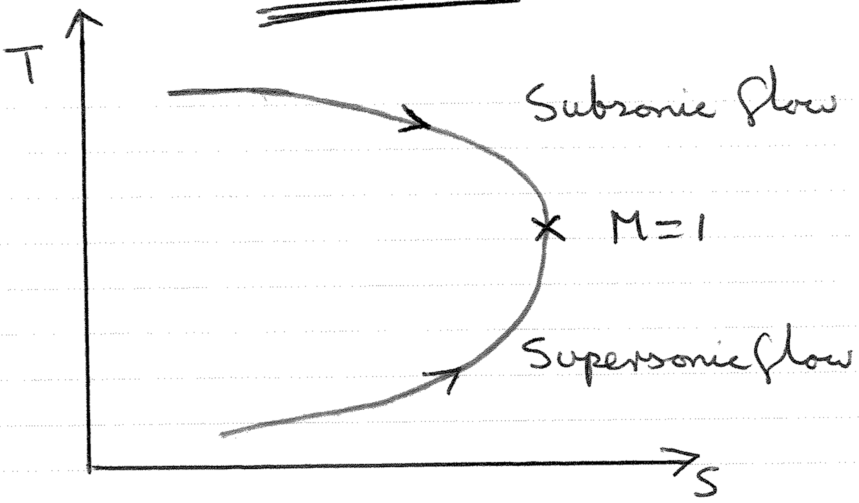
J.D.DENTON

PAPER 3A3 - 2003

Q1 (a)



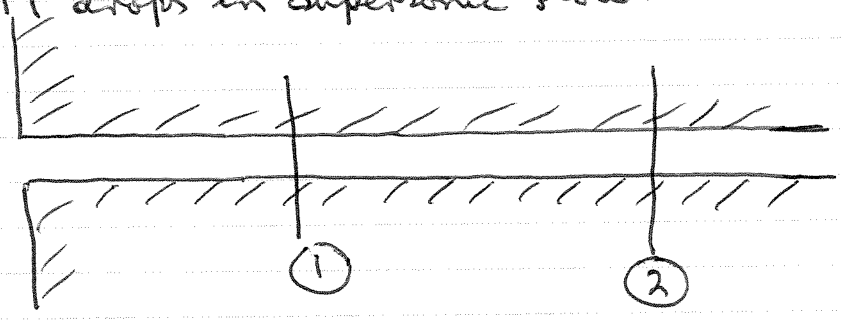
$$d\left(\frac{F}{A}\right) = P(M^2-1)\frac{dV}{V}.$$

Friction must always cause the impulse function to decrease. Hence if $M>1$, dV/V must be $-$ve. If $M<1$ dV/V must be $+$ve.

The entropy must always increase in the direction of flow, T_0 is constant so the stagnation pressure must decrease.

The variation of static pressure is most easily seen by considering the function $\dot{m}\sqrt{C_pT_0}/AP_s$. This increases continuously + M. T_0 is constant and so P falls as M rises in subsonic flow and P rises as M drops in supersonic flow. (6)

b)



At (1) $P/P_0 = 0.8$, $M_1 = 0.57$

and $\dot{m}\sqrt{C_pT_0}/AP_{01} = 1.048$

$\therefore\ \dot{m} = 44.9$ Kg/sec (4)

At (2) $P/P_0 = .666$, $M_2 = .78$

From tables. At (1) $4C_f\frac{L}{D} = .609$

Q1 Cont'd At (2) $4 C_f \frac{L}{D} = .088$

From (1) to (2) = 10 m.

$$\therefore \Delta\left(4 C_f \frac{L}{D}\right) = .609 - .088$$

$$= 4 \times C_f \times \frac{10}{.5}$$

$$\rightarrow \underline{\underline{C_f = .0065}}$$

(4)

At the pipe entry

$$4 C_f \frac{L}{D} = \text{value at (1)} + \frac{4 \times .0065 \times 150}{.5}$$

$$= .609 + 7.8 = 8.409.$$

Tables $\rightarrow \underline{\underline{M_1 \simeq 0.25}}$

$$\rightarrow \frac{\dot{m}\sqrt{C_p T_0}}{A\ P_{0_1}} = .533$$

$$\rightarrow \underline{\underline{P_{0_1} = 235\ \text{kPa}}}$$

(6)

~~1 (b) Cont'd.~~

$$\therefore \Delta\left(\frac{4fL}{D}\right) = .587$$

$$\therefore \Delta L = \frac{.587 \times .1}{4 \times .003} = \underline{\underline{4.89\ m}}$$

