

$$1. (a) \quad g = h - Ts$$

$$dg = dh - Tds - sdT$$

$$= vdp - sdT \quad (\text{using } Tds = dh - vdp)$$

$$\therefore \left(\frac{\partial g}{\partial p}\right)_T = v \quad ; \quad \left(\frac{\partial g}{\partial T}\right)_p = -s$$

$$\therefore \frac{\partial^2 g}{\partial p \partial T} = \left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$$

Taking $h = h(T, p)$:

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

$$\text{Also } dh = Tds + vdp$$

$$= T\left(\frac{\partial s}{\partial T}\right)_p dT + \left[T\left(\frac{\partial s}{\partial p}\right)_T + v\right] dp$$

Equating coefficients of dp gives:

$$\left(\frac{\partial h}{\partial p}\right)_T = v + T\left(\frac{\partial s}{\partial p}\right)_T$$

$$\therefore \left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p \quad (\text{using Maxwell's relation})$$

Triple product rule is :

$$\left(\frac{\partial h}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_h \left(\frac{\partial T}{\partial h}\right)_p = -1$$

$$\therefore \left(\frac{\partial T}{\partial p}\right)_h = -\frac{(\partial h/\partial p)_T}{(\partial h/\partial T)_p} \quad \left(\frac{\partial h}{\partial T}\right)_p = c_p \text{ by definition.}$$

$$\therefore \left(\frac{\partial T}{\partial p}\right)_h = -\frac{1}{c_p} \left[v - T\left(\frac{\partial v}{\partial T}\right)_p \right]$$

(b) A throttling process is isenthalpic. To find the change in temperature for a given change in pressure, at constant h , we need $(\partial T/\partial p)_h$. From the given equation of state,

$$p(v-b) = RT$$

$$\therefore p \left(\frac{\partial v}{\partial T} \right)_p = R$$

$$\therefore v - T \left(\frac{\partial v}{\partial T} \right)_p = \left(\frac{RT}{p} + b \right) - \left(\frac{RT}{p} \right) = b$$

$$\therefore \left(\frac{\partial T}{\partial p} \right)_h = -\frac{b}{C_p} = \text{constant.}$$

$$\text{Hence, } (T_2 - T_1) = -\frac{b}{C_p} (p_2 - p_1)$$

$$\begin{aligned} \therefore T_2 &= 600 - \frac{0.04}{1.75 \times 10^3} (2 \times 10^5 - 10 \times 10^5) \\ &= 618.3 \text{ K} \end{aligned}$$

(c) (i) From the Maxwell relation,

$$\left(\frac{\partial S}{\partial p} \right)_T = - \left(\frac{\partial v}{\partial T} \right)_p = -\frac{R}{p}$$

$$\therefore S = -R \ln p + f(T)$$

$$\text{Now, from } Tds = dh - vdp, \quad C_p = \left(\frac{\partial h}{\partial T} \right)_p = T \left(\frac{\partial S}{\partial T} \right)_p$$

$$\therefore C_p = T \frac{df}{dT}$$

$$\frac{df}{dT} = \frac{C_p}{T} \quad \longrightarrow \quad f(T) = C_p \ln T + \text{constant}$$

$$\text{Thus, } S = C_p \ln T - R \ln p + \text{constant.}$$

(Just happens to be the same as for a perfect gas)

(iii) Change in steady-flow specific exergy :

$$\begin{aligned} e_2 - e_1 &= (h_2 - T_0 s_2) - (h_1 - T_0 s_1) \\ &= T_0 (s_1 - s_2) \end{aligned}$$

because $h_2 = h_1$ in a throttling process. Thus

$$\begin{aligned} e_2 - e_1 &= T_0 \left[c_p \ln \frac{T_1}{T_2} - R \ln \frac{P_1}{P_2} \right] \\ &= 300 \left[1.75 \ln \left(\frac{600}{618.3} \right) - 0.45 \ln \left(\frac{10}{2} \right) \right] \\ &= 300 [-0.0526 - 0.7242] \\ &= -233.1 \text{ kJ/kg} \end{aligned}$$

For a perfect gas, the temperature in a throttling process does not change. Hence, for the same value of R ,

$$e_2 - e_1 = -300 \times 0.7242 = -217.3 \text{ kJ/kg.}$$

2. (a) Definition of γ_c is

$$\gamma_c = \frac{dh_{is}}{dh} = \frac{v dp}{c_p dT}$$

For a semi-perfect gas, $pv = RT$, $c_p = c_p(T)$:

$$R \frac{dp}{p} = \gamma_c \frac{c_p(T)}{T} dT$$

$$\rightarrow R \ln\left(\frac{P_2}{P_1}\right) = \gamma_c \int_{T_1}^{T_2} \frac{c_p(T)}{T} dT$$

For an expansion process $3 \rightarrow 4$:

$$R \ln\left(\frac{P_3}{P_4}\right) = \frac{1}{\gamma_c} \int_{T_4}^{T_3} \frac{c_p(T)}{T} dT$$

Reason for preferring polytropic to isentropic efficiency:

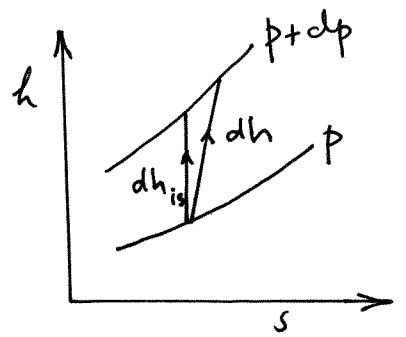
If a compressor is split into several stages and each stage has the same polytropic efficiency, then a stage-by-stage calculation will give the same final compressor exit temperature as a single, overall calculation. This will not be so for isentropic efficiencies.

$$\begin{aligned} \text{(b)} \quad \int_{T_1}^{T_2} \frac{c_p(T)}{T} dT &= \frac{c_{p0}}{T_0^n} \int_{T_1}^{T_2} T^{n-1} dT \\ &= \frac{c_{p0}}{T_0^n} \left[\frac{T^n}{n} \right]_{T_1}^{T_2} \\ &= \frac{c_{p0}}{n} \left[\left(\frac{T_2}{T_0}\right)^n - \left(\frac{T_1}{T_0}\right)^n \right] \end{aligned}$$

Thus:

$$0.287 \ln\left(\frac{30}{1}\right) = 0.9 \times \frac{1.010}{0.12} \left[\left(\frac{T_2}{T_0}\right)^{0.12} - 1 \right]$$

$$\therefore \frac{T_2}{T_0} = 2.746 \rightarrow T_2 = 818.7 \text{ K}$$



$$\begin{aligned}
 (h_2 - h_1) &= \int_{T_1}^{T_2} c_p(T) dT = \frac{C_{p0}}{T_0^n} \int_{T_1}^{T_2} T^n dT \\
 &= \frac{C_{p0}}{T_0^n} \left[\frac{T^{n+1}}{n+1} \right]_{T_1}^{T_2} \\
 &= \frac{C_{p0} T_0}{n+1} \left[\left(\frac{T_2}{T_0} \right)^{n+1} - \left(\frac{T_1}{T_0} \right)^{n+1} \right] \\
 &= \frac{1.01 \times 298.15}{1.12} \left[2.746^{1.12} - 1 \right] \\
 &= 564.6 \text{ kJ/kg air.}
 \end{aligned}$$

(c) SFEE for combustion chamber

$$(m_a + m_f) h_{p3} - m_a h_{a2} - m_f h_{f0} = 0$$

$$(A+1) h_{p3} - A h_{a2} - h_{f0} = 0$$

$$(A+1)(h_{p3} - h_{p0}) - A(h_{a2} - h_{a0}) = h_{f0} + A h_{a2} - (A+1) h_{p0} = \text{LCV}$$

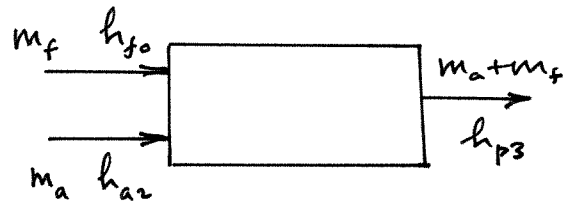
Now, $(h_{a2} - h_{a0}) = 564.6 \text{ kJ/kg}$

$$\begin{aligned}
 (h_{p3} - h_{p0}) &= \frac{1.05 \times 298.15}{1.13} \left[\left(\frac{1500}{298.15} \right)^{1.13} - 1 \right] \\
 &= 1442.5 \text{ kJ/kg}
 \end{aligned}$$

$$\therefore (1442.5 - 564.6)A = 45000 - 1442.5$$

Air/fuel ratio, $A = 49.62$

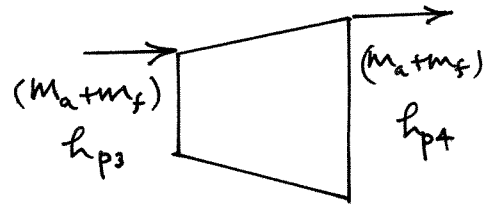
$$(f = 1/A = 0.02015)$$



$$(A = \frac{m_a}{m_f} = \text{air/fuel ratio})$$

For the turbine:

$$p_3 = 0.95 p_2 = 28.5 \text{ bar}$$



$$0.30 \ln\left(\frac{28.5}{1}\right) = \frac{1}{0.85} \frac{1.05}{0.13} \left[\left(\frac{1500}{298.15}\right)^{0.13} - \left(\frac{T_4}{298.15}\right)^{0.13} \right]$$

$$\therefore \frac{T_4}{T_0} = 2.525 \quad \rightarrow \quad T_4 = 752.8 \text{ K}$$

$$\begin{aligned} (h_3 - h_4) &= \frac{1.05 \times 298.15}{1.13} \left[\left(\frac{1500}{298.15}\right)^{1.13} - \left(\frac{752.8}{298.15}\right)^{1.13} \right] \\ &= 930.6 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Gas turbine overall efficiency} &= \frac{1.02015 \times 930.6 - 564.6}{0.02015 \times 45000} \\ &= 0.424 \end{aligned}$$

$$3. (a) \quad S_{4s} = S_3 = 6.939 \text{ kJ/kg K}$$

$$x_{4s} = \frac{6.939 - 0.437}{8.452 - 0.437} = 0.8112$$

$$\text{Heat input} = q_r = h_3 - h_1 = 3331.2 - 125.7 = 3205.5 \text{ kJ/kg}$$

$$\text{Work output} = w = \eta_{se} q_r = 0.33 \times 3205.5 = 1057.8 \text{ kJ/kg}$$

