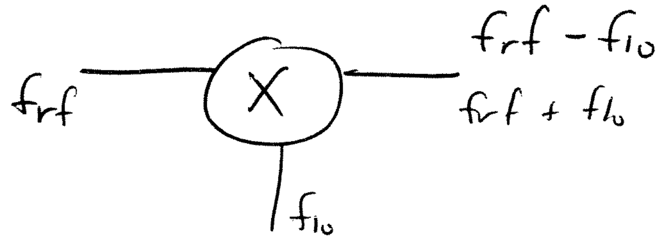
T.D. WILKINSON

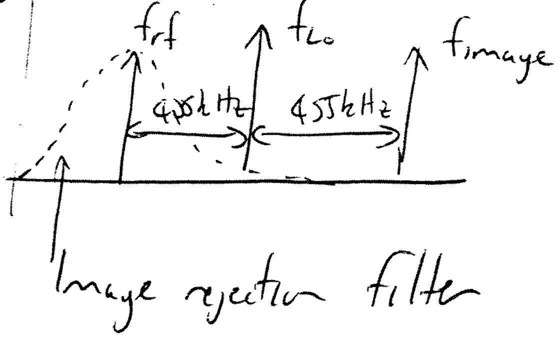
PART IIA MODULE 3B1 2003 SOLUTIONS

Q1 a) Image rejection is an important part of a Superhet AM radio. The mixer produces two possible combinations of the input frequencies: the sum & the difference



The sum frequency $f_{rf} + f_{lo}$ is removed by the IF filter stage, however there is still a second RF frequency 455 kHz above the f_{LO} which will be mixed down to $f_{lo} - f_{rf} = 455\,kHz$ which is the image frequency.

The image frequency is removed by tuning the input RF signal through a filter ganged to the LO.

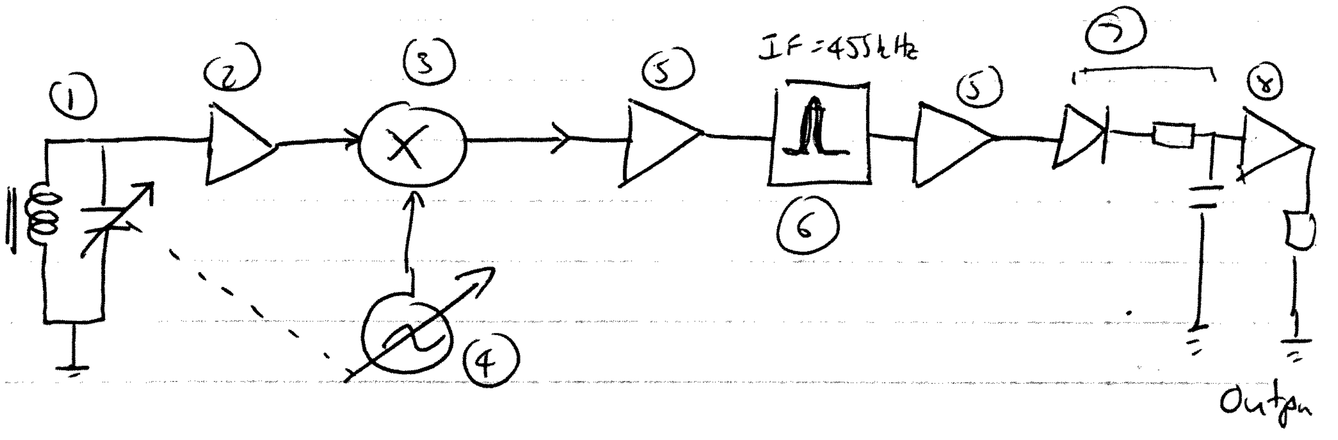


$f_{lo} = 1.6\,MHz$

$\Rightarrow$ radio station $= f_{lo} - 455$
$= 1.145\,MHz$

Image frequency $= f_{lo} + 455\,kHz$
$= 2.055\,MHz$

b)



① Aerial (inductor) + tuned capacitor ganged to LO. This forms the Image rejection filter

(2) RF amp to boost the RF signal & to match impedances (buffer)

(3) Mixer (analogue multiplier) forms the analogue product of the ~~?~~ two inputs (sum & difference)

(4) Local oscillator (LO) provides a strong oscillation ganged to the RF filter.

(5) IF amps to boost IF ~~frequency~~ signal, narrow band at 455 kHz.

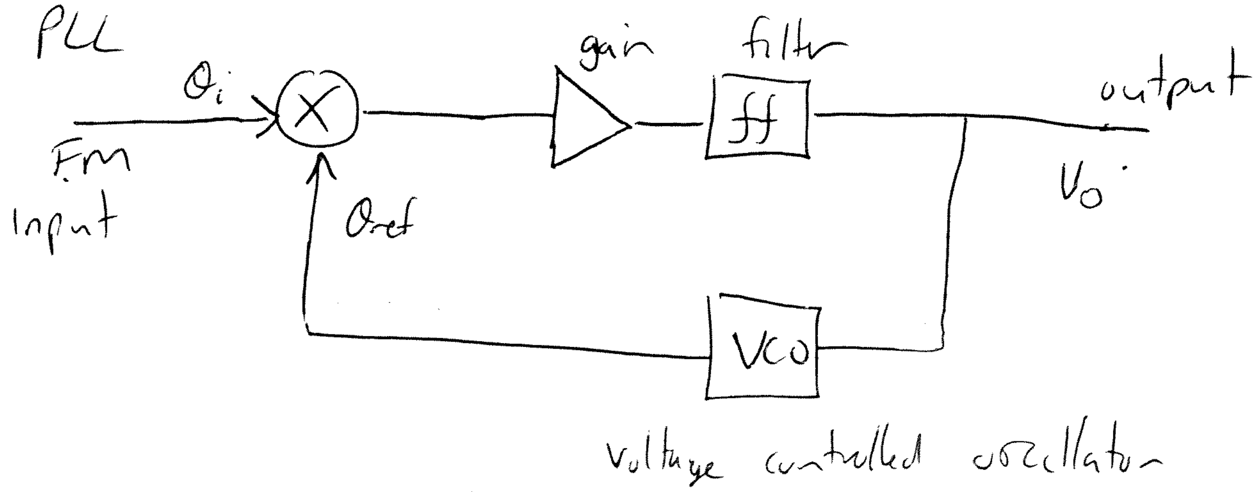
(6) IF filter. Removes sum frequency & other harmonics generated in the LO & mixer ~~?~~ plus removes residual LO signal

