J. A. WILLIAMS

Engineering Tripos Part IIA - 2003 - Module 3C3

1. (a) Bookwork - but entraining velocity & viscosity required to draw fluid into conjunction. Convergent geometry & continuity generate positive pressures.

(b) When $K = 0.5$, surfaces parallel so that

$$W^* \Rightarrow 0 \; ; \quad Q^* \Rightarrow 0.5 \text{ and } F^* = 1.0$$

(c) (i) $W^* = \dfrac{\bar{p} h_o^2}{\eta U L}$ so if $\bar{p}, \eta, U$ and L fixed

$$h_o \propto \sqrt{W^*}$$

and h_o will have max when W^* has max.

Tabulated values suggest this when $K = 0.6$.

$$\mu = \frac{F}{W} = \frac{f L^2}{\bar{p} L^2} = \frac{f}{\bar{p}} \qquad \text{But } f = \frac{\eta U F^*}{h_o} \text{ \& } h_o^2 = \frac{\eta U L W^*}{\bar{p}}$$

$$\therefore \mu = \frac{\eta U F^*}{\bar{p}} \times \sqrt{\frac{\bar{p}}{\eta U L W^*}} \Rightarrow \frac{F^*}{\sqrt{W^*}} \sqrt{\frac{\eta U}{\bar{p} L}}$$

when $K = 0.60$ $\quad \dfrac{F^*}{\sqrt{W^*}} = \dfrac{0.700}{\sqrt{.0703}} = 2.64$

$$\therefore \mu = 2.64 \sqrt{\frac{\eta U}{\bar{p} L}}$$

(ii) Energy balance

$$F . U = \Phi . c_p . \Delta T$$

$$\text{ie. } \frac{\eta U F^* L^2}{h_o} . U = U h_o L \, Q^* \, c_p \, \Delta T$$

$$\text{ie. } c_p \Delta T = \frac{F^*}{Q^*} \frac{\eta U L}{h_o^2} \qquad \text{but } h_o^2 = \frac{\eta U L W^*}{\bar{p}}$$

So that $\Delta T = \frac{\bar{p}}{c\rho}\left(\frac{F^*}{\phi^* W^*}\right)$

Thus in moving from $\chi = 0.55$ to 0.65

ΔT is changed by factor $\frac{.565}{.0585 \times 1.37} \times \frac{.0585 \times .650}{.870}$

i.e. $\times\ 0.308$

(iii) If pad surface remains planar central pivot ($\chi = 0.5$) gives no load support. Crowning pads, however, will enable positive hydrodynamic pressures to be generated for motion in either direction.

Question 1: Hydrodynamic pad bearing
Attempts: 16, average mark 12.0, maximum 19, minimum 6
Generally well done. Several scripts in part (c) (iii) showed evidence of reading beyond the lecture notes.

---

2 (a) Bookwork - 'Full Sommerfeld' solution suggests existence of both positive hydrodynamic pressures in region $0<\theta<\pi$ and equal and opposite negative pressures from $\pi<\theta<2\pi$. Solution is of the form $p = f(\theta)\sin\theta$. Clearly these negative pressures are unrealistic in presence of cavitation. Simply neglecting negative pressures, the 'half Sommerfeld' solution provides reasonable numerical estimates but at cost of infringing continuity of mass or volume flow at $\theta = \pi$. The 'Reynolds' soln' overcomes this.

(b) In an air bearing operating in ambient atmospheric pressure p_a, hydrodynamic or aerodynamic pressures have to be superposed on p_a. If $p < p_a$ then in the divergent region absolute pressures are still positive, so Full Sommerfeld conditions more appropriate.