(Feed pump work has been neglected)

$$\therefore h_4 = 3331.2 - 1057.8 = 2273.4 \text{ kJ/kg}$$

$$h_{4s} = (1 - 0.8112) \times 125.7 + 0.8112 \times 2555.5 = 2096.8 \text{ kJ/kg}$$

$$\text{Turbine isentropic efficiency} = \eta_t = \frac{3331.2 - 2273.4}{3331.2 - 2096.8} = 0.857$$

$$(b) \quad q_r = (h_3 - h_1) \quad (h_3 \text{ is constant})$$

$$dq_r = -dh_1 = -C_{pf} dT_1 = -C_{pf} dT_4$$

$$w = (h_3 - h_4) = \eta_t (h_3 - h_{4s}) \quad (\eta_t, h_3 \text{ are constant})$$

$$dw = -\eta_t dh_{4s}$$

The change dh_{4s} is at constant entropy so $T_4 ds_{4s} = dh_{4s} - v_{4s} dp = 0$

Note that the Tds equation is true for a pure substance and there can be more than one phase present:

$$v_{4s} = (1 - x_{4s})v_f + x_{4s}v_g \quad (v_f, v_g \text{ evaluated at } T_4)$$

$$\approx x_{4s}v_g \quad (\text{neglect liquid volume})$$

$$\therefore dw = -\eta_t x_{4s} v_g dp$$

Now, dp is the pressure change associated with the temperature change dT_4 and, as the steam remains saturated, the Clausius - Clapeyron equation can be applied to give:

$$\frac{dp}{dT_4} = \frac{h_{fg}}{v_f T_4} \approx \frac{h_{fg}}{v_g T_4} \quad (v_f \ll v_g)$$

$$\therefore dw = -\eta_t x_{4s} h_{fg} \frac{dT_4}{T_4}$$

(c) η_b is the HRSG efficiency: heat transferred from GT exhaust to steam cycle divided by maximum possible heat transfer (when stack temperature = 25°C)

$$\eta_b = \frac{q}{q_{\max}}$$

$$\eta_{cc} = 0.35 + 0.80 \times 0.33 \times 0.65 = 0.522 \quad (52.2\%)$$

$$(d) \quad \eta_{st} = \frac{w}{q}$$

$$\therefore \frac{d\eta_{st}}{\eta_{st}} = \frac{dw}{w} - \frac{dq}{q} = \frac{-\eta_t x_{4s} h_{fg}}{w T_4} dT_4 + \frac{C_{pf}}{q} dT_4$$

$$= \frac{-0.857 \times 0.8112 \times 2429.8 \times 10}{1057.8 \times 303.15} + \frac{4.18 \times 10}{3205.5}$$

$$= -0.05268 + 0.01304$$

$$= -0.03964$$

$$d\eta_{st} = -0.33 \times 0.03964 = -0.0131$$

i.e., a decrease in η_{st} of 1.31 percentage points.

(This is why power station testing always takes place in mid-winter!)

$$\frac{d\eta_b}{\eta_b} = \frac{dq_r}{q}$$

(No change in q_{max} because GT exhaust temperature and pinch point conditions do not change.)

$$\therefore d\eta_b = -0.80 \times 0.01304 = -0.0104$$

i.e., a decrease in η_b of 1.04 percentage points because less heat is transferred to the steam cycle.

$$\eta_{cc} = \eta_{gt} + \eta_b \eta_{st} (1 - \eta_{gt})$$

$$d\eta_{cc} = (1 - \eta_{gt}) \eta_b \eta_{st} \left[\frac{d\eta_{st}}{\eta_{st}} + \frac{d\eta_b}{\eta_b} \right]$$

$$= (\eta_{cc} - \eta_{gt}) \left[\frac{d\eta_{st}}{\eta_{st}} + \frac{d\eta_b}{\eta_b} \right]$$

$$= -(\eta_{cc} - \eta_{gt}) \left[\frac{d\omega}{\omega} \right]$$

$$= -(0.522 - 0.350) \times 0.05268$$

$$= -0.0091$$

i.e., a decrease in η_{cc} of 0.91 percentage points.

(The steam cycle is less important when it is part of a combined cycle.)

4 (a) The junction loss is the loss associated with the flow of minority carriers at the n-p semiconductor junction in the photovoltaic cell - holes to the p-type material, electrons to the n-type material. In the equation for the net current density at the junction

$$J = J_0 \left[\exp\left(\frac{eV_L}{kT}\right) - 1 \right]$$

the junction loss is represented by the -1 term.