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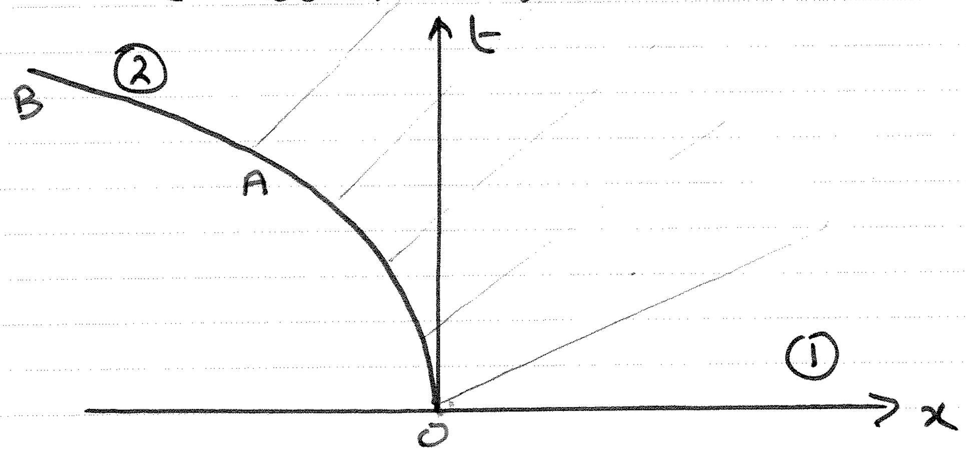
2) (a) See lecture notes (8)

(b)



Across each characteristic the flow is isentropic and

$$V_1 - \frac{2a_1}{\gamma-1} = V_2 - \frac{2a_2}{\gamma-1}$$

$V_1 = 0$

Since the flow is isentropic $\frac{a_2}{a_1} = \sqrt{\frac{T_2}{T_1}}$

$$= \left(\frac{\rho_2}{\rho_1}\right)^{\frac{\gamma-1}{2}} . \quad \frac{\rho_2}{\rho_1} = 0.5 \rightarrow \frac{a_2}{a_1} = 0.8705$$

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \underline{\underline{.379}}$$

$$\therefore V_2 - \frac{2 \times .8705\, a_1}{\gamma-1} = -\frac{2a_1}{\gamma-1}$$

$$V_2/a_1 = \underline{\underline{-0.647}}$$

(8)

When $V_2 = a_2$, $\frac{-2a_1}{\gamma-1} = -a_2 - \frac{2a_2}{\gamma-1}$

$$= \frac{-(\gamma+1)}{\gamma-1} a_2 \quad . \quad \therefore \frac{a_2}{a_1} = \frac{2}{\gamma+1}$$

Q2 Cont'd.

$$\frac{a_2}{a_1} = \sqrt{\frac{T_2}{T_1}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{2\gamma}}$$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\cancel{\frac{\gamma-1}{2\gamma}}\ \frac{2\gamma}{\gamma-1}}$$

$$\rightarrow \quad \underline{\underline{\frac{P_2}{P_1} = 0.279}}$$

6

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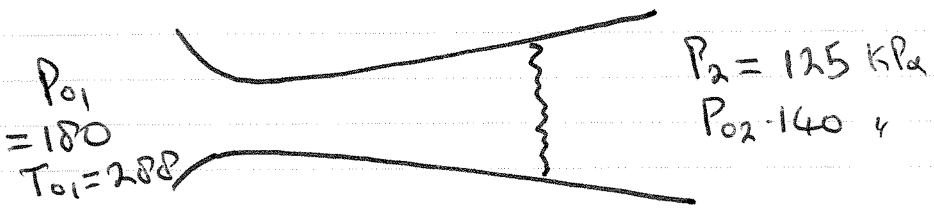
3) (a) See lecture notes

6



Since the P_0 drops there must be a shock

$\therefore$ The nozzle must be choked.

$\therefore$ At throat $\quad \dfrac{\dot{m}\sqrt{C_p T_0}}{A\,P_0} = 1.281$

$$\rightarrow \quad \dot{m} = \underline{\underline{4.286 \text{ Kg/s.}}}$$

3

At nozzle exit $\quad P/P_0 = \dfrac{125}{140} = .8928$

$\rightarrow \quad M_e = 0.41$

$$\rightarrow \quad \frac{\dot{m}\sqrt{C_p T_0}}{A_e\,P_{0e}} = .822$$

$$\rightarrow \quad \underline{\underline{A_e = .0200 \text{ m}^2}}$$

3

Across the shock, $\quad P_{0s}/P_{01} = \dfrac{140}{180} = .778$

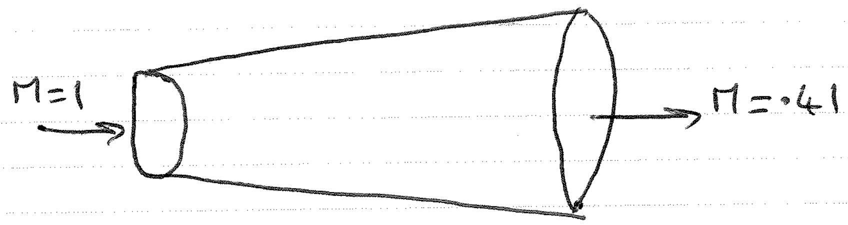
$$\rightarrow \quad \underline{\underline{M_s = 1.88}}$$

$$\rightarrow \quad \frac{\dot{m}\sqrt{C_p T_0}}{A_s\,P_{01}} = .837$$

Q3 Cont'd

$$\therefore \frac{A_s}{A_t} = \frac{1.281}{.837}$$

$$\rightarrow \underline{\underline{A_s = .0153 \ m^2}}$$ (4)



