text{Slope of circulation distribution along the wing.}} dz$$

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b) Total lift $\quad L = \rho V \int_{-s}^{s} \Gamma(z)\, dz$ (data card)

Parabolic

$$L = \rho V \Gamma_{OP} \int_{-s}^{s} \left(1 - \frac{z^2}{s^2}\right) dz$$

$$= \rho V \Gamma_{OP} \left(2s - \frac{2s^3}{3s^2}\right)$$

$$= \frac{4}{3} \rho V \Gamma_{OP}\, s$$

Elliptic:

$$L = \frac{\pi}{4} \rho V 2s\, \Gamma_{OE}$$

$$\Rightarrow \quad \Gamma_{OP} = \frac{3\pi}{8} \Gamma_{OE}$$

c) Applying Biot-Savart to trailing vortex sheet and integrating over the wing gives for the down-wash at $z=0$

$$|W_p| = \frac{1}{4\pi}\int_{-s}^{s} \frac{d\Gamma}{dz}\,\frac{dz}{z}$$

here: $$\frac{d\Gamma_p}{dz} = -\Gamma_{op}\,\frac{2z}{s^2}$$

$$|W_p| = \frac{2}{4\pi s^2}\int_{-s}^{s} \Gamma_{op}\,\frac{z}{z}\,dz = \frac{4s}{4\pi s^2}\,\Gamma_{oP}$$

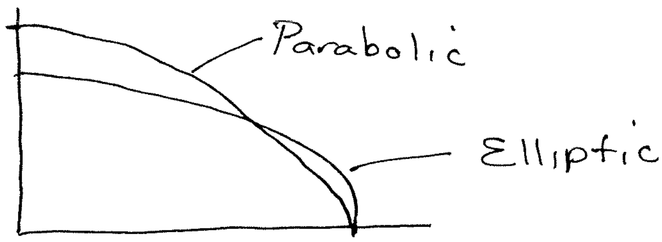
$$= \frac{\Gamma_{oP}}{\pi s}$$

for elliptic wing (from data card)

$$|W_E| = \frac{\Gamma_{oE}}{4s}$$

$$\frac{W_P}{W_E} = \frac{3}{2} \quad (50\% \text{ larger}).$$

(d)



Parabolic is larger near $z=0$ and drops quicker towards the tips:

A straight wing would need more chord at the root and less at the tips

~~At the~~

The angle of attack would be greater due to downwash and need to generate more lift at the centre.

6 (a) $u(x,y) = A\sin(By)$

$u = 0$ at $y = 0$ is ok.

$u = U_e$ at $y = \delta$

$$U_e = A\sin(B\delta)$$

Also $\frac{\partial U}{\partial y} = 0$ at $y = \delta$

$$AB\cos(By) = 0 \quad \text{at } y = \delta$$

$$AB\cos(B\delta) = 0$$

we assume that $A \neq 0$, $B \neq 0$

$$B\delta = \frac{\pi}{2} \quad B = \frac{\pi}{2\delta}$$

$$\sin(B\delta) = 1$$

$$A = U_e$$

$$\frac{u}{U_e} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

→ Define $\eta = \frac{y}{\delta}$
To simplify algebra and save writing

$$\frac{u}{U_e} = \sin\left(\frac{\pi\eta}{2}\right)$$

6 (b)

(i) $\delta^* = \int_0^\delta 1 - \frac{u}{U_e} \, dy$

$= \delta \int_0^1 1 - \frac{u}{U_e} \, d\eta$

$= \delta \int_0^1 1 - \sin(\frac{\pi}{2}\eta) \, d\eta$

$= \delta \left[\eta + \frac{2}{\pi} \cos(\frac{\pi}{2}\eta) \right]_0^1$

$= (1 - \frac{2}{\pi}) \delta$

(ii) $\theta = \int_0^\delta \frac{u}{U_e} \left(1 - \frac{u}{U_e}\right) dy$

$= \delta \int_0^1 \frac{u}{U_e} \left(1 - \frac{u}{U_e}\right) d\eta$

$= \delta \int_0^1 \sin(\frac{\pi}{2}\eta) \left(1 - \sin(\frac{\pi}{2}\eta)\right)$

$= \delta \left[\left[-\frac{2}{\pi} \cos(\frac{\pi}{2}\eta) \right]_0^1 - \int_0^1 \sin^2(\frac{\pi}{2}\eta) \, d\eta \right]$

$= \delta \left(\frac{2}{\pi} - \frac{1}{2} \int_0^1 1 - \cos(\pi\eta) \, d\eta \right)$

$= \delta \left(\frac{2}{\pi} - \frac{1}{2} \left[\eta - \frac{1}{\pi} \sin(\pi\eta) \right]_0^1 \right)$

$= \delta \left(\frac{2}{\pi} - \frac{1}{2} \right)$. (0.1375)

6 (b) continued.

