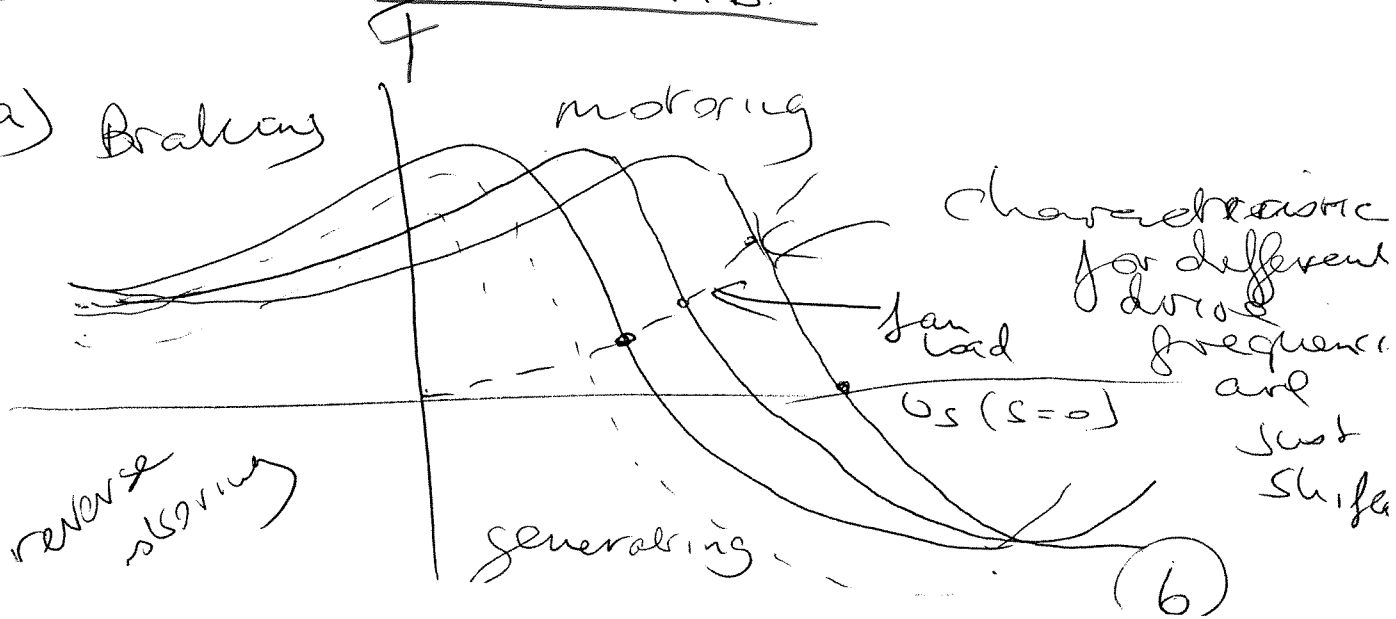
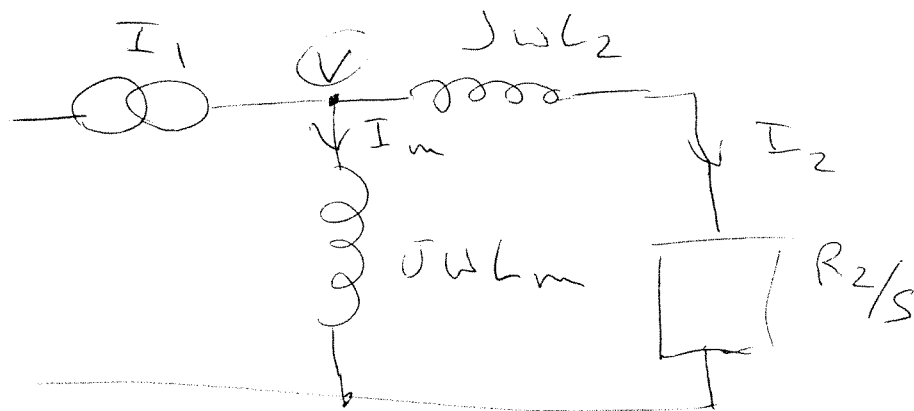


3B4 2003 exam crib.