The force on the diverging part of the nozzle is the change in impulse function

$F = (PA + \rho A V^2)$

From Tables. At throat $\frac{F}{\dot{m}\sqrt{C_p T_o}} = 0.99$

at exit $F/\dot{m}\sqrt{C_p T_o} = 1.34$

$$\therefore \text{Force} = (1.34 - 0.99) * \dot{m}\sqrt{C_p T_{o1}}$$

$$= 0.35 \times 4.286 \sqrt{1005 \times 288}$$

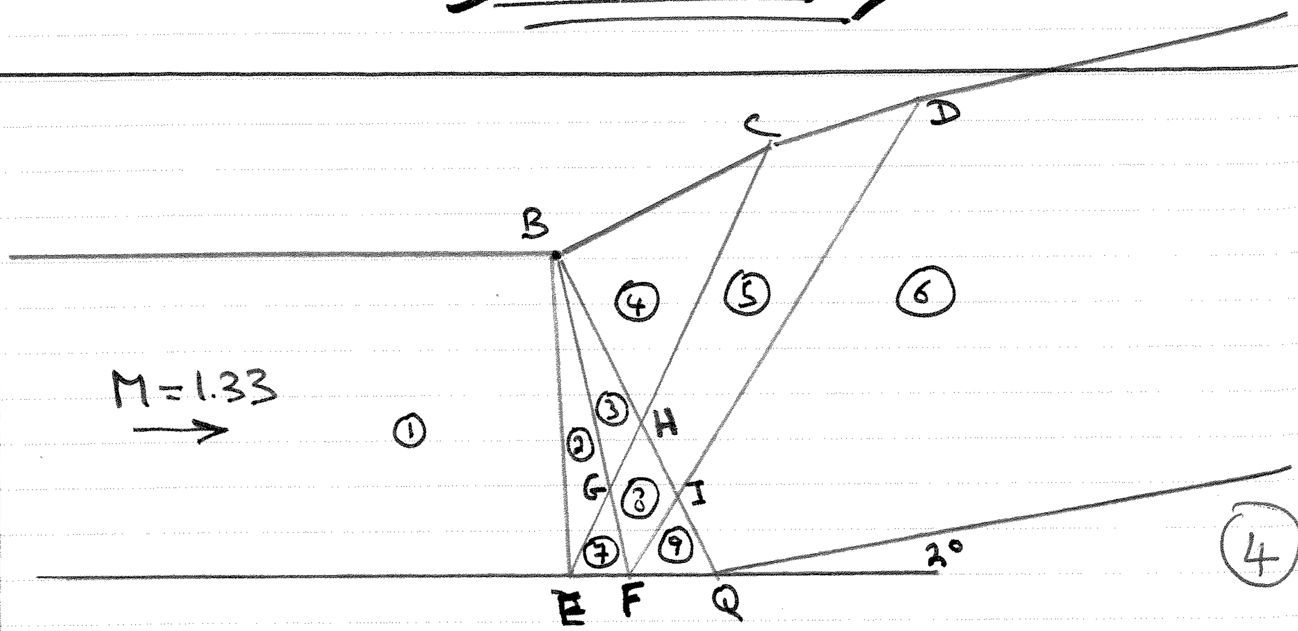
$$= \underline{\underline{807.\ N.}}$$ – acting upstream on the nozzle. (4)

Q3 cont'd.

$$\therefore \frac{A_s}{A_t} = \frac{1.281}{.837}$$

$$\rightarrow A_s = .0 \quad m^2$$

Q4



$$M_1 = 1.33 \quad , \quad \frac{A}{A^*} = 1.079 \quad , \quad \nu_1 = 6.99^\circ$$

$$\therefore \frac{A_6}{A^*} = 1.215 \times 1.079 \rightarrow 1.311$$

$$\rightarrow M_6 = 1.67 \quad , \quad \nu_6 = 16.89 .$$

From ① to ④ $\quad \nu - \theta = \text{const}$

$$\nu_1 - \theta_1 = \nu_4 - \theta_4$$

From ④ to ⑥ $\quad \nu + \theta = \text{const}$

$$\nu_4 + \theta_4 = \nu_6 + \theta_6$$

$$\rightarrow 2\nu_4 = \nu_1 + \nu_6 - \theta_1 + \theta_6$$

$$\theta_6 - \theta_1 = 2^\circ$$

$$\therefore \nu_4 = \frac{\nu_1 + \nu_6}{2} + 1^\circ$$

$$\rightarrow \nu_4 = 12.94^\circ , \quad \underline{M_4 = 1.535}$$

Q4 Cont'd.

$\nu_4 = 12.94 \quad \therefore \quad \underline{\theta_4 = 5.95^\circ}$ (6)

Say $\theta_4 = 6^\circ$ and use 2° intervals for the characteristics.