(c) This soln from one candidate cannot be bettered:

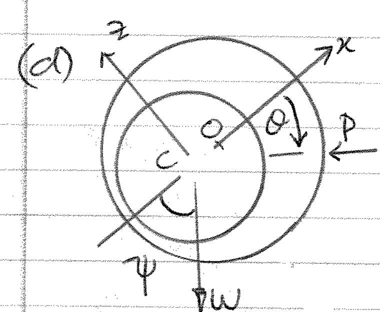
$$h_{min} = c - e = c(1-\varepsilon)$$

$$\frac{1}{S} = \frac{6\pi\varepsilon}{(1-\varepsilon^2)^{1/2}(2+\varepsilon^2)} \quad \therefore \quad \frac{1}{S^2} = \frac{36\pi^2\varepsilon^2}{(1-\varepsilon)(1+\varepsilon)(2+\varepsilon^2)^2}$$

$$\therefore \; c(1-\varepsilon) = \frac{36\pi^2\varepsilon^2 \cdot S^2 c}{(1+\varepsilon)(2+\varepsilon^2)^2}$$

But if $\varepsilon \Rightarrow 1$ $\quad h_{min} \simeq \dfrac{36\pi^2 S^2 c}{(1+1)\,3^2} = 2\pi^2 S^2 c$

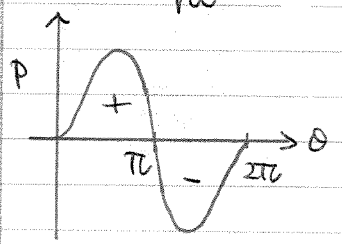
(d)



Full Sommerfeld solution is of form $p = f(\theta)\sin\theta$; $f(\theta)$ - even functio

By inspection $W_x \Rightarrow 0$

and $W_z > 0$



hence attitude angle $\psi = \pi/2$

i.e. OC is $\perp$r to load vector

This could be seen by considering the integrals

$$\frac{W_x}{L} = R\int_0^{2\pi} f(\theta)\sin\theta\cos\theta \cdot d\theta$$

$$\Rightarrow \frac{R}{2}\int f(\theta)\sin 2\theta \, d\theta$$

$$\Rightarrow -\frac{R}{2}\underbrace{\left[f(\theta)\cos 2\theta\right]_0^{2\pi}}_{0} + \underbrace{\int_0^{2\pi}\cos 2\theta \, f'(\theta)\, d\theta}_{0}$$

$$\frac{W_z}{L} = R\int_0^{2\pi} f(\theta)\sin\theta \cdot \sin\theta \, d\theta$$

$$\Rightarrow R\int_0^{2\pi} f(\theta)\sin^2\theta \, d\theta$$

+ ve

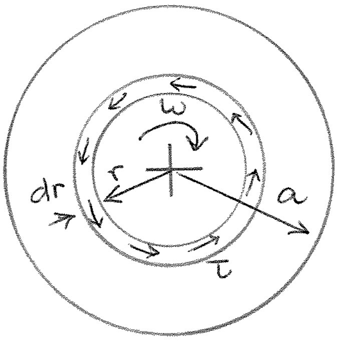
Question 2: Journal bearing

Attempts: 23, average mark 12.0, maximum 18, minimum 6

The most popular question. Some candidates got lost in the algebra of part (c) but there were several solutions more elegant than the examiner's.

3(a) Bookwork – smooth, friction free surfaces, no discontinuities in slope, no close conformity, linear elasticity.

(b) Also Bookwork
$$p = p_0\left\{1 - \frac{r^2}{a^2}\right\}^{1/2}$$ Hertz point contact



$$\therefore \quad \tau = \mu p = \mu p_0\left\{1 - \frac{r^2}{a^2}\right\}^{1/2}$$

But $dM = 2\pi r\,\tau . r\,dr = 2\pi r^2 \mu p\,dr$

$$\therefore \quad M = 2\pi \mu p_0 \int_0^a r^2\left(1 - r^2/a^2\right)^{1/2} dr$$