The spectrum loss is associated with the energy spectrum of the incident sunlight and the excitation energy E_g of the semiconductor. Any photon with energy $< E_g$ cannot produce an ion pair so its energy is lost (as thermal energy). Photons with energy $> E_g$ will normally produce one ion pair and the excess energy will be lost (as thermal energy).

The two most significant of the other (minor) losses are the reflection of light off the photovoltaic cell without entering it, and the recombination of ion pairs before they reach the p-n junction.

(b) The power output from the cell (per unit area) is

$$P = J_L V_L = J_S V_L - J_0 V_L \left[\exp\left(\frac{eV_L}{kT}\right) - 1 \right]$$

P is maximised when $dP/dV_L = 0$.

$$\frac{dP}{dV_L} = J_S - J_0 \left[\exp\left(\frac{eV_L}{kT}\right) - 1 \right] - J_0 V_L \frac{e}{kT} \exp\left(\frac{eV_L}{kT}\right)$$

So $dP/dV_L = 0$ when

$$J_0 \exp\left(\frac{eV_L}{kT}\right) \left[1 + \frac{eV_L}{kT} \right] = J_S + J_0$$

$$\therefore \exp\left(\frac{eV_L}{kT}\right) = \frac{1 + J_S/J_0}{1 + eV_L/kT} \quad (1)$$

4 (b) continued

If $V_L = 0.504 \text{ V}$ and $T = 10^\circ\text{C} = 283 \text{ K}$

$$\frac{eV_L}{kT} = \frac{1.6 \times 10^{-19} \times 0.504}{1.381 \times 10^{-23} \times 283} = 20.63$$

$$\text{LHS of (1)} = \exp(20.63) = 914 \times 10^6$$

$$\text{RHS of (1)} = \frac{1 + 200/10^{-8}}{1 + 20.63} = 925 \times 10^6$$

This is close enough given the nature of (1), so the power output is maximised when $V_L = 0.504 \text{ V}$.

For $V_L = 0.504 \text{ V}$

$$\begin{aligned} J_L &= J_s - J_0 \left(\exp\left(\frac{eV_L}{kT}\right) - 1 \right) \\ &= 200 - 10^{-8} (\exp(20.63) - 1) = 190.9 \text{ A/m}^2 \end{aligned}$$

$$\therefore P_L = J_L V_L = 190.9 \times 0.504 = \underline{\underline{96.2 \text{ W/m}^2}}$$

$$R_L = V_L / J_L = 0.504 / 190.9 = \underline{\underline{2.64 \times 10^{-3} \Omega\text{m}^2}}$$

(c) Equation (1) still gives the conditions for the optimum.

This must be solved iteratively or by trial and error.

A suitable iterative scheme is to write $\frac{eV_L}{kT}$ as X and then rearrange (1) as

$$X = \ln \left[\frac{1 + J_s/J_0}{1 + X} \right]$$

This can be used as a basis for iterative solution by

$$X_{n+1} = \ln \left[\frac{1 + J_s/J_0}{1 + X_n} \right]$$

For the values given

$$X_{n+1} = \ln \left[\frac{4 \times 10^{10}}{1 + X_n} \right]$$

Starting from $X_0 = 0$ successive iterands are:

$$0, 24.412, 21.177, 21.313, 21.307, 21.307$$

4(c) continued

$$\text{Hence } \frac{eV_L}{kT} = 21.307$$

$$\Rightarrow V_L = \frac{21.307 \times 1.381 \times 10^{-23} \times 303}{1.6 \times 10^{-19}} = \underline{\underline{0.557 \text{ V}}}$$

$$\begin{aligned} \therefore J_L &= J_s - J_0 \left(\exp\left(\frac{eV_L}{kT}\right) - 1 \right) \\ &= 400 - 10^{-8} \left(\exp(21.307) - 1 \right) = 382.1 \text{ A/m}^2 \end{aligned}$$

$$\therefore P_L = J_L V_L = 382.1 \times 0.557 = \underline{\underline{212.8 \text{ W/m}^2}}$$

$$R_L = V_L / J_L = 0.557 / 382.1 = \underline{\underline{1.46 \times 10^{-3} \Omega \text{m}^2}}$$

The increase in J_s with the change in insolation (J_s doubles here) enables the power output to be increased by a factor of 2.2, but in order to achieve this the load resistance must be decreased. Thus for optimal photovoltaic cell performance over the course of a day the load resistance must be varied as the level of insolation varies.

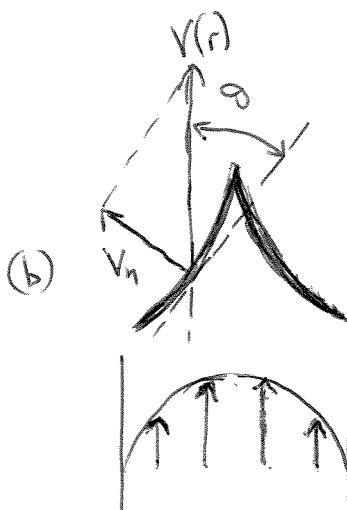
5. (a) • laminar burning velocity: the speed with which a steady, planar, laminar flame propagates against stationary reactants.