(7) AM demodulator. Performs evelope detection on the IF signal & filters out the sum frequency, leaving the base band signal

(8) Base band amplifier. (audio amp)

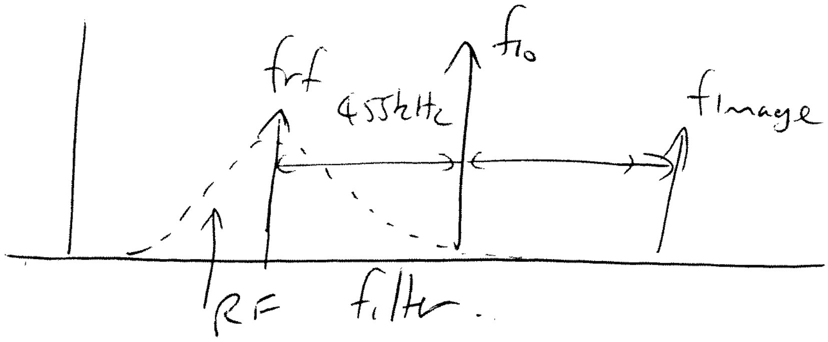
<u>Advantages</u>
* More selective than the RF radio
* IF amplifier & filter work at fixed frequency ⇒ are simple to design and narrow band
* Easier to produce large signal ~~?~~ (IF) for the AM demodulator ⇒ more linear.
* More resistant to noise.

c) FM demodulator can be made from a PLL

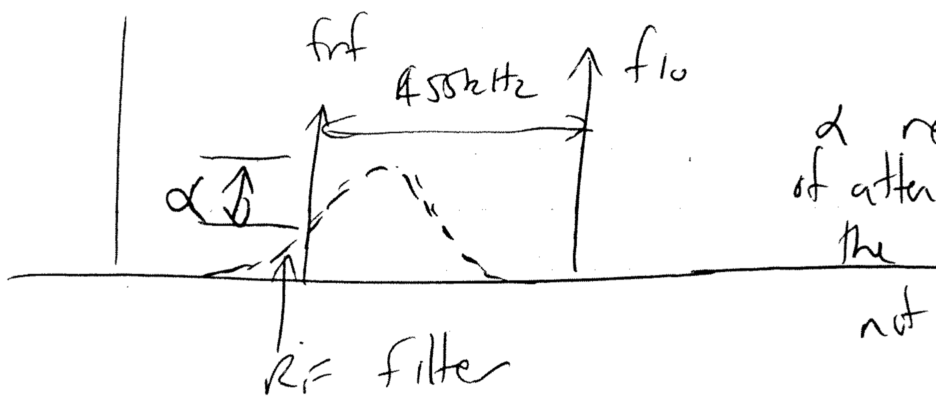


PLL changes the frequency of θ_{ref} so that the difference $\theta_i - \theta_{ref}$ will be zero. Hence the voltage into the VCO is proportional to the frequency of the input signal. If the response time of the PLL is fast enough, then V_o will be the FM demodulated version of the input signal.

d) The LO & RF filter must track in a superhet radio so that the image frequency corresponding to the LO frequency will be rejected. If a static RF filter is used, ~~&~~ then the radio response will follow the filter, giving a narrow tuning range for the radio.

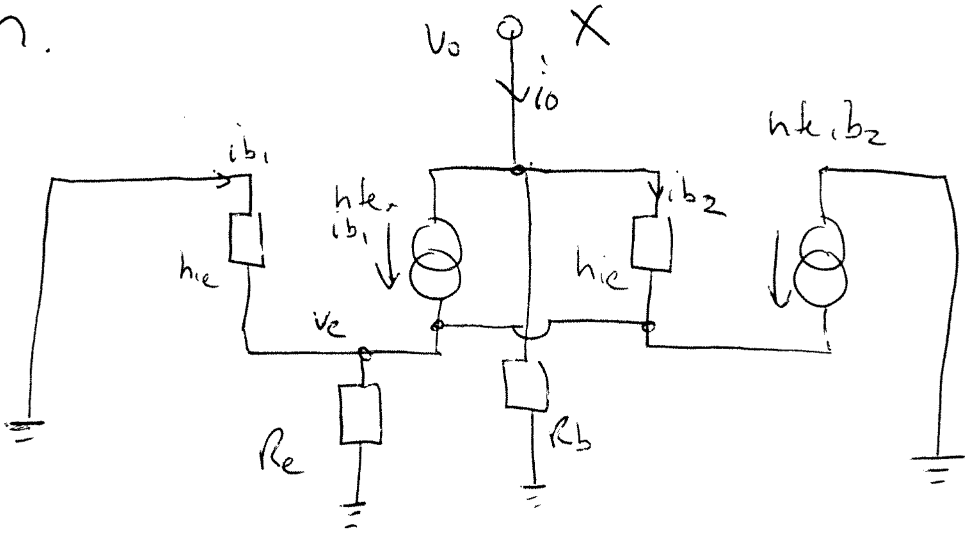


We always want the frequency frf which is 455 kHz less than f_{LO} to be at the peak of the RF filter hence it must be ganged to f_{LO}



α represents the amount of attenuation due to the RF filter if not properly ganged.