(iii)

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

$$= \mu \frac{\partial}{\partial y}\left(\sin\left(\frac{\pi\eta}{2}\right)\right) U_e \bigg|_{\eta=0}$$

$$= \mu \frac{U_e}{\delta}\frac{\partial}{\partial \eta}\left(\sin\left(\frac{\pi\eta}{2}\right)\right) \bigg|_{\eta=0}$$

$$= \mu \frac{U_e}{\delta} \frac{\pi}{2} \cos\left(\frac{\pi\eta}{2}\right) \bigg|_{\eta=0}$$

$$= \frac{\pi}{2} \frac{\rho\nu U_e}{\delta}$$

(c) $$\frac{d\theta}{dx} = \frac{C_f'}{2}$$

It expresses the fact that in a zero pressure gradient flow the skin friction balances the rate of change of momentum of the fluid near the wall. This is an integrated effect over the whole boundary layer.

θ grows since it is a measure of the difference in momentum between the ideal flow and the boundary layer flow

6 (d)

$$\frac{d\theta}{dx} = \frac{c_f'}{2}$$

$$= \frac{\tau_w}{\rho U_e^2}$$

$$= \frac{\pi}{2}\frac{\nu}{\delta U_e}$$

$$\theta = \left(\frac{2}{\pi} - \frac{1}{2}\right)\delta$$

$$\frac{d\theta}{dx} = \left(\frac{2}{\pi} - \frac{1}{2}\right)\frac{d\delta}{dx}$$

$$\left(\frac{2}{\pi} - \frac{1}{2}\right)\frac{d\delta}{dx} = \frac{\pi}{2}\frac{\nu}{\delta U_e}$$

$$\delta\, d\delta = \frac{\pi}{\left(\frac{4}{\pi} - 1\right)}\frac{\nu}{U_e}dx$$

$$\frac{1}{2}\delta^2 = \frac{\pi}{\left(\frac{4}{\pi} - 1\right)}\frac{\nu x}{U_e} + C$$

→ assume $\delta = 0$ at $x = 0$ → $C = 0$

$$\delta^2 = \frac{2\pi}{\left(\frac{4}{\pi} - 1\right)}\frac{\nu x}{U_e}$$

$$\delta = \sqrt{\frac{2\pi}{\left(\frac{4}{\pi} - 1\right)}}\sqrt{\frac{\nu x}{U_e}}$$

$$\delta = x\sqrt{\frac{2\pi}{\left(\frac{4}{\pi} - 1\right)}}\frac{1}{\sqrt{R_x}}$$

$$C_f' = \frac{\pi \nu}{\delta U_e}$$

$$= \frac{\dfrac{\pi \nu}{U_e}}{x\sqrt{\dfrac{2\pi}{\left(\frac{4}{\pi}-1\right)}}\dfrac{1}{\sqrt{R_x}}}$$

$$= \sqrt{\frac{\left(\frac{4}{\pi}-1\right)}{2\pi}}\,\pi\,\frac{\nu}{U_e x}\sqrt{R_x}$$

$$\simeq \frac{0{\cdot}655}{\sqrt{R_x}}$$

3A1 EXAM CRIBS 2002/3

7 (a) Momentum equation

$$-\frac{\partial P}{\partial x} - \sigma B^2 U + \rho\nu \frac{\partial^2 U}{\partial z^2} = 0$$

$$\frac{\partial^2 U}{\partial z^2} - \frac{\sigma B^2}{\rho\nu} U - \frac{1}{\rho\nu}\frac{\partial P}{\partial x} = 0 \qquad (1)$$

(b) Choose a velocity scale U_0 (which might be velocity on the centreline)

$$\frac{\partial^2 U}{\partial z^2} \sim \frac{U_0}{Z^2} = \frac{\sigma B^2}{\rho\nu} U_0$$ for equal forces

$$L = Z = \sqrt{\frac{\rho\nu}{\sigma B^2}}$$

(c) Solution of (1)

$$U = \cosh(Az) + C$$

$$\frac{\partial U}{\partial z} = A\cosh(Az)$$

$$\frac{\partial^2 U}{\partial z^2} = A^2\cosh(Az)$$

Substitute in

7 (c)

$$A^2\cosh(Az) - \frac{\sigma B^2}{\rho\nu}\cosh(Az) - C\frac{\sigma B^2}{\rho\nu} - \frac{1}{\rho\nu}\frac{\partial P}{\partial x} = 0$$