1 a) Braking



b) ~~4~~



(3)

$$V = j\omega L_m I_m$$

$$I_2 = \frac{V}{(j\omega L_2 + R_2/s)} = \frac{j\omega L_m I_m}{(j\omega L_2 + R_2/s)}$$

$$I_1 = I_m + I_2 = I_m + \frac{j\omega L_m I_m}{(j\omega L_2 + R_2/s)}$$

(4)

$$I_1 = I_m \frac{j\omega L_2 + R_2/s + j\omega L_m}{j\omega L_2 + R_2/s} \Rightarrow I_1 = I_m \frac{\sqrt{(R_2/s)^2 + \omega^2 L_2^2 + L_m}}{\sqrt{(R_2/s)^2 + (\omega L_2)^2}}$$

$$c) \quad I_{\text{mag}} = \frac{6.6 \times 10^3}{\sqrt{3} \cdot 60} = 63.5 \text{ A.}$$

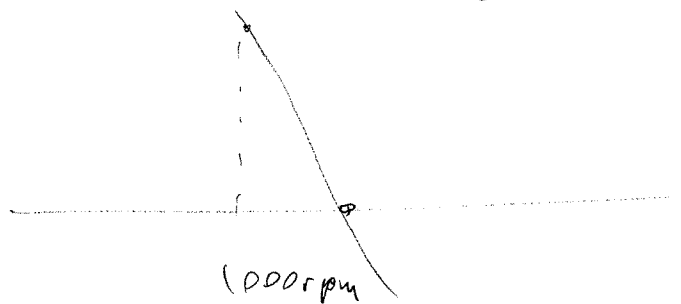
$$400 = I_{\text{mag}} \sqrt{\frac{0.1^2 + \omega_2^2 \left(\frac{62.4}{1000}\right)^2}{0.1^2 + \omega_2^2 \left(\frac{2.4}{1000}\right)^2}}$$

$$400 = 63.5 \sqrt{\frac{0.1^2 + \omega_2^2 \left(\frac{62.4}{1000}\right)^2}{0.1^2 + \omega_2^2 \left(\frac{2.4}{1000}\right)^2}}$$

$$\Rightarrow \omega_2 = 3.227$$

Assume a straight line characteristic

Solve for s
when $\omega_2 = s\omega$



7

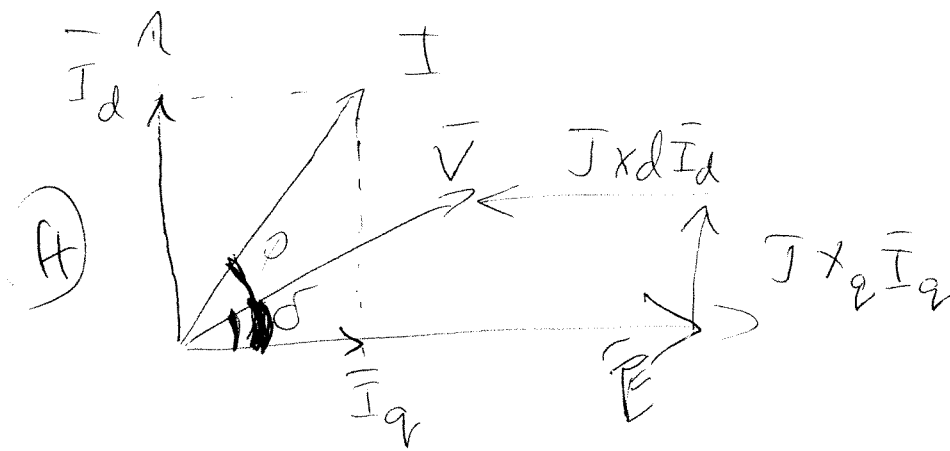
$$(1-s)\omega = \frac{1000 \times 2\pi}{60}$$

$$s\omega = 3.23$$

$$\Rightarrow \omega = 107.9$$

$$f = 17.2 \text{ Hz.}$$

2a)