Q2 SSM.



Assume that h_{ie} is the same for both transistors and that C is low impedance.

$$\Rightarrow \; i_{b1} = \frac{0 - V_e}{h_{ie}} = \frac{-V_e}{h_{ie}} \qquad i_{b2} = \frac{V_o - V_e}{h_{ie}}$$

$$V_e = (\cancel{1+}h_{fe})(i_{b1} + i_{b2})R_e$$

$$i_o = h_{fe} i_{b1} + i_{b2} + \frac{V_o}{R_b} \qquad \text{sub for } i_{b1} + i_{b2}$$

$$= -\frac{h_{fe} V_e}{h_{ie}} + \frac{V_o - V_e}{h_{ie}} + \frac{V_o}{R_b} \qquad (1)$$

$$V_e = \left[-\frac{h_{fe} V_e}{h_{ie}} + \frac{h_{fe} V_o}{h_{ie}} - \frac{h_{fe} V_e}{h_{ie}} \right] R_e = \frac{h_{fe} R_e V_o}{h_{ie}} - \frac{2 h_{fe} V_e}{h_{ie}}$$

$$\Rightarrow V_e = \frac{V_o}{2 + \frac{h_{ie}}{h_{fe} R_e}} \qquad (2) \qquad \text{sub (2) into (1)}$$

$$\Rightarrow i_o = \frac{-h_{fe}}{h_{ie}} \frac{V_o}{2 + \frac{h_{ie}}{h_{fe} R_e}} + \frac{V_o}{h_{ie}} - \frac{1}{h_{ie}} \frac{V_o}{2 + \frac{h_{ie}}{h_{fe} R_e}} + \frac{V_o}{R_b}$$

$$= \frac{V_o}{h_{ie}} + \frac{V_o}{R_b} - \frac{V_o(\cancel{1+}h_{fe})}{h_{ie}\left(2 + \frac{h_{ie}}{h_{fe} R_e}\right)}$$

if $h_{fe} R_e \gg h_{ie}$ then $h_{ie}/h_{fe}R_e \to 0$

$$\Rightarrow i_o \approx \frac{V_o}{h_{ie}} + \frac{V_o}{R_b} - \frac{V_o h_{fe}}{2h_{ie}}$$

$$\Rightarrow Z_o = \frac{V_o}{i_o} = \left[\frac{1}{h_{ie}} + \frac{1}{R_b} - \frac{h_{fe}}{2h_{ie}}\right]^{-1} = R_b // h_{ie} // \left(-\frac{2h_{ie}}{h_{fe}}\right)$$

$$\equiv (-2r_e)$$

Hence with the correct choice of R_b + R_e, Z_o will be negative

<u>Check</u> $V_{SS} = -5V$ $\Rightarrow V_E = -0.75V$

$$\Rightarrow 2I_E = \frac{-0.75 - -5}{22k} = 0.193\,mA \quad I_E = 0.1\,mA$$

$$\Rightarrow I_C = h_{fe} I_B = \frac{h_{fe} I_E}{1+h_{fe}} \approx I_E \quad \Rightarrow r_e = \frac{25}{I_C} = \frac{25}{0.1} \text{ (in mA)}$$

$$= 250\ \Omega$$

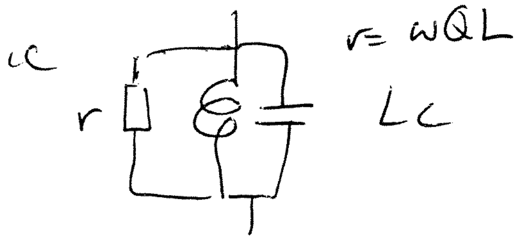
$h_{ie} = h_{fe} r_e = 50k\Omega$

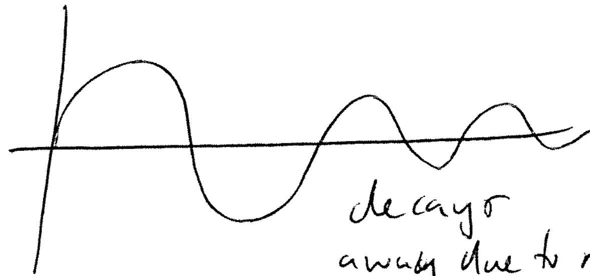
$\Rightarrow h_{fe} \times R_e = 200 \times 22k = 4.4M\Omega$ $(\gg h_{ie})$

$\Rightarrow$ approximation is OK

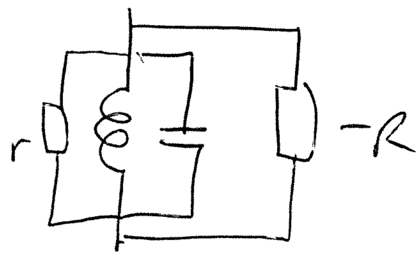
$$Z_o = 10k // 50k // (-2 \times 250) = -532\,\Omega$$