• Flammability limits: the % by vol of fuel in a premixed mixture between which a flame can propagate. No flame propagation if $\phi < \phi_{lean}$ or if $\phi > \phi_{rich}$. (Flammability limits are also expressed in terms of equivalence ratio.)

• Quenching distance: the diameter of the smallest circular pipe in which a flame can propagate.

• Lean limit: low $\phi \Rightarrow$ low $T \Rightarrow$ chain branching reactions fail to produce enough radicals \Rightarrow flame not possible

Rich limit: high $\phi \Rightarrow$ low(ish) T , no O_2 , lots of fuel \Rightarrow not enough radicals ($H + O_2 \rightarrow OH + O$ fails) \Rightarrow flame not possible.

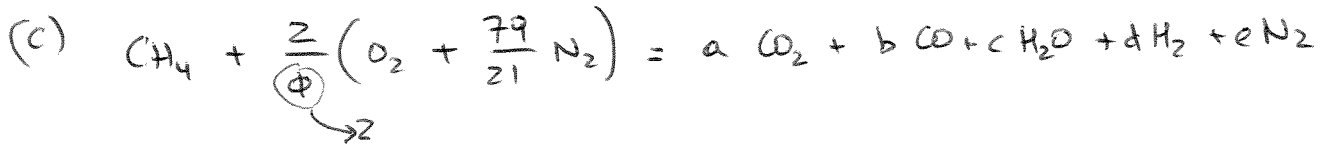


V_n must be equal to S_L

$$\Rightarrow V(r) \sin \theta = S_L$$

$$\Rightarrow \sin \theta = \frac{S_L}{V_0 \left(1 - \frac{r^2}{R^2}\right)}$$

Flame is not conical because V is not uniform.



Conservation of atoms:

$$\text{C: } 1 = a + b \quad \Rightarrow \quad 1 = a + b \quad (1)$$

$$\text{H: } 4 = 2c + 2d \quad \Rightarrow \quad 2 = c + d \quad (2)$$

$$\text{O: } 2 = 2a + b + c \quad \Rightarrow \quad 2 = 2a + b + c \quad (3)$$

$$\text{N: } 2 \cdot \frac{79}{21} = 2 \cdot e \quad \Rightarrow \quad e = \frac{79}{21} \quad (4)$$

5 species in products, 4 conservation eqns \Rightarrow
we need 1 equilibrium reaction.

Rich combustion ($\phi=2$) \Rightarrow pick water-gas shift reaction
(no O_2 in products \Rightarrow no other alternative).



$$K_p = \frac{(P_{\text{H}_2}/P^0)(P_{\text{CO}_2}/P^0)}{(P_{\text{CO}}/P^0)(P_{\text{H}_2\text{O}}/P^0)}$$

$$= \frac{X_{\text{H}_2} \cdot X_{\text{CO}_2}}{X_{\text{CO}} \cdot X_{\text{H}_2\text{O}}} \frac{P/P^0 \cdot P/P^0}{P/P^0 \cdot P/P^0} = \frac{a \cdot d}{b \cdot c} \quad (5)$$

$$K_p = \exp(-1.091) \text{ at } 1600 \text{ K.}$$

Solution: From (1), (2), & (3): $\underline{a=1-b}$
 $d=2-c \Rightarrow d=2-(2-2a-b)$
 $\Rightarrow \underline{d=2-b}$
 $c=2-2a-b \Rightarrow \underline{c=b}$