$$I_q = I \cos(\phi + \delta)$$

$$I_d = I \sin(\phi + \delta)$$

$$\tan \delta = \frac{x_q I_q}{V \cos \delta} = \frac{x_q I \cos(\phi + \delta)}{V \cos \delta}$$

$$\tan \delta = \frac{x_q I (\cos \phi - \frac{\sin \phi \sin \delta}{\cos \delta})}{V}$$

$$\Rightarrow \tan \delta \left(\frac{V}{x_q I} \right) + \frac{1}{\cos \delta} (\sin \phi) = \frac{x_q I \cos \phi}{V}$$

$$\Rightarrow \tan \delta = \left(\frac{I x_q \cos \phi}{V + I x_q \sin \phi} \right)$$

$$b) P = 3 V I \cos \phi$$

$$= \frac{3 \times 6.6 \times 10^3}{\sqrt{3}} I \times 1 = 11.7 \times 10^6$$

$$\Rightarrow I = \frac{11.7 \times 10^6}{\sqrt{3}} = 875 \text{ A}$$

$$\delta = \tan^{-1} \left(\frac{x_q \times I}{V} \right) = 16.6^\circ$$

from a)

$$E = V \cos \delta + x_d I_d = 3651 + 1.8 \times 875 \times \sin 16.6$$

$$= 4101 \text{ V}$$

2 dots.

$$b) \quad P = 3(V I_q \cos \delta + V I_d \sin \delta)$$

$$\text{But: } X_q I_q = V \sin \delta$$

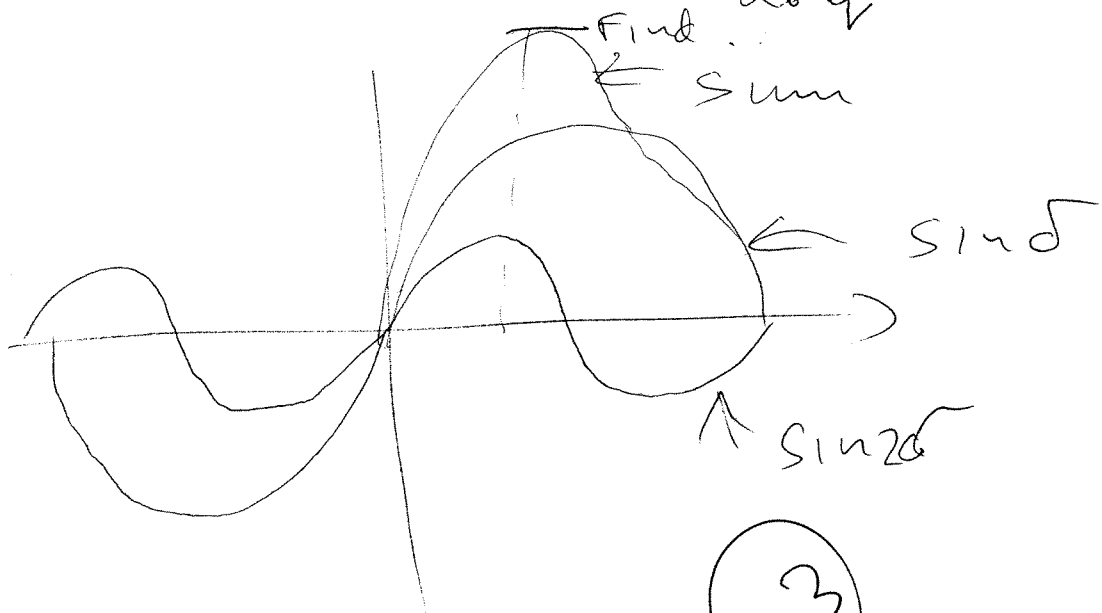
$$\text{and } X_d I_d = E - V \cos \delta$$

$$\Rightarrow P = 3V \left(\frac{V \sin \delta \cos \delta}{X_q} + \frac{(E - V \cos \delta) \sin \delta}{X_d} \right)$$