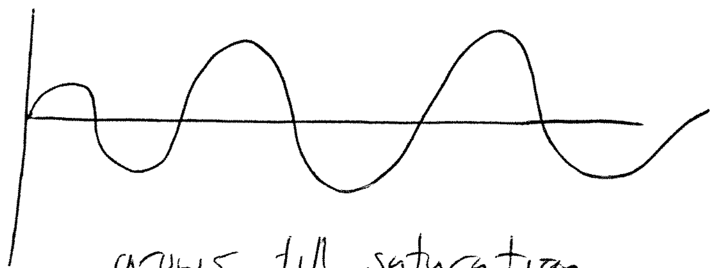
b) If a -ve R is combined with an LC resonant tank, then the result will be unstable as long as the -ve R // r at resonance is still negative





decays away due to r





grows till saturation when -R // r = 0

⇒ oscillator

1 MHz oscillation ⇒ $\omega^2 = \frac{1}{LC} = (2\pi f)^2$

⇒ choose $C \approx 50pF$ typical for a tuneable capacitor

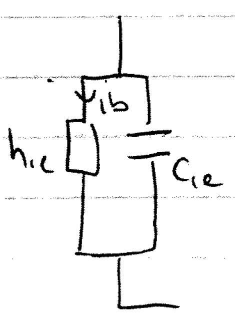
$\Rightarrow L = \frac{1}{C(2\pi f^2)} = 507\ \mu H$

$r = \omega QL = 2\pi \times 1 \times 10^6 \times 50 \times 507 \times 10^{-6}$

$= 160k\ (> |-R|)$

c) Approx $f_T = \frac{1}{2\pi C_{ie} r_e}$ ⇒ $C_{ie} = \frac{1}{2\pi \times 250 \times 250 \times 10^6}$

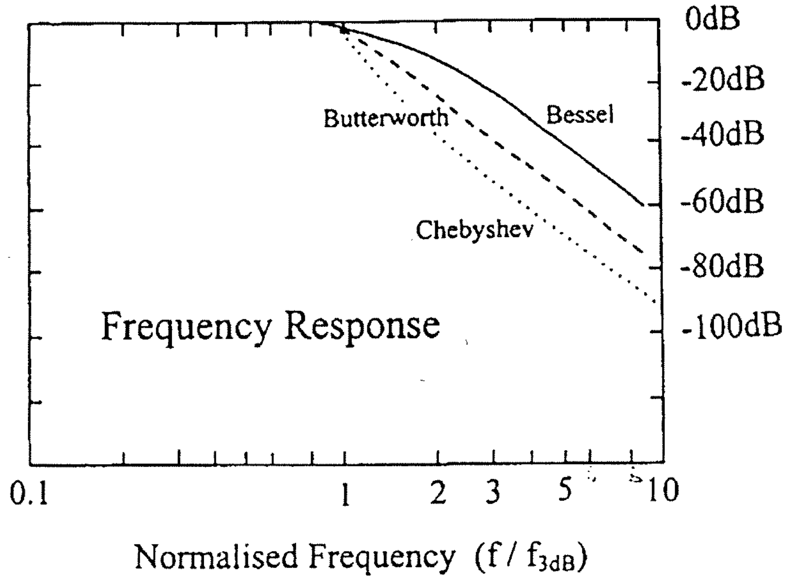
$= 2.5pF$

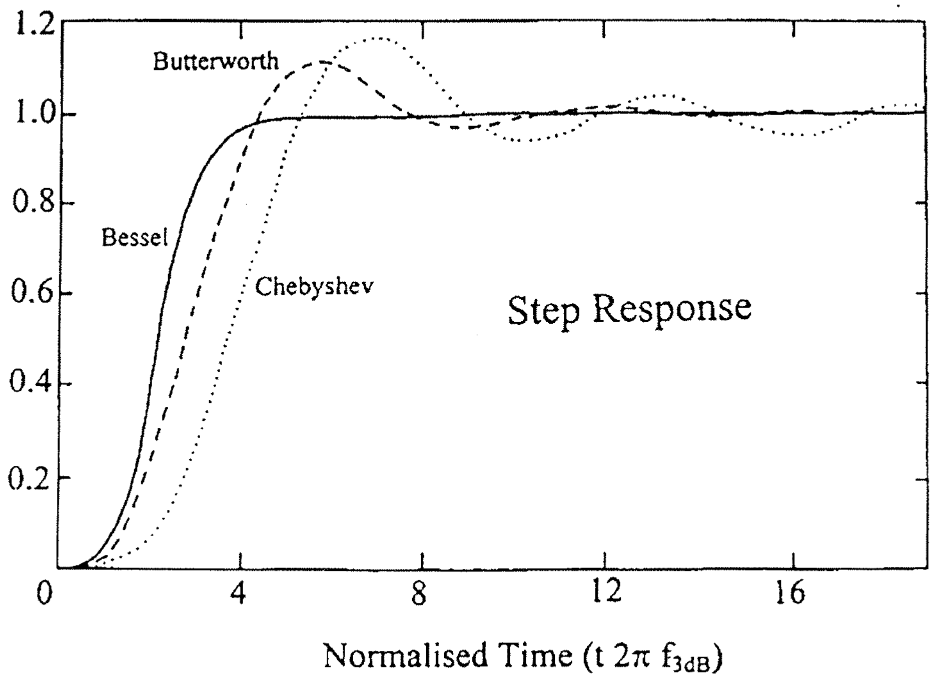


⇒ $\frac{1}{\sqrt{2}}$ point for i_b = $\frac{1}{2\pi C_{ie} h_{ie}}$

= 1.27 MHz ⇒ OK (just)