Solution of the Homogeneous equation gives

$$A^2 = \frac{\sigma B^2}{\rho\nu} \qquad A = \sqrt{\frac{\sigma B^2}{\rho\nu}} = \frac{1}{L}$$

To simplify notation we will continue to use A

Now this leaves

$$-CA - \frac{1}{\rho\nu}\frac{\partial P}{\partial x} = 0$$

$$\frac{\partial P}{\partial x} = -\rho\nu C A = -\rho\nu C\sqrt{\frac{\sigma B^2}{\rho\nu}}$$

What is C. Use boundary conditions

$$U = \cosh(Az) + C$$

$$z = -\frac{h}{2} \text{ (at wall)}$$

$$U = 0$$

$$0 = \cosh\left(-A\frac{h}{2}\right) + C \qquad C = -\cosh\left(-A\frac{h}{2}\right)$$

7 (C) continued

Check other B/C.

$$z = +\frac{h}{2}$$

$$U = \left(\cosh\left(A\frac{h}{2}\right) - \cosh\left(-A\frac{h}{2}\right)\right) = 0$$

cosh is an even function

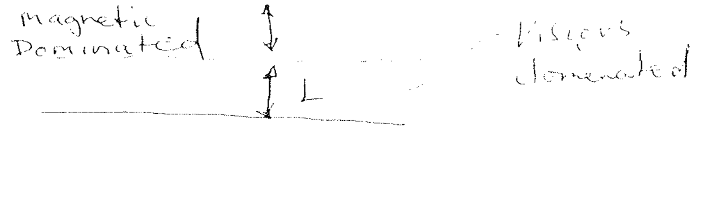
$$\cosh(-x) = \cosh(x)$$

So it <u>WORKS</u>!

$$U = \left(\cosh\left(\frac{z}{L}\right) - \cosh\left(\frac{h}{2L}\right)\right)$$

So our L enters as the appropriate length to scale z.

This is because L defines which region is dominated by viscosity and which is dominated by magnetic field



7 (d) Viscous force $\rho\nu \frac{\partial^2 U}{\partial z^2}$

Lorentz force $-\sigma B^2 U$

$$-\cancel{\sigma B^2}\left(\cosh\left(\frac{z}{L}\right) - \cosh\left(\frac{h}{2L}\right)\right) = \cancel{\sigma B^2}\cosh\left(\frac{z}{L}\right)$$

$$2\cosh\left(\frac{z}{L}\right) = \cosh\left(\frac{h}{2L}\right)$$

$$h = 4L$$

$$\cosh\left(\frac{z}{L}\right) = \frac{1}{2}\cosh(2)$$

NOW WE NEED DISTANCE FROM WALL

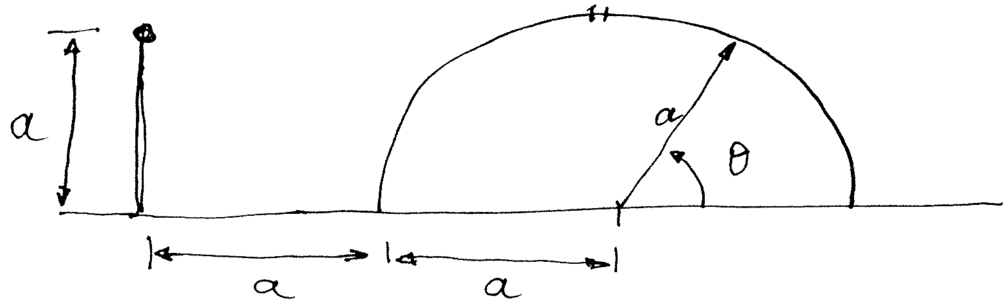
i.e. $$\frac{\frac{h}{2} - z}{L} = 0.75$$

$$\frac{z}{L} = 1.25$$

$$\cosh(1.25) = 1.8884$$

$$\frac{1}{2}\cosh(2) = 1.8811$$

8



a) $\psi = U\sin\theta\left(r - \frac{a^2}{r}\right)$

b) $u = \frac{\partial\psi}{\partial y} = \frac{\partial}{\partial y}\left[U\sin\theta\left(r - \frac{a^2}{r}\right)\right]$

$\sin\theta = \frac{y}{r} \qquad r = \sqrt{x^2+y^2}$

$$\frac{\partial\psi}{\partial y} = \frac{\partial}{\partial y}\left[U\left(y - y\frac{a^2}{x^2+y^2}\right)\right]$$

$$= U\left(1 - \frac{a^2(x^2-y^2)}{(x^2+y^2)^2}\right)$$

$x = -2a \quad y = a$

$$= U\left(1 - \frac{3a^4}{25a^4}\right)$$

$$= U\left(1 - \frac{12}{100}\right)$$ 12% error

c) Since we want velocity on surface its most sensible to choose polar form since $u_r = 0$, on surface.