$$\therefore K_p = \frac{(1-b)(2-b)}{b^2} \Rightarrow (K_p+1)b^2 + 3b - 2 = 0$$

$$\Rightarrow b = 0.8129 \text{ or } \underline{3.7042}$$

\hookrightarrow impossible because $d < 0$

Hence: $a = 0.1871$

$$b = 0.8129$$

$$c = 0.8129$$

$$d = 1.1871$$

$$e = 3.7619$$

$$h = 6.7558$$

$$X_{CO_2} = 2.77\%$$

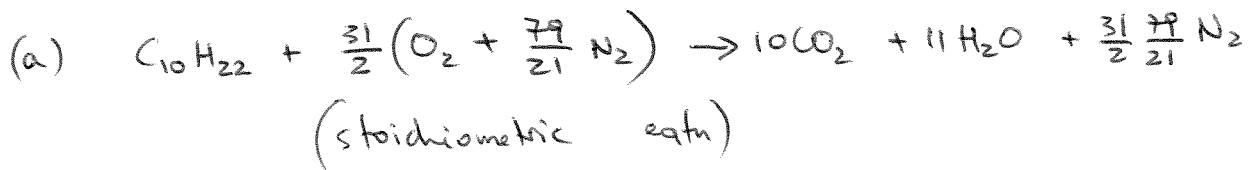
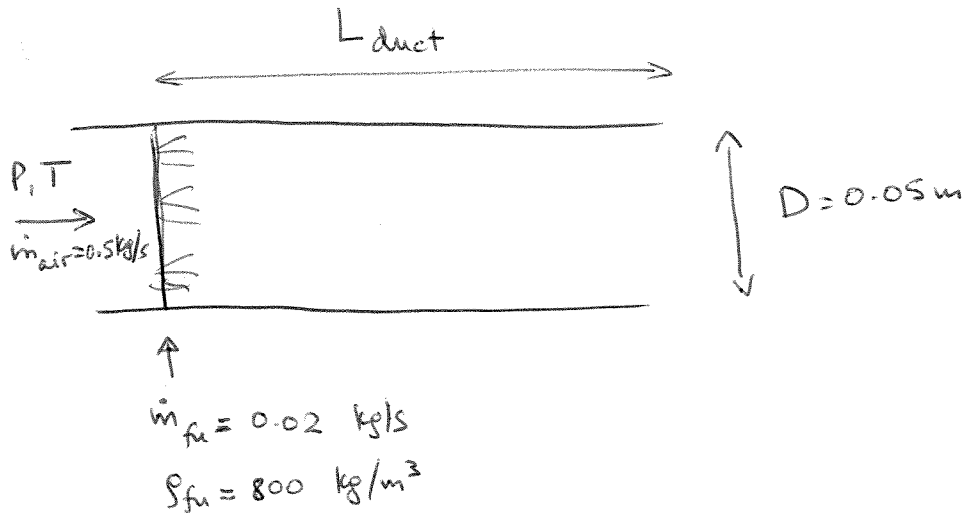
$$X_{CO} = X_{H_2O} = 12.03\%$$

\Rightarrow

$$X_{H_2} = 17.57\%$$

$$X_{N_2} = 55.68\%$$

6



$$\phi = \frac{(\dot{m}_{fu}/\dot{m}_{air})}{(\dot{m}_{fu}/\dot{m}_{air})_{st}}$$

$$\left(\frac{\dot{m}_{fu}}{\dot{m}_{air}} \right)_{st} = \frac{1 + (10 + 12 + 22 + 1)}{\frac{31}{2} \left(32 + \frac{79}{21} \cdot 28 \right)} = \frac{142}{2129} = 0.0667$$

$$\Rightarrow \phi = \frac{(0.02/0.5)}{0.0667} = \underline{\underline{0.6}}$$

(b)
$$\frac{d}{dt} (\text{mass of droplet}) = - \dot{m}'' \cdot (\text{surface area}) \Rightarrow \frac{d}{dt} \left[\rho_{fu} \frac{4}{3} \pi \left(\frac{d}{2} \right)^3 \right] = - \frac{\rho_{fu} \cdot \beta}{4d} \cdot 4\pi \frac{d^2}{4}$$

$$\Rightarrow \frac{d}{dt} (d) = - \frac{\beta}{2d} \quad (\Rightarrow) \quad \left[\frac{d^2}{2} \right]_{d_{in}}^{d} = - \frac{\beta t}{2}$$

$$\Rightarrow d_{in}^2 - d^2 = \beta t$$

$$\therefore t_{evap} \text{ when } d = 0 \Rightarrow t_{evap} = \frac{(40 \times 10^{-6})^2}{1 \times 10^6} = \underline{\underline{1.6 \times 10^{-3} \text{ s}}}$$

To find L_{duct} for complete evaporation, we need U :

$$\dot{m} = \rho U A \Rightarrow U \approx \frac{\dot{m}_{air}}{\frac{\pi}{4} D^2 \cdot \frac{\rho \cdot MW}{RT}} \approx \frac{0.5}{\frac{\pi}{4} (0.05)^2 \cdot \frac{10 \cdot 10^5 \cdot 288}{8314 \cdot 1000}}$$

$$\Rightarrow U \approx 73.5 \text{ m/s}$$

$$\Rightarrow L_{duct} = U \cdot t_{evap} = \underline{\underline{0.12 \text{ m}}}$$

(c) $\tau_{ign} \propto \exp\left(\frac{E}{R \cdot T}\right)$ (important dependence, worth remembering)
 $\propto \exp\left(\frac{T_{act}}{T}\right)$

$$\frac{\tau_{ign}(1000)}{\tau_{ign}(950)} = \frac{\exp\left(\frac{15000}{1000}\right)}{\exp\left(\frac{15000}{950}\right)} = 0.454$$

If U same, $L_{ign}(950 \text{ K}) = 2.2 L_{ign}(1000 \text{ K})$

\Rightarrow OK, outside duct.