Q3 a)





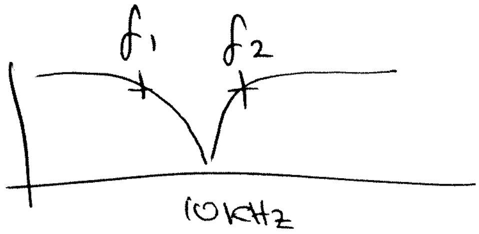
The optimum choice of filter for any particular application will depend on the relative importance of frequency and time performance. The **Bessel** filter is best for **passing transients undistorted,** the **Chebyshev** is best for **sharp frequency cut-off** and the **Butterworth** offers a fair compromise between the two as a **general all-rounder.**

Bessel filter is best when no overshoot is required such as a filter for a chart recorder pen

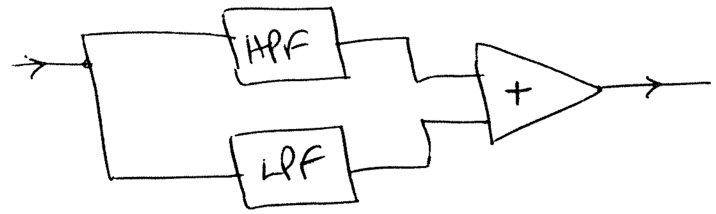
Chebyshev is best when sharp response is desired such as in an IF amp.

Butterworth is a good compromise between the two. ie audio filter in a CD player.

Q3 b) Band stop filter



Can be made with a LPF and HPF in parallel and adding outputs (assume phase shifts ok for this)



$\Rightarrow$ 5 op-amps $\therefore$ 2 op-amps per filter available $= 4$ poles $(n=4)$

Butterworth filter atten. $= \left[1 + \left(f/f_c\right)^{2n}\right]^{-1/2}$

$$\Rightarrow \quad \frac{1}{100} = \left[1+\left(f/f_c\right)^{2\times 4}\right]^{-1/2}$$

$$\Rightarrow \quad \frac{1}{10^4} = \frac{1}{\underbrace{\cancel{1}}_{\text{neglect.}} + \left(f/f_c\right)^8} \quad \Rightarrow \quad \frac{1}{10} = \left(f/f_c\right)^{-2}$$

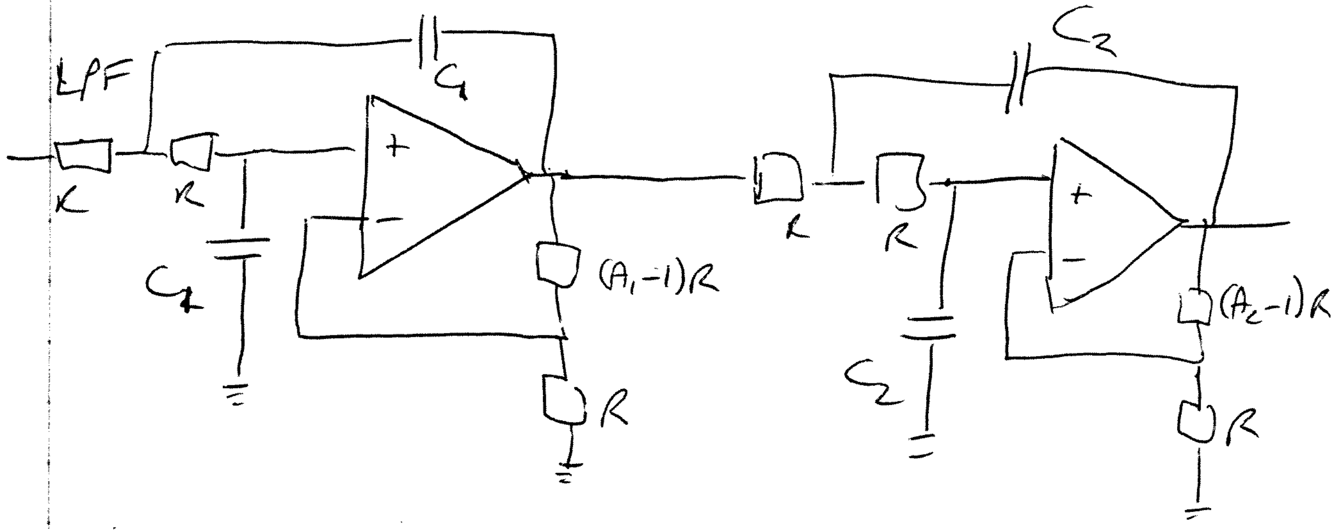
$$\therefore \quad \left(f/f_c\right)^2 = 10 \qquad \therefore \quad f/f_c = \sqrt{10} = 3.16$$

$$\Rightarrow \text{LPF} \quad f_1 = \frac{10\text{K}}{3.16} = 3.16 \text{ kHz}$$

$$\text{HPF} \quad f_2 = 3.16 \times 10\text{K} = 31.6 \text{ kHz}$$

$\therefore$ band stop width $= 31.6 - 3.16 = \underline{28.4 \text{ kHz}}$

⇒ VCVS 4pole (2 op-amps)



Choose $R = 10k$

$\Rightarrow \quad A_1 = 1.586 \qquad (A_1 - 1)R = 5.86\,k\Omega \quad (5.6\,k\Omega)$