$$u_\theta = -\frac{\partial \psi}{\partial r}$$

$$= -U \sin\theta \left(1 + \frac{a^2}{r^2}\right)$$

and at $r = a$.

$$u_\theta = -2U\sin\theta$$

Pressure by Bernoulli

$$p + \tfrac{1}{2}\rho u_\theta^2 = \underbrace{p_a + \tfrac{1}{2}\rho U^2}_{\text{far upstream.}}$$

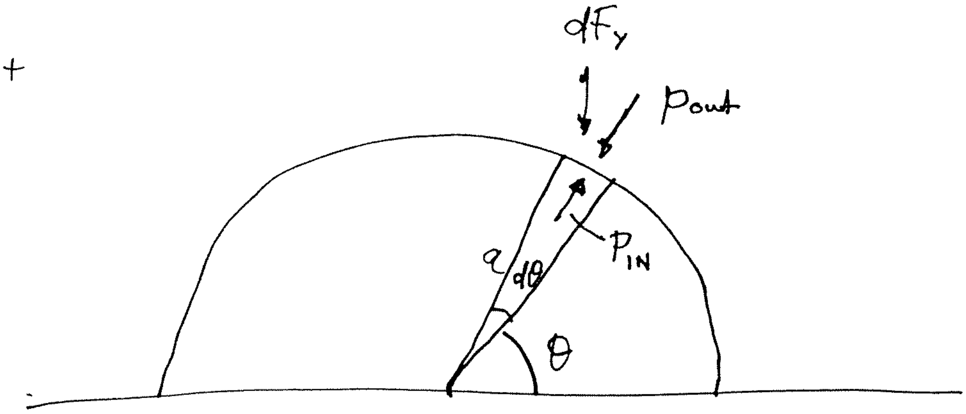
$$p = p_a + \tfrac{1}{2}\rho U^2 (1 - 4\sin^2\theta)$$

Now at the top where the vent is $\theta = \frac{\pi}{2}$, $r = a$ (and hence the pressure inside)

$$p_{IN} = p_a - \tfrac{3}{2}\rho U^2$$

c) continued

Lift



$$\text{Force} = p\,dA = p\,a\,d\theta$$

$$dF_{radial} = (p_{IN} - p_{out})\,a\,d\theta$$

$$dF_y = dF_{radial}\sin\theta$$

$$F_y = \tfrac{1}{2}\rho U^2 \int_0^{\pi} a(-4 + 4\sin^2\theta)\sin\theta\,d\theta$$

$$= -\tfrac{1}{2}\rho a U^2 \int_0^{\pi} 4(1-\sin^2\theta)\sin\theta\,d\theta$$

$$= -2\rho a U^2 \int_0^{\pi} \cos^2\theta\sin\theta\,d\theta$$

$$= -2\rho a U^2 \left[-\tfrac{1}{3}\cos^3\theta\right]_0^{\pi}$$

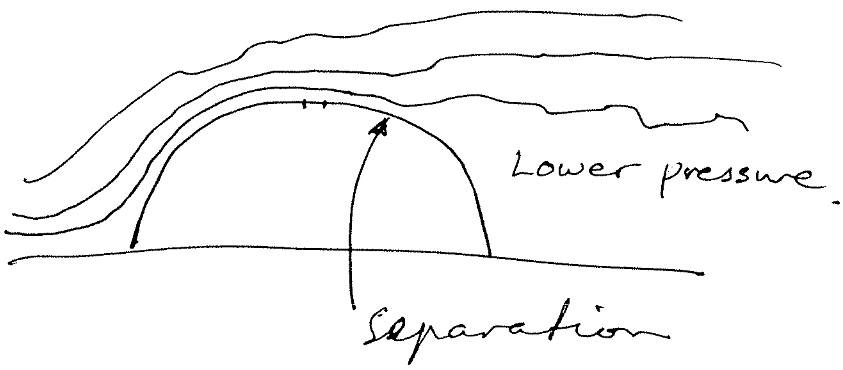
$$= -\tfrac{4}{3}\rho a U^2$$

So there is a downforce which explains the reason for the vent.

For pressure inside $= p_a$ $\quad 4\sin^2\theta = 1$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

8 d) Viscosity will lead to separation on the downstream side of the hut. This leads to a lower pressure on the outside than in the inviscid case (less pressure recovery.) So the down-force will be less. In the case where the force is up (lift) then this will be larger



Of course this also leads to a large drag force which may be important.