Region	θ	ν	M	μ
1	0	6.99	1.33	
2	2	8.99	1.40	45.6
3	4	10.99	1.47	42.9
4	6	12.99	1.53	40.8
5	4	14.99	1.605	38.5
6	2	16.89	1.67	36.8
7	0	10.99	1.47	42.9
8	2	12.99	1.53	40.8
9	0	14.99	1.605	38.5

(4)

The angles of the characteristics to the inlet flow direction are $\theta \pm \mu$

WAVE	θ	μ	ANGLE
BE	0	48.2	48.2
BF	2	45.6	43.6
BH	4	42.9	38.9
EC	6	40.8	46.8
FD	4	38.5	42.5
EG	2	45.6	47.6
FI	2	40.8	42.8
GF	0	42.9	42.9
IQ	0	38.5	38.5
HI	2	40.8	38.8

(4)

In a real flow the expansion from B is continuous – not in 2° increments. Hence the corners at Q, C and D would need to be "rounded" to achieve cancellation.

If the sharp corners were used in practice they would generate weak shock waves.

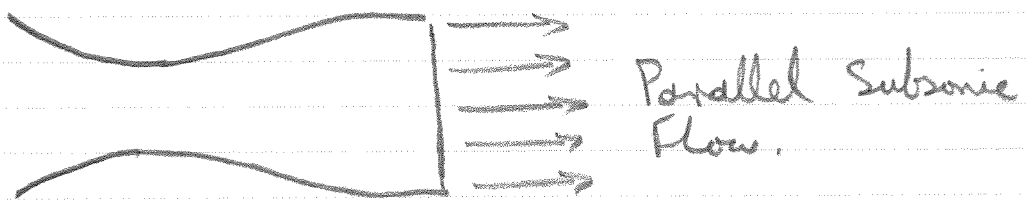
(2)

Q5

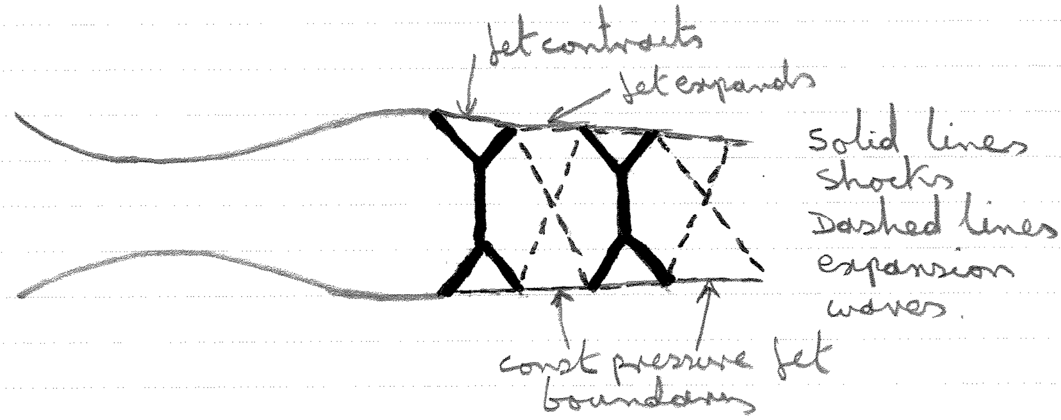
a) The flows appear in the order (iv), (ii), (i), (iii) as the back pressure is reduced. (2)

b)

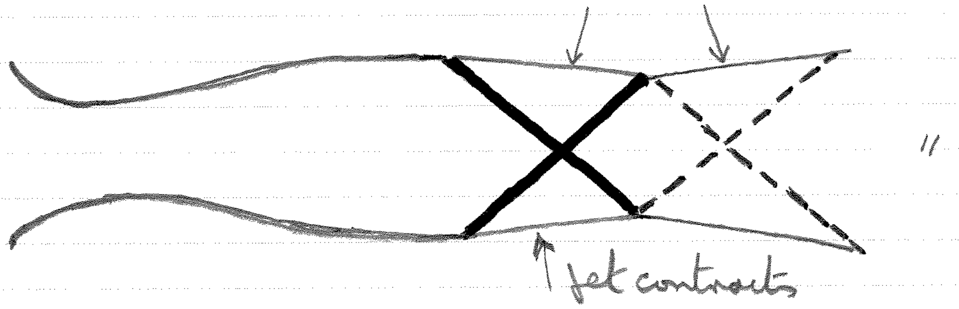
(iv)