$f_n = 1 \Rightarrow f_1 = \dfrac{1}{2\pi R C_1 \times 1} = 5.5\,kHz$

$\Rightarrow C_1 = 2.9\,nF \quad (2.7\,nF)$

$A_2 = 2.235 \qquad (A_2 - 1)R = 12.35\,k\Omega \quad (12\,k\Omega)$

$f_n = 1$ for all n $\Rightarrow C_2 = C_1 = 2.9\,nF$

For HPF swap around R + C's in cct above and use $1/f_n = 1$

$\Rightarrow \quad A_1 = 1.586 \qquad (A_1 - 1)R = 5.86\,k\Omega \quad (5.6\,k\Omega)$

$1/f_n = 1 \qquad f_2 = \dfrac{1}{2\pi R C_3} = 18\,kHz$

$\Rightarrow C_3 = 0.88\,nF \quad (820\,pF)$

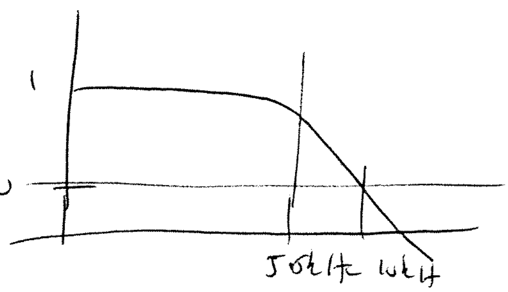
$(A_2 - 1)R = 12.35k\Omega$ $(12k\Omega)$

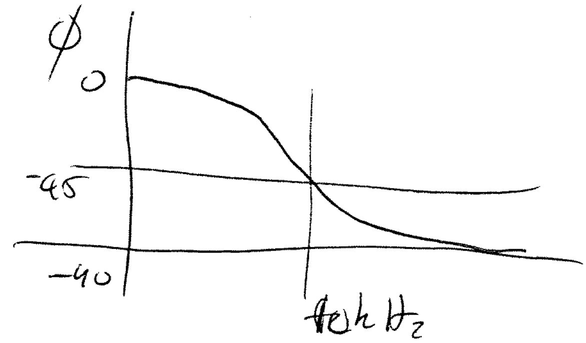
$C_4 = C_3 = 820pF$

Final Op/amp can be used to add together the two filter to give the final band-stop.

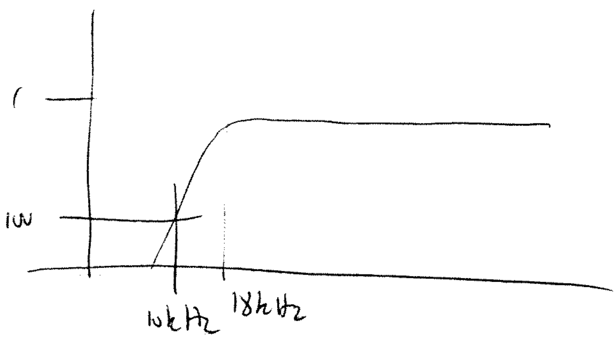
c) Design so that at 10kHz, the attenuation will be 100.