But $r = a\sin\theta$, $dr = a\cos\theta\,d\theta$

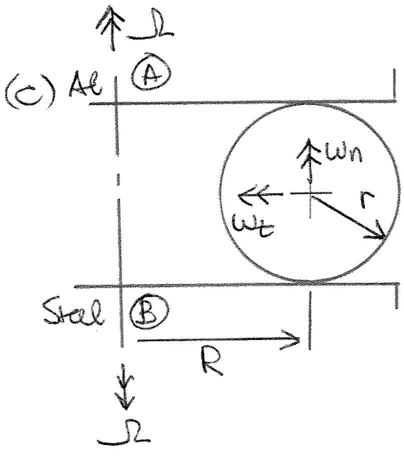
$$M = 2\pi\mu p_0 a^3 \int_0^{\pi/2} \sin^2\theta . \cos^2\theta\,d\theta$$

ie. $M = 2\pi\mu p_0 a^3 . \frac{\pi}{16} = \underline{\frac{\pi^2}{8}\mu p_0 a^3}$

But from data sheet $p_0 = \frac{3W}{2\pi a^2}$

thus $M = \frac{\pi^2}{8} . \mu \frac{3W}{2\pi a^2} . a^3 = \underline{\frac{3\pi}{16}\mu P a}$

(c)



(i) For no slip $r\omega_t = R\Omega$

Spin at contact with plate (A)

$$\omega_{spin} = \omega_n - \Omega$$

Spin at (B) $= \omega_n - (-\Omega)$

cannot both be zero.

(ii) At Hertz point contact $a = \left(\frac{3Pr}{4E^*}\right)^{1/3}$ <u>Data sheet</u>

$$\therefore \quad \frac{M_A}{M_B} = \frac{\mu_A}{\mu_B}\left(\frac{E_B^*}{E_A^*}\right)^{1/3}$$ since P same at both.

$$E_A^* = \left(\frac{1-.3^2}{210} + \frac{1-.33}{70}\right)^{-1} = \underline{58.6\,GPa}$$

$$E_B^* = \left(\frac{1-.3^2}{210} + \frac{1-.3^2}{210}\right)^{-1} = \underline{115\,GPa}$$

$$\therefore \quad \frac{M_A}{M_B} = \frac{0.18}{0.2}\left(\frac{115}{58.6}\right)^{1/3} = 1.13$$

Thus spin most likely at steel/steel contact B.

(iii) If there is no spin at A then $\omega_n = \Omega$

$\therefore$ spin angular vely at B is 2Ω.

So if reqd. torque is T, $2T.\Omega = 3M_B.2\Omega$

i.e. $T = 3M_B$ and $P = 5/3\,N$

$$= 3 \times \frac{3\pi}{16} \times 0.2 \times \frac{5}{3} \times \left(\frac{3 \times 5/3 \times .005}{4 \times 115 \times 10^9}\right)^{1/3}$$

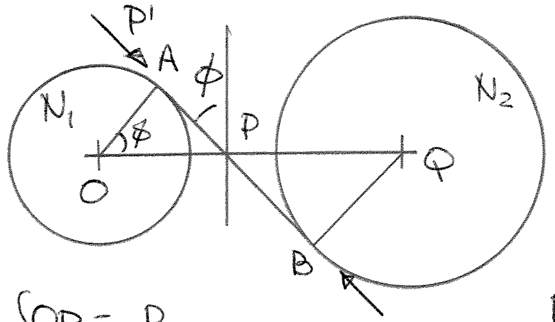
$$= \underline{2.23 \times 10^{-5}\,Nm}$$

Question 3: Hertzian contact

Attempts: 21, average mark 12.8, maximum 19, minimum 3

The 'book-work' of parts (a), (b) and (c)(i) was well done. Several candidates assumed that slip will occur at the points of contact with the lower values of μ rather than thinking about the spin torque – which was rather the point of the question.