To avoid autoignition: decrease T } although these
 decrease P } will be limited
 by thermodynamic
 considerations

decrease $d_{in} \Rightarrow$ shorter time
 needed for evaporation
 \Rightarrow safer!

$$7. (a) \quad m_f Q = m_{TOT} C_v (T_3 - T_2)$$

$T_3 = T$ after combustion

(First law)

$T_2 = T$ before combustion

$$m_{TOT} = m_f + m_a + m_r$$

$m_r =$ mass residual gas

$$x_r = \frac{m_r}{m_{TOT}}$$

$$\Rightarrow m_a + m_f = (1 - x_r) m_{TOT}$$

$$\frac{m_f}{m} = \frac{m_f}{m_a} \frac{m_a}{m_{TOT}} = \frac{m_f}{m_a} \frac{m_{TOT} - m_f - m_r}{m_{TOT}} = \frac{m_f}{m} \left(1 - \frac{m_f}{m_{TOT}} - x_r\right)$$

Assuming $\frac{m_f}{m_{TOT}} \ll 1$, $\frac{m_f}{m_{TOT}} \ll x_r$

$$\Rightarrow \frac{m_f}{m_{TOT}} \approx \frac{m_f}{m_a} (1 - x_r)$$

Substituting in First law: $T_3 - T_2 = \frac{m_f}{m_a} \frac{Q(1-x_r)}{C_v}$ QED.

$$(b) \quad \frac{(T_3 - T_2)_B}{(T_3 - T_2)_A} = \frac{(m_f/m_a)_B}{(m_f/m_a)_A} \cdot \frac{(1-x_r)_B}{(1-x_r)_A} = \frac{1-0.3}{1-0.1} = 0.778$$

$\left(\frac{m_f}{m_a} \right)$ in both cases equal because operation remains stoichiometric

$$\frac{T_2}{T_1} = \Gamma_c^{\gamma-1} = 9^{0.4} = 2.408$$

$$\Rightarrow T_2 = 2.408 + 300 = 722 \text{ K}$$

$$\Rightarrow T_{3A} = 722 + 2100 = 2822 \text{ K}$$

$$(T_3 - T_2)_B = 0.778 (T_3 - T_2)_A = 0.778 \cdot 2100 = 1634 \text{ K}$$

$$\Rightarrow \underline{\underline{T_{3B} = 2356 \text{ K}}}$$

$$\begin{aligned}
 (c) \quad W_{NET} &= \eta_{f,i} m_f Q \\
 &= [1 - r_c^{-(\gamma-1)}] m_f Q \\
 &= [1 - r_c^{-(\gamma-1)}] \frac{m_f}{m_a} Q m_a \quad \left(\left(\frac{m_f}{m_a} \right)_A = \left(\frac{m_f}{m_a} \right)_B \right)
 \end{aligned}$$

$$\Rightarrow \frac{W_{NET, B}}{W_{NET, A}} = \frac{m_{aB}}{m_{aA}} \approx \frac{(1 - x_r)_B}{(1 - x_r)_A} = 0.778$$

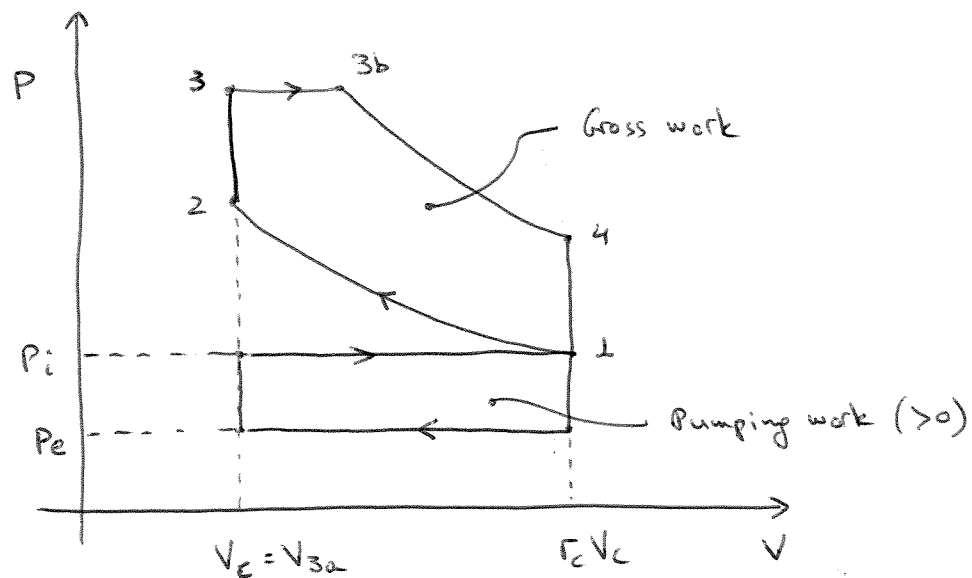
There is a net reduction in total charge admitted and a net reduction in work per cycle, with a proportional reduction in fuelling rate.

(d) Fuel conversion efficiency is unchanged under ideal conditions

$$\eta_{f,i} = 1 - r_c^{-(\gamma-1)} = 0.585$$

(e) To increase work, while keeping EGR, mass of charge introduced in cylinder must be increased, e.g. by turbocharging.