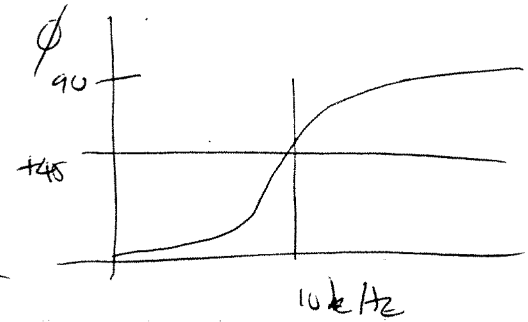
LPF



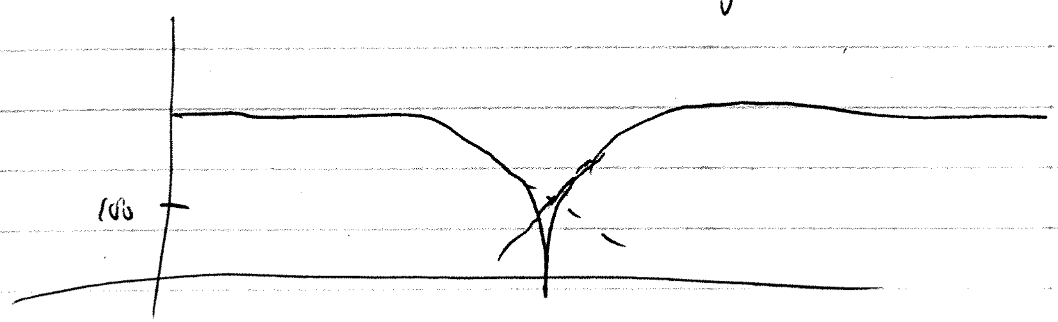


HPF

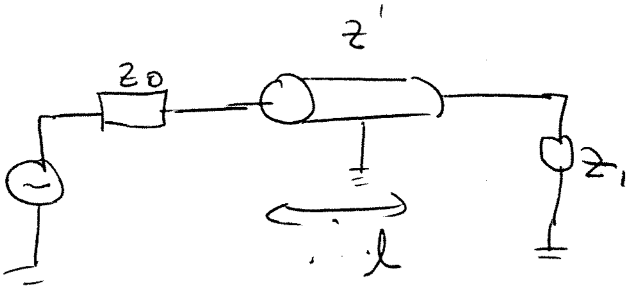




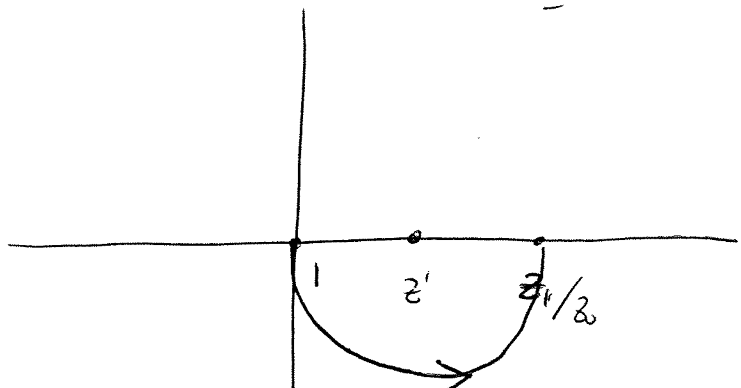
LPF & HPF are out of phase at $f_c = 10kHz$. Hence there will be strong deconstructive interference at the f_c frequency when the two filter are added together. This gives a much stronger attenuation than the we designed for at 10kHz.



Q4 a) Quarter wave match



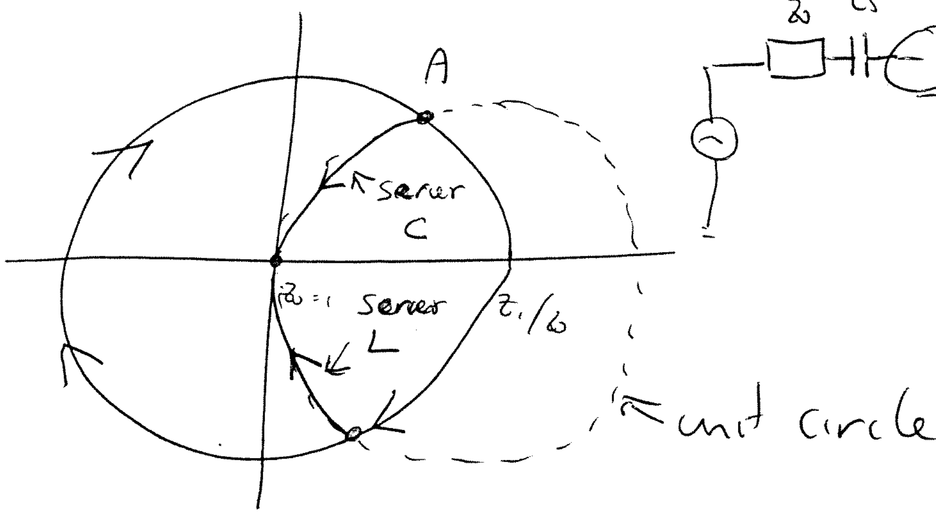
$z_1 > z_0$

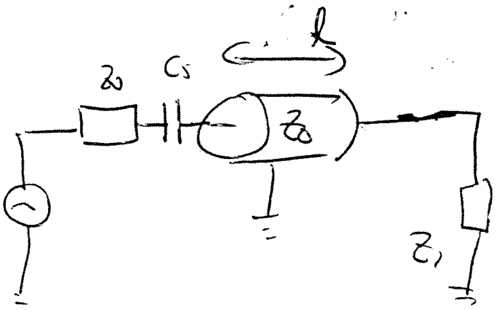


$z' = \sqrt{z_0 z_1}$

$l = \lambda/4$ length towards load

ii) Series Inductor/capacitor





Point A gives the intersection of the transmission line path with the unit circle. This defines the series capacitance needed to match the line

Point B gives the intersection of which defines the series inductance

b) Capacitance $C = \frac{A\epsilon}{d} = \frac{(w+2d)\epsilon_0 \epsilon_r}{d}$ per m

The 2d component allows for fringing field effects

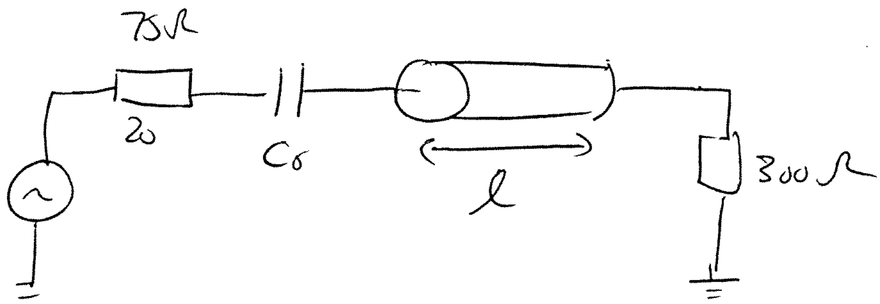
$$Z_0 = \sqrt{\frac{L}{C}} \qquad v = \frac{1}{\sqrt{LC}} \qquad c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Speed $v = \frac{1}{\sqrt{\epsilon_0 \epsilon_r \mu_0}} = \frac{c_0}{\sqrt{\epsilon_r}}$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{vC} = \frac{d}{\epsilon_0 \epsilon_r (w+2d)} \times \frac{\sqrt{\epsilon_r}}{c_0}$$

$$= \frac{d}{(w+2d)\, c_0 \epsilon_0 \sqrt{\epsilon_r}}$$

c)



choose $Z_0 = 75\Omega$

$$75 = \frac{0.5 \times 10^{-3}}{(w + 1\times10^{-3})\, 3\times10^{8} \times 8.85\times10^{-12} \times \sqrt{3.3}}$$

$\Rightarrow$ $w = 0.38$ mm (nb/ 300Ω cannot be made in this system)