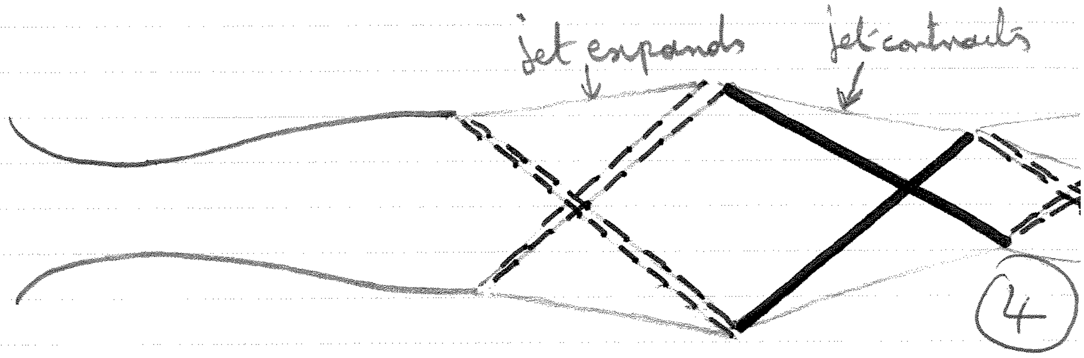
(ii)



(i)



(iii)



(4)

c) Case ii occurs when the Mach number after the first shock is too low to allow flow turning back to a parallel flow at

Q5 cont'd

the second shock. i.e if the flow turning is δ_1° at the first shock and the Mach No is M_2 downstream of the first shock, then δ_1° is greater than the maximum possible turning of an oblique shock with upstream Mach No M_2. 4

d) A soon as the pressure is lowered below that in case (iv), case (ii) will occur i.e as soon as $P_b < P_{01} \times \frac{P_s}{P_{01}}$ at $M_1 = 2$.

ie $P_b / P_{01} < \frac{P_s}{P_1} \times \frac{P_1}{P_{01}}$ at $M_1 = 2$

ie $P_b / P_{01} < 4.5031 \times 0.128$

$P_{01} / P_b > \underline{\underline{1.735}}$

Case (ii) turns into case (i) when

$\delta_{max\,M_2} < \delta_1$ for $M_1 = 2.0$

From Tables.

δ_1	M_2	$\delta_{max\,M_2}$
10°	1.6395	$\sim 15^\circ$
12°	1.5639	$\sim 13^\circ$
14°	1.4851	$\sim 12^\circ$
16°	1.4017	$\sim 9.5^\circ$

So the answer is between $\delta_1 = 12^\circ$ and 14°

Q5 cont'd.

Interpolating in the tables more accurately

$\delta_1 = 12^\circ \quad M_2 = 1.5639 \quad \delta_{max} = 13.72$

$\delta_1 = 14^\circ \quad M_2 = 1.4851 \quad \delta_{max} = 11.69^\circ$

So the solution is approx $\delta_1 = 13^\circ$, $M_2 \simeq 1.52$

For this M_2, $P_s / P_1 = 1.99$ (Houghton & Brock)

and $\frac{P_1}{P_{01}} = \cdot 128$

So $P_{01} / P_b = 3.93$

$\therefore$ Case (ii) can occur for:

$$1.735 < \frac{P_{01}}{P_b} < 3.93$$

10

Q6 a) See Lecture Notes (10)

b) Backswept blades give lower work input for same tip speed but the pressure rise in the diffuser is relatively less → better efficiency. Radial blades are better for stress at high tip speeds since the "centrifugal forces" gives no bending stress. Also backswept blades give a steeper P:m characteristic → more stable operation. (4)

$$\text{Tip speed} = \frac{48000}{60} \times 2\pi \times \frac{.155}{2} = 389.5 \text{ m/sec}$$

$$V_{\theta 2} = \sigma U = 331.1 \text{ m/sec}$$

$$\Delta h_0 = U V_{\theta 2} = \underline{128.9 \text{ KJ/Kg.}}$$

$$\Delta T_0 = \Delta h_0 / C_p = 128.3 \text{ K.}$$

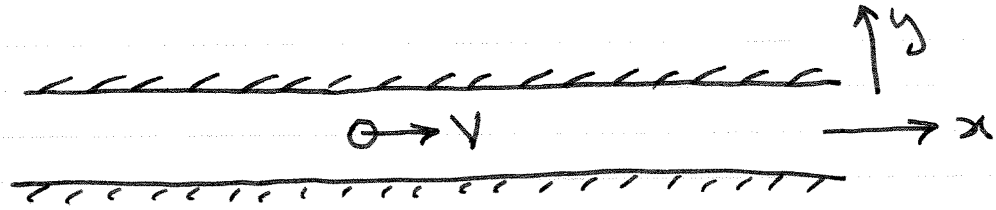
$$\Delta T_{01S} = 128.3 \times .8 = 102.6$$

$$\frac{T_{02 IS}}{T_{01}} = \frac{390.6}{288}$$

$$\rightarrow \left(\frac{P_{02}}{P_{01}}\right) = \left(\frac{T_{02IS}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = \underline{2.906} \quad (6)$$

~~Q6~~ Q7

a)



Because the plates are effectively of infinite extent there can be no pressure gradient in the x direction and no velocity gradient in that direction.