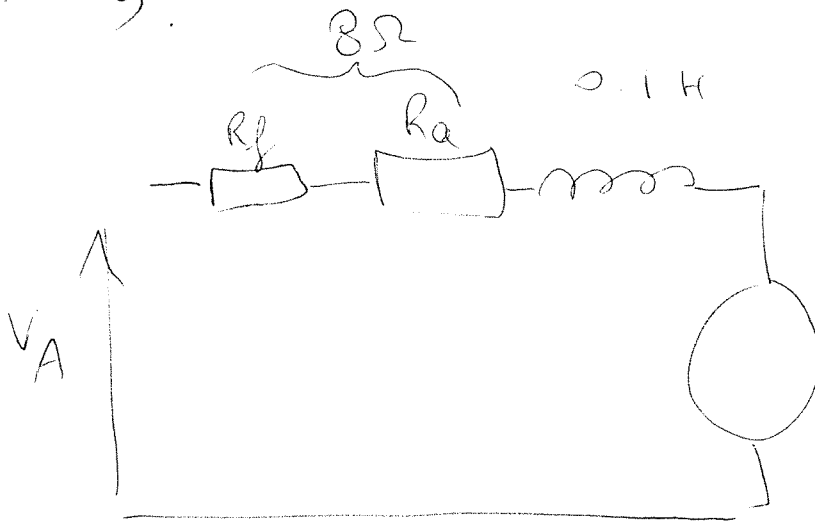
$$\textcircled{4} = \frac{3VE \sin \delta}{X_d} + 3V^2 \left(\frac{\sin \delta \cos \delta}{X_q} - \frac{\sin \delta \cos \delta}{X_d} \right)$$

$$T W_s = \frac{3VE \sin \delta}{X_d} + \frac{3V^2}{2X_d X_q} (X_d - X_q) \sin 2\delta$$

(11)



3. a)



$$e_a = k' I_f \omega$$

$$= k' \times 2.8 \times \frac{4500 \times 55 \times 2}{60}$$

$$e_a = 240 - 2.8 \times 8 = 217.6$$

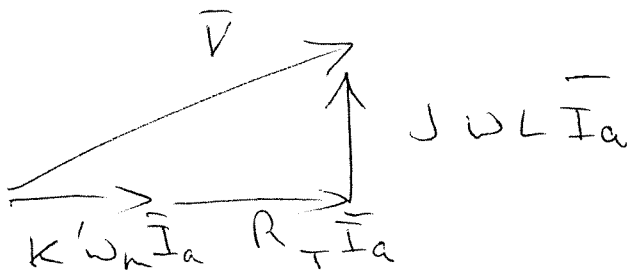
$$\Rightarrow k' = 0.165 \text{ Nm/A}^2$$

$$P_{out} = e_a i_a = 609 \text{ W} = T \omega$$

$$\Rightarrow T = 1.3 \text{ Nm}$$

$$b) \bar{V}_a = \bar{e}_a + j\omega L \bar{I}_a + (R_f + R_a) \bar{I}_a$$

$$= k' \bar{I}_a \omega_{mech} + j\omega L \bar{I}_a + (R_f + R_a) \bar{I}_a$$



from the diagram.

$$240^2 = ((10055 \times 0.1) \times 2.8)^2 + (0.165 \omega \times 2.8 + 8 \times 2.8)^2$$

$$i) \Rightarrow \omega_{mech} = 435 \text{ rad s}^{-1} \approx 4151 \text{ rpm}$$

$$ii) \Rightarrow \text{Torque is unchanged i.e. } 1.3 \text{ Nm}$$

$$\begin{aligned}
 \text{iii) } \cos \phi &= \frac{(R_f + R_a) I_a + k' \omega_m I_a}{V} \\
 &= \frac{2 \times 2.8 \times 0.165 \times 435 + 2.8}{240} \\
 &= 0.931 \text{ lagging} \quad (2)
 \end{aligned}$$

$$\text{iv) Power}_m = \omega T = 566$$

$$\begin{aligned}
 \text{Power efficiency} &= \frac{566}{566 + 2.8^2 \times 8} \times 100 \\
 &= 90\% \quad (1)
 \end{aligned}$$