8. (a)



(b)

Point 1:

$$P_1 = P_i = 2 \text{ bar}$$

$$V_1 = r_c V_c$$

$$V_{\text{displ}} = r_c V_c - V_c = (r_c - 1) V_c \Rightarrow V_c = \frac{V_{\text{displ}}}{r_c - 1} = \frac{0.5 \times 10^{-3}}{14 - 1}$$

$$= 0.0385 \times 10^{-3} \text{ m}^3$$

$$V_1 = r_c V_c = 0.538 \times 10^{-3} \text{ m}^3$$

$$T_1 = T_i = 325 \text{ K}$$

Point 2:

$$V_2 = V_c = 0.0385 \times 10^{-3} \text{ m}^3$$

$$P_2 = P_1 r_c^\gamma = 2 \cdot 14^{1.4} = 80.46 \text{ bar}$$

$$T_2 = T_1 r_c^{\gamma-1} = 325 \cdot 14^{0.4} = 934 \text{ K}$$

Point 3a:

$$m_a (T_{3a} - T_2) = m_f Q \cdot \epsilon \quad \epsilon = 80\%$$

$$\Rightarrow T_{3a} = T_2 + \frac{m_f Q \cdot \epsilon}{m_a} = T_2 + \frac{m_f}{m_a + m_f} \frac{Q \cdot \epsilon}{c_w}$$

$$= T_2 + \frac{m_f/m_a}{1 + m_f/m_a} \frac{Q \cdot \epsilon}{c_w} = 934 + \frac{0.03}{1 + 0.03} \cdot \frac{43000 \cdot 0.8}{0.8}$$

$$\Rightarrow T_{3a} = 2186 \text{ K}$$

Hence $P_{3a} = P_2 \frac{T_{3a}}{T_2} = 188.3 \text{ bar}$

$$V_{3a} = V_C = 0.0385 \times 10^{-3} \text{ m}^3$$

Point 3b :

$$m c_p (T_{3b} - T_{3a}) = m Q (1 - \epsilon)$$

$$\Rightarrow T_{3b} = T_{3a} + \frac{m_f/m_a}{1 + m_f/m_a} \frac{Q (1 - \epsilon)}{\gamma C_v}$$

$$\Rightarrow T_{3b} = 2410 \text{ K}$$

$$P_{3b} = P_{3a} = 188.3 \text{ bar}$$

$$\frac{V_{3b}}{V_{3a}} = \frac{T_{3b}}{T_{3a}} \Rightarrow V_{3b} = 0.0424 \times 10^{-3} \text{ m}^3$$

Point 4 :

$$V_4 = r_c V_C = 0.538 \times 10^{-3} \text{ m}^3$$

$$T_4 = T_{3b} \left[\frac{V_4}{V_{3b}} \right]^{(\gamma-1)} = 2410 \left(\frac{0.538}{0.0424} \right)^{-0.4} = 872.3 \text{ K}$$

$$P_4 = P_{3b} \left[\frac{V_4}{V_{3b}} \right]^{\gamma} = 188.3 \left(\frac{0.538}{0.0424} \right)^{-1.4} = 5.37 \text{ bar}$$

(c) $GIMEPF = \frac{W_{NET}}{V_d}$; $W_{NET} = W_{3a \rightarrow 3b} + W_{3b \rightarrow 4} + W_{12}$

$$W_{3a \rightarrow 3b} = P_3 (V_{3b} - V_{3a}) = 73.4 \text{ J}$$

$$\begin{aligned} W_{3b \rightarrow 4} &= m c_v (T_{3b} - T_4) = \frac{P_1 V_1}{R T_1} C_v (T_{3b} - T_4) \\ &= \frac{P_1 V_1}{T_1} \frac{1}{\gamma - 1} (T_{3b} - T_4) = 1272.9 \text{ J} \end{aligned}$$

$$W_{12} = -m c_v (T_2 - T_1) = -\frac{P_1 V_1}{T_1} \frac{1}{\gamma - 1} (T_2 - T_1) = -504 \text{ J}$$

$$\Rightarrow W_{NET} = 73.4 + 1272.9 - 504 = \underline{\underline{842 \text{ J}}}$$

$$GIMEPF = 842 / 0.5 \times 10^{-3} = \underline{\underline{16.85 \text{ bar}}}$$

Net indicated mean effective pressure

$$= \text{GIMEP} - (P_e - P_i) = 16.85 - (1.8 - 2.0) \\ = \underline{\underline{16.65}} \text{ bar}$$

positive pumping work by pressurized manifold

(d) Compressor \geq turbine work must match

$$\Rightarrow W_c = W_T$$

$$\Rightarrow \dot{m}_a C_p (T_{c,out} - T_{c,in}) = (\dot{m}_f + \dot{m}_a) C_p (T_{T,in} - T_{T,out}) \quad \eta_M \rightarrow \approx 100\%$$

$$\Rightarrow T_{c,in} \frac{r^{(\gamma-1)/\gamma} - 1}{\eta_c} = \left(\frac{\dot{m}_f}{\dot{m}_a} + 1 \right) T_{T,in} \left[1 - r_{PT}^{-\frac{(\gamma-1)}{\gamma}} \right] \eta_T$$

$$\Rightarrow 285 \cdot \frac{(2/1)^{\frac{0.4}{1.4}} - 1}{0.8} = 1.03 \cdot 700 \cdot \left[1 - \left(\frac{1.8}{1.0} \right)^{-\frac{0.4}{1.4}} \right] \eta_T$$

$$\Rightarrow \eta_T = 0.70$$