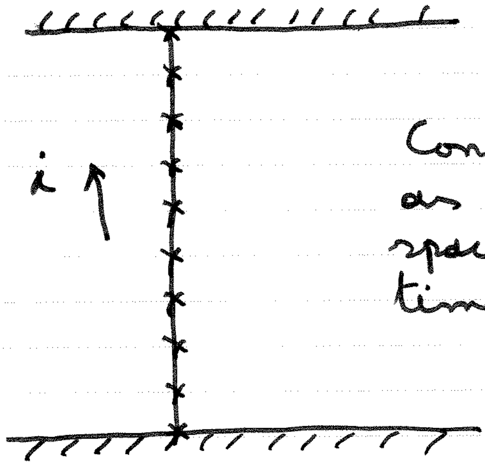
The viscous force per unit volume

$$F_v = \mu \frac{\partial^2 V}{\partial y^2}$$

This is the only force producing the fluid acceleration and there is no $V\frac{\partial V}{\partial x}$ term in it.

$$\therefore \mu \frac{\partial^2 V}{\partial y^2} = \rho \frac{\partial V}{\partial t}$$ 3

b)



Consider a grid as shown. i = the space index, n = the time index

In time

$$V_i^{n+1} + V_i^n + \frac{\partial V_i}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 V_i}{\partial t^2}\Delta t^2 + \ldots$$

So a 1st order accurate approximation to $\frac{\partial V}{\partial t}$ is

$$\frac{\partial V}{\partial t} = \frac{V_i^{n+1} - V_i^n}{\Delta t}$$

In space
$$V_{i+1}^n = V_i^n + \frac{\partial V}{\partial y}\Delta y + \frac{1}{2}\frac{\partial^2 V}{\partial y^2}\Delta y^2 + \ldots$$
$$V_{i-1}^n = V_i^n - \frac{\partial V}{\partial y}\Delta y + \frac{1}{2}\frac{\partial V^2}{\partial y^2}\Delta y^2 + \ldots$$

Q7 contd.

Adding $\rightarrow$ $V_{i+1}^n + V_{i-1}^n = 2V_i^n + \frac{\partial^2 V}{\partial y^2}\Delta y^2 + \cdots$

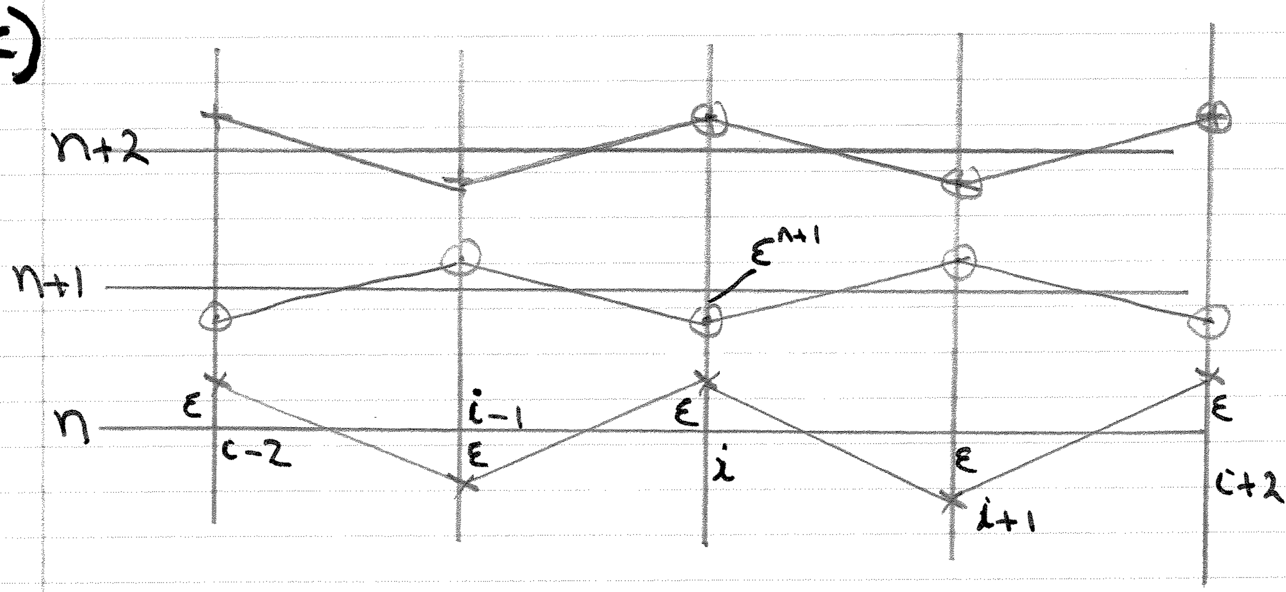
So a second order accurate approximation to

$$\frac{\partial^2 V}{\partial y^2} \text{ is } \frac{\partial^2 V}{\partial y^2} = \frac{V_{i+1}^n + V_{i-1}^n - 2V_i^n}{\Delta y^2}$$ 5

Hence the finite difference eqn is

$$\frac{V_i^{n+1} - V_i^n}{\Delta t} = \frac{\nu}{\Delta y^2}\left(V_{i+1}^n + V_{i-1}^n - 2V_i^n\right)$$

c)