DC motor gives greater Torque because it has a lower series impedance. Hence current is greater for the same speed (ie the same back emf).

$$\begin{aligned}
 \text{c) Power dissipated as heat} &= 2.8^2 \times R \\
 P_m &= 63 \text{ W.}
 \end{aligned}$$

$$\begin{aligned}
 C \int_{40}^{70} \frac{d\theta}{-P_m + k(\theta - 18)} &= 90 \\
 -\frac{C}{k} \left(\ln \left(-\frac{63}{k} + \theta - 18 \right) \right) \Big|_{40}^{70} &= 90
 \end{aligned}$$

$$-\frac{C}{k} \ln \left(\frac{70 - \frac{63}{k} - 18}{40 - \frac{63}{k} - 18} \right) = 90$$

Also (when switched off)

$$-\frac{C}{k} \left[\ln(\theta - 18) \right]_{70}^{40} = 45$$

$$-\frac{C}{k} \left(\ln \frac{22}{52} \right) = 45$$

$$\Rightarrow C/k = 52.3$$

$$52 - \frac{63}{k} = \left(22 - \frac{63}{k} \right) e^{-(90/52.3)}$$

$$= \left(22 - \frac{63}{k} \right) \times 0.179$$

$$\Rightarrow k = 1.075$$

$$C = 55.9$$

$$\theta = 22.2$$

(6)

$$e^{-(90/52.3)} + \frac{63}{k} \left(1 - e^{-(90/52.3)} \right)$$

(4)

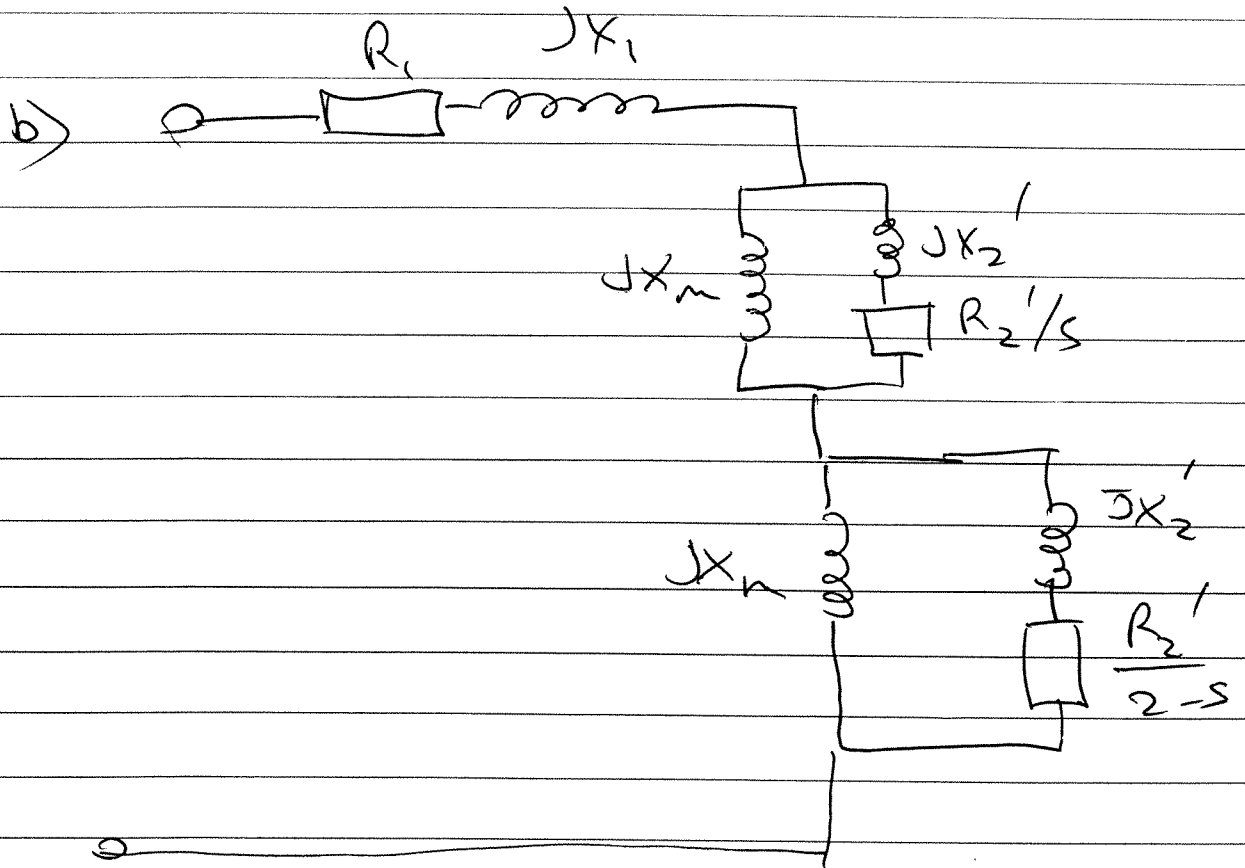
$$a) F(\theta, I) = \frac{2NI}{s} \cos(\rho\theta)$$