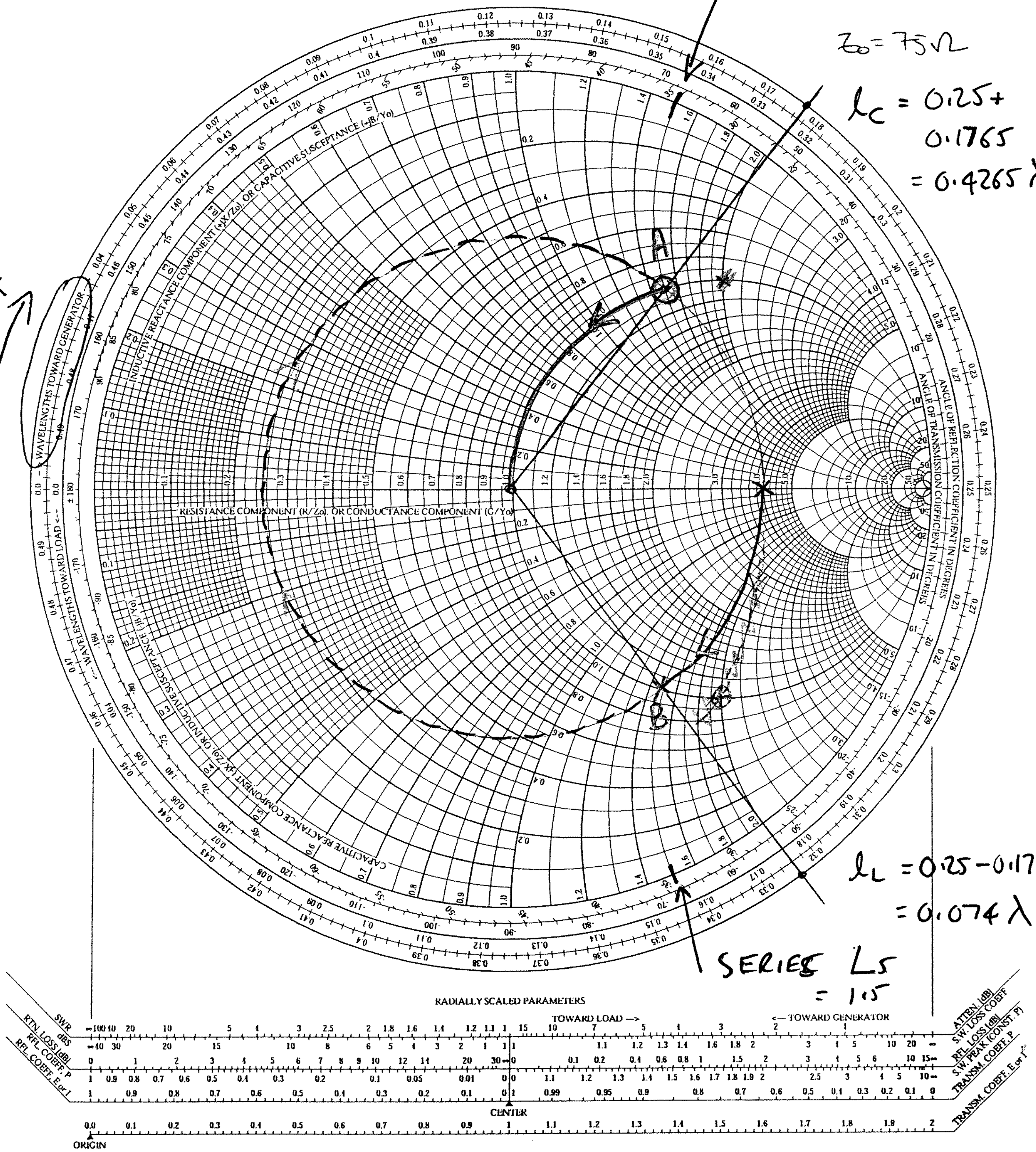
$Z_0 = 75\Omega$ and $z_L = \frac{300}{75} = 4$

* See Smith Chart

The Complete Smith Chart

Black Magic Design

SERIES C_S = 1.5.



$Z_0 = 75\Omega$

$l_C = 0.25 + 0.1765 = 0.4265\lambda$

$l_L = 0.25 - 0.176 = 0.074\lambda$

SERIES L_S = 1.5

c) cont For Series Capacitor

$$X_s = 1.5 \times 75 = 112.5\,\Omega = \frac{1}{\omega C_s}$$

$$\Rightarrow C_s = \frac{1}{\omega X_s} = \frac{1}{2\pi \times 1 \times 10^9 \times 112.5} = 1.4\,pF$$

length $l_c = 0.4265\lambda$ $\quad \lambda = \frac{v}{f} = \frac{c_0}{f\sqrt{\varepsilon_r}} = \frac{3\times\cancel{10^8}}{1\times\cancel{10^9}\sqrt{3.3}}$

$$= 0.165\,m$$

$$\Rightarrow l_c = 0.4265 \times 0.165 = 0.07\,m \quad (70\,mm)$$

For dc path you must have a series inductor.

$$\Rightarrow X_L = 1.5 \times 75 = 112.5\,\Omega = \omega L_s$$

$$\Rightarrow L_s = \frac{112.5}{2\pi \times 10^9} = 18\,nH$$

$$l_c = 0.074\lambda = 0.074 \times 0.165 = 0.012\,m$$

$(12\,mm)$

If $\lambda/4$ matching were to be used, then

$$Z_0 = \sqrt{75 \times 300} = 150\,\Omega$$

$\Rightarrow w = -0.3\,mm$ which means that this characteristic impedance is not possible

$\Rightarrow \lambda/4$ matching will not work.