Consider the perturbation

$$V_i = \varepsilon^n(-1)^i, \quad \text{at time } n=0$$

RHS of the finite diff eqn is then

$$\frac{\nu}{\Delta y^2}\left(V_{i+1}^n + V_{i-1}^n - 2V_i^n\right) = -\frac{\nu}{\Delta y^2}\,4\varepsilon_i^n$$

Hence the eqn gives.

$$\varepsilon_i^{n+1} = \varepsilon_i^n - \frac{\nu\Delta t}{\Delta y^2}\cdot 4\varepsilon_i^n$$

$$= \varepsilon_i^n\left(1 - \frac{4\nu\Delta t}{\Delta y^2}\right)$$

Q7 Cont'd

For stability $\left|\varepsilon_i^{n+1}\right| < \left|\varepsilon_i^n\right|$

i.e $\left|1 - 4\frac{\nu \Delta t}{\Delta y^2}\right| < 1$

ie $1 - 4\frac{\nu \Delta t}{\Delta y^2} > -1$

$\rightarrow \frac{\nu \Delta t}{\Delta y^2} < 0.5$ (4)

d) If we use $\Delta V = 1.5 \left.\frac{\partial V}{\partial t}\right)^n \Delta t - 0.5 \left.\frac{\partial V}{\partial t}\right)^{n-1} \Delta t$

$$\rightarrow \Delta V = \left.\frac{\partial V}{\partial t}\right)^n \Delta t + \frac{1}{2} \frac{\left[\left.\frac{\partial V}{\partial t}\right)^n - \left.\frac{\partial V}{\partial t}\right)^{n-1}\right]}{\Delta t} \Delta t^2$$

$$\rightarrow \Delta V = \left.\frac{\partial V}{\partial t}\right)^n \Delta t + \frac{1}{2} \frac{\partial^2 V}{\partial t^2} \Delta t^2$$

which is a second order accurate approx'm in time. (3)

e)

Consider a perturbation $\Delta V^n = \varepsilon (-1)^i (-1)^n$ applied to this scheme. The form of the perturbation is as shown on the previous sketch

As before $\left.\frac{\partial V}{\partial t}\right)_i^n = -\frac{\nu}{\Delta y^2} 4 \varepsilon_i$

and by simply reversing the sign

$$\left.\frac{\partial V}{\partial t}\right)_i^{n-1} = +\frac{\nu}{\Delta y^2} 4 \varepsilon_i$$

Applying these to the above equation

$$\varepsilon_i^{n+1} = \varepsilon_i - 1.5 \frac{\nu \Delta t}{\Delta y^2} \cdot 4 \varepsilon_i - 0.5 \frac{\nu \Delta t}{\Delta y^2} 4 \varepsilon_i$$

Q7 Cont'd

$$\rightarrow \quad \varepsilon_i^{n+1} = \varepsilon_i \left(1 - 8\frac{\nu\Delta t}{\Delta y^2}\right)$$

Again for stability $|\varepsilon_i^{n+1}| < |\varepsilon_i|$

i.e. $\left|1 - \dfrac{8\nu\Delta t}{\Delta y^2}\right| < 1$

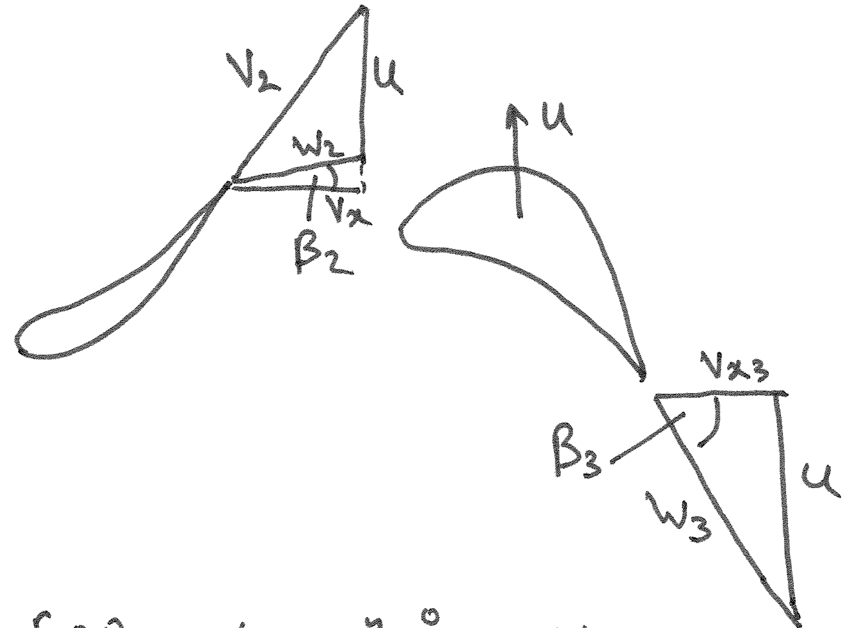
i.e. $1 - \dfrac{8\nu\Delta t}{\Delta y^2} > -1$

$$\rightarrow \quad \frac{\nu\Delta t}{\Delta y^2} < 0.25$$ (5)

So the more time accurate scheme only allows half the time step of the first order scheme.