4(a)



$$\begin{cases} OP = R_1 \\ QP = R_2 \end{cases}$$

When contact a pitch point

$$AP = R_1 \sin\phi \; ; \; BP = R_2 \sin\phi$$

$$\therefore \frac{1}{R} = \frac{1}{AP} + \frac{1}{BP} = \frac{1}{\sin\phi}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

But $m = \dfrac{2R_1}{N_1} = \dfrac{2R_2}{N_2}$

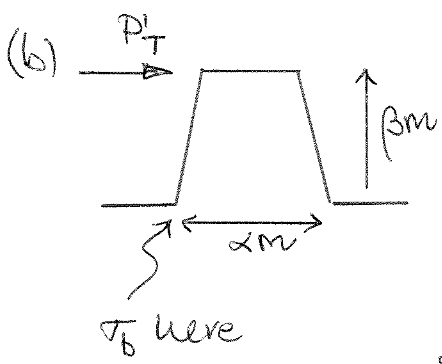
$$\therefore \frac{1}{R} = \frac{2}{m\sin\phi}\left(\frac{1}{N_1} + \frac{1}{N_2}\right)$$

Hertz line contact

$$p_0 = \sqrt{\frac{P'E^*}{\pi R}}$$

$$\therefore p_0 = \sqrt{\frac{P'E^*}{\pi} \cdot \frac{2}{m\sin\phi} \cdot \frac{N_1 + N_2}{N_1 N_2}}$$

(b)



By dimensional analysis

$$\sigma_b = f(\alpha, \beta, m, P'_T)$$

i.e. $\dfrac{\sigma_b m}{P'_T} = f(\alpha, \beta)$ const. $= \dfrac{1}{J}$

Simple beam calculation must give same result. A more detailed calculation is required to evaluate stress concentration factor.

In practice J is a function not just of tooth shape i.e. α and β but also of where the peak load acts. The worst case assumes no load sharing between tooth pairs and gives lower curve (smaller values of J). If there is load sharing then the maximum load no longer acts at the tip of the tooth and J depends on the ratio of tooth numbers.

(c) Power $= 20\,kW$ i.e. $P' \times OA \times \omega_1 = 20\times10^3$

$$\therefore P' = \frac{20\times10^3\times2}{15m\cos\phi\, b} \times \frac{1}{40\pi}$$

$\omega_1 = 1200$ rpm $= 40\pi\, s^{-1}$

$$\therefore P' = \frac{22.58}{mb}$$

b = tooth width
m = tooth module

Choose "square" pinion so $b = Nm = 15m$

$$\therefore P' = \frac{22.58}{15m^2} = \frac{1.51}{m^2}$$

Suppose design limited by contact stress, then check bending stress:

$$p_0 = \sqrt{\frac{1.51}{m^2}\,\frac{E^*}{\pi}\cdot\frac{2}{m\sin20}\cdot\frac{15+45}{15\cdot45}}$$

and E^* for steel on steel $= 115\times10^9$ Pa

hence $p_0 = 170\times10^3\, m^{-3/2}$

If $p_0 = 590$ MPa Data sheet

then $m = \left(\dfrac{170\times10^3}{590\times10^6}\right)^{2/3} \Rightarrow$ 4.35 mm

Best to choose next standard size say 4.5 mm

$$\sigma_b = \frac{P_T'}{Jm} = \frac{P'\cos\phi}{Jm} = \frac{1.51\times\cos20}{J\,m^3} = \frac{1.51\times\cos20}{0.22\times0.0045^3}$$

Data sheet curve.

$\Rightarrow$ 71 MPa

This is well below suggested limit value of 170 MPa.

So Design summary module 4.5 mm; face width 67.5 mm
standard teeth $\phi = 20°$; $a = m$, ded. $= 1.25a$
Mat'l: through hardened & tempered steel.
pinion 15 teeth; wheel 45.

(d) Possibilities include: use of helical teeth, better quality material, increase in tooth numbers to give a more balanced design. Probably not cost-effective on comparatively low-tech application. But some care would be necessary in design of connection of gears to shaft, couplings etc.

Question 4: Gear design

Attempts: 21, average mark 13.7, maximum 18, minimum 4

Again the 'book-work' was done well and there were a number of sensible design solutions to part (c).

JAW
V/03