$$I = \hat{I} \cos \omega t$$

$$F = \frac{2NI\hat{I}}{s} \cos(\omega t) \cos(\rho\theta)$$

(4)

$$\Rightarrow F = \frac{NI\hat{I}}{s} (\underbrace{\cos(\omega t - \rho\theta)}_{\text{forward}} + \underbrace{\cos(\omega t + \rho\theta)}_{\text{backward}})$$



(7)

$$\text{Forward} = R_2' / s$$

$$\text{Backward} = R_2' / (2-s)$$

ie for small s
forward impedance
is large backward
is small

When the motor is stationary the two are equal.

$$\begin{aligned}
 c) \quad \bar{Z}_f &= \frac{jX_m (jX_2' + R_2/s)}{jX_m \cancel{1} + jX_2' + R_2/s} \\
 &= \frac{\cancel{jX_m (jX_2' + R_2/s)}}{(j \frac{X_m R_2}{s} - X_m X_2') \times (j(X_m + X_2') - R_2/s)} \\
 &\quad - \left((X_m + X_2')^2 + \left(\frac{R_2}{s}\right)^2 \right)
 \end{aligned}$$

$$\text{Re}(\bar{Z}_f) = \frac{s^2 X_m^2 R_2'}{s^2 (X_m + X_2')^2 + \left(\frac{R_2}{s}\right)^2} \quad (2)$$

$$\text{Re}(\bar{Z}_b) = \frac{(2-s) X_m^2 R_2'}{(2-s)^2 (X_m + X_2')^2 + (R_2')^2}$$

$$P = 3I^2 (\text{Re}(\bar{Z}_f) - \text{Re}(\bar{Z}_b)) \quad (1)$$

When $P = 0$; Torque = 0.

$$\Rightarrow 0 = \frac{s}{s^2 (X_m + X_2')^2 + R_2'^2} - \frac{(2-s)}{(2-s)^2 (X_m + X_2')^2 + R_2'^2}$$

Use $A = (R_2')^2$ $B = (X_m + X_2')^2$

$$\Rightarrow 0 = \frac{s}{s^2 B + A} - \frac{(2-s)}{(2-s)^2 B + A} \quad (9)$$

$$\Rightarrow 0 = 4sB - 4Bs^2 + s^3B + As$$

$$- 2Bs^2 + Bs^3 - 2A + sA$$

collecting terms we get.

$$0 = -2A + s(A+4B) - ~~s^2(4B)~~ 6Bs^2 + 2Bs^3$$

we know that $(s-1) = 0$ is one of the solutions.

$$\Rightarrow 0 = (s-1)(s^2 + 2s + A/B) = 0$$

$$\Rightarrow s = \pm \sqrt{\frac{4 - 4A/B}{4}}$$

$$\Rightarrow s = 1 \pm \sqrt{1 - \left(\frac{R_2'}{x_m + x_2'}\right)^2} \quad (3)$$

Due to the backward wave
T is actually negative at synchronous
Speed