Q8

a)



$V_2 = 500, \quad \alpha_2 = 70^\circ \rightarrow V_x = 171, \quad V_{\theta_2} = 469.8$

$V_{\theta_2 rel} = 469.8 - 400 = 69.8$

$\tan \beta_2 = \frac{69.8}{171}, \quad \underline{\underline{\beta_2 = 22.2^\circ}}$

$\tan \beta_3 = \frac{400}{171}, \quad \underline{\underline{\beta_3 = 66.85^\circ}}$ — not asked for (4)

$\Delta h_0 = U . V_{\theta_2} = \underline{\underline{187.92}}$ KJ/Kg.

$W_2 = V_x \sec \beta_2 = 184.69$ m/s

$W_3 = V_x \sec \beta_3 = 434.96$ m/s.

$\Delta h_{rot} = \frac{1}{2}(W_3^2 - W_2^2) = 77.54$ KJ/Kg

$\text{Reaction} = \frac{\Delta h_{rot}}{\Delta h_{o_r}} = \underline{\underline{41.25\,\%}}$ (4)

(since $\Delta h \equiv \Delta h_0$)

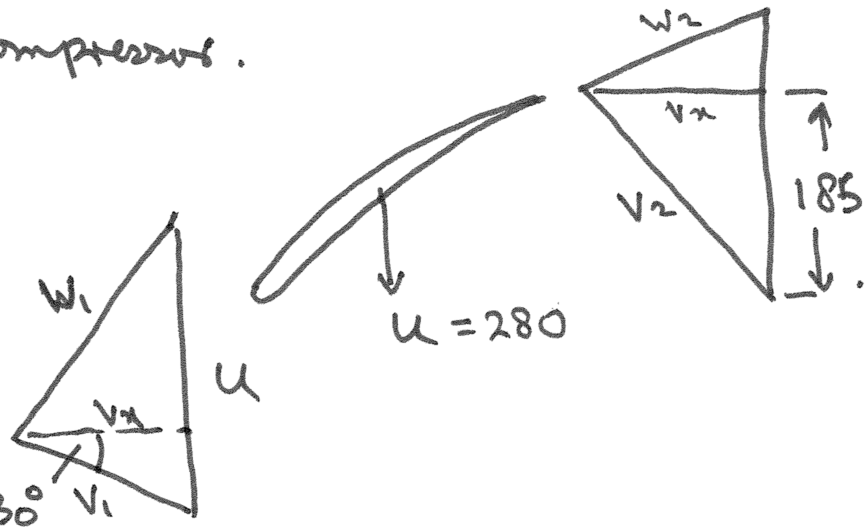
$\Delta T_0 = \frac{\Delta h_0}{C_p} = 187\,K.$

$\Delta T_{01s} = \Delta T_0 / \eta_{1s} = 210.11\,K.$

$T_{021s} = 1350 - 210.11 = 1139.88\,K.$

$P_{02}/P_{01} = \left(\frac{T_{021s}}{T_{01}}\right)^{\gamma/\gamma-1} = \underline{\underline{0.553}}$ (4)

8 Cont. b) Compressor.



$$V_x = U \cdot \phi = 280 \times .45 = 126 \text{ m/s.}$$

$$V_{\theta_1} = V_x \tan 30^\circ = 72.75 \text{ m/s.}$$

$$V_{\theta_2} = 185 \text{ – given}$$

$$\Delta h_0 = U(V_{\theta_2} - V_{\theta_1}) = \underline{\underline{31.43}} \text{ KJ/Kg.}$$

$$\therefore \text{No of stages} = \frac{\text{Turbine } \Delta h_0}{\text{Comp. } \Delta h_0 \text{stage}} = \underline{\underline{6}} \quad (3)$$

$$W_1^2 = V_x^2 + (U - V_{\theta_1})^2 = 242.55 \text{ m/s.}$$

$$V_1^2 = V_x^2 + V_{\theta_1}^2 = 145.49 \text{ m/s.}$$

$$T_1 = T_{01} - \tfrac{1}{2} V_1^2 / C_p = 289.47 \text{ K.}$$

$$M_{1rel} = W_1 / \sqrt{\gamma R T_1} = \underline{\underline{0.71}} \text{ – (not used)}$$

$$T_{01rel} = T_1 + \tfrac{1}{2} W_1^2 / C_p = 318.74 \text{ K.}$$

$$P_{01rel} = P_1 \times (T_{01rel} / T_1)^{\frac{\gamma}{\gamma-1}} = 1.40 \times P_1$$

$$P_1 = P_{01} \times (T_1 / T_{01})^{\gamma/\gamma-1} = 0.882 \text{ bar. } P_{01rel} = 1.235 \text{ bar}$$

$$\Delta P_{0rel} = Y_p * (P_{01rel} - P_1) = .04(1.4 - 1) * .882$$

$$\Delta P_{0rel} = \underline{\underline{0.014 \text{ bar}}} \quad (